Supplementary Appendix

to

Local Mispricing and Microstructural Noise:

A Parametric Perspective

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1 Monte Carlo Evidence on Local Autocorrelation Tests

1.1 Monte Carlo Design

We explore whether the evidence of persistent autocorrelation regimes may be an artifact of MMN or price jumps in the high-frequency return series through simulations based on a standard one-factor stochastic volatility model used in prior studies, e.g., Huang and Tauchen (2005), Barndorff-Nielsen et al. (2008), and Goncalves and Meddahi (2009).

Normalizing a day to $t \in [0, 1]$, the efficient log-price, $y(t)$, evolves according to,

$$
\begin{aligned}
dy(t) &= \mu_y \, dt + \sigma_y(t) \, dW_y(t) + dJ(t) \, , \\
\sigma_y(t) &= \sigma_d(t) \, \sigma_s(t) \, , \\
\sigma_s(t) &= \exp\big(\beta_0 + \beta_1 \tau(t)\big) \, , \\
d\tau(t) &= \alpha \, \tau(t) dt + dW_\tau \, ,
\end{aligned}
$$

where $W_y(t)$ and $W_\tau(t)$ are correlated Brownian motions with corr$(dW_y, dW_\tau) = \rho$.

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We rely on the following parametrization: $\mu_y = 0.03$, $\beta_1 = 0.125$, $\alpha = -0.025$, and $\rho = -0.3$. Following Barndorff-Nielsen et al. (2008), we set $\beta_0 = \beta_1^2/(2\alpha)$ to normalize the integrated variance, ensuring that $\mathbb{E}[\sigma_s^2(t)] = 1$.

The efficient price volatility, $\sigma_y(t\,)$, has a two-component multiplicative structure. Beyond the stochastic component $\sigma_s(t)$, we introduce a deterministic component $\sigma_d(t)$ to capture the empirically documented diurnal J-shape volatility pattern. We model it as an exponential polynomial, along the lines of Hasbrouck (1999) and Andersen et al. (2012),

$$\sigma_d(t) \;=\; C \,+\, Ae^{-at} \,+\, Be^{-b(1-t)}\,,$$

with parameters set to $A = 0.75$, $B = 0.25$, $C = 0.88929198$, and $a = b = 10$. The specification ensures a sensible normalization of the diurnal effect, implying $\int_0^1 \sigma_d(t)dt = 1$.

Jumps are introduced through a sum of two compound Poisson processes, $J(t) = J_s(t) + J_\ell(t)$, independent of $W_y(t)$ and $W_\tau(t)$. J_s refers to small frequent jumps and J_ℓ to large jumps. If a small jump occurs at time t, its size depends on spot volatility, as it is drawn from, $N\big(0, 25\,\sigma^2(t)\Delta t\big)$ with $\Delta t = 1/23400$ or one second. The intensity of small jump arrivals is one per three minutes, while large jumps occur on average once a week, and each large jump contributes 25% of the expected daily integrated variance.

Letting the efficient log-price be contaminated by noise, $u(t)$, the observed mid-quote is,

$$p(t) \;=\; y(t) \,+\, u(t)\,.$$

The noise component is i.i.d. $N\big(0,\, 0.1\,\sigma^2(t)\Delta t\big)$ at time t, whenever $p(t)$ is observed. This standard specification generates an MA-structure for the observed returns, with a negative autocorrelation and a noise-to-signal ratio that varies with the sampling frequency.

Using an Euler discretization scheme, we simulate daily price series on a second-to-second basis, generating T=23,400 recorded mid-quotes. To align the setting with our empirical analysis, we simulate 61 trading days and break them into sequences of intraday intervals. We then compute the first-order autocorrelations within each local interval and repeat the statistical tests applied to the individual stock price series in Section 2 in the main manuscript. We repeat this procedure 1,000 times and report the percentage of local intervals for which the null hypotheses is rejected.

1.2 Simulation Results

Table 1 reports the same statistics as Table 1 in the main manuscript, but based on *simulated* data. In the scenario without jumps and microstructure noise (top panel), the tests using the average absolute or signed autocorrelations tend to over-reject the null hypothesis of zero autocorrelation. The size distortion is most prevalent for short local intervals and low sampling frequencies. Conversely, for longer intervals and higher sampling frequencies, the test size approaches its nominal value. This suggests that the size distortion arises from a finite-sample bias; the tests eventually behave as prescribed by the Kokoszka and Politis (2011) asymptotic approximation.

The second panel reports on a simulation scenario *with* jumps. The jumps do not induce serial dependence in the returns, but they can seriously affect the distribution of $\hat{\rho}_k$ and, consequently, the statistical properties of the test. In fact, we observe a slight increase in the rejection rates compared to the scenario with no jumps. Again, this size distortion tends to vanish when considering combinations of T and Δ implying a higher number of observations. Finally, we note that, in the absence of microstructure noise, the simulated price series generates an equal proportions of local intervals with positive and negative estimated return autocorrelations.

The two bottom panels provide results based on scenarios *with* MMN. The presence of (independent) noise induces negative return autocorrelation. Accordingly, for sampling frequencies of 2 seconds and 3 seconds, nearly all tests reject the null hypothesis across all simulation designs. When the test is applied to the subset of intervals with exclusively positive or exclusively negative estimated autocorrelations then, as expected, the (one-sided) tests correctly reject the null only for the negative correlation regimes (columns (I_-)).

Table 2 is the counterpart to Table 2 in the main manuscript, reporting results for Wald-Wolfowitz run tests on simulated data. Since none of the scenarios introduces any dependence structure in the return autocorrelations, we expect test size to be satisfactory. In fact – even in the presence of noise and jumps – the rejection rates are well aligned with their nominal values for all designs, interval lengths and observation frequencies. However, rejections from the left tail occur slightly more often than from the right, suggesting that the test statistic is mildly skewed in small samples.

Finally, Table 3 mimics Table 3 in the main manuscript, reporting on the persistence in the sign of the autocorrelation regimes. The noise has a small, yet notable, impact on the rejection patterns. We observe a moderate over-rejection in the transition rate from intervals with positive autocorrelation (left panel) and, conversely, an under-rejection from intervals with negative autocorrelations (right panel). Since the noise process induces *negative* autocorrelation, the simulations generate only a few instances of (estimated) positive serial correlation and, as a result, $\widehat{p}_{00}$ and $\widehat{p}_{11}$ are imprecisely estimated, rendering size distortions more common.[1] Nonetheless, rejections occur equally often in the left and right tail, so the test (correctly) does

[1]This conjecture is corroborated by the fact that size distortions grow more pronounced for higher frequencies and longer intraday intervals. In these settings, we obtain even more negative estimates for $\widehat{\rho}_k$, further reducing the number of transitions between different regimes.

	T = 10 min							T = 15 min							T = 20 min						
	(I)		(I_+)		(I_-)			(I)		(I_+)		(I_-)			(I)		(I_+)		(I_-)		
Δ	$\%_{5\%}$	$\%_{1\%}$	$\%_{5\%}$	$\%_{1\%}$	$\%_{5\%}$	$\%_{1\%}$	$\bar{\pi}_+$	$\%_{5\%}$	$\%_{1\%}$	$\%_{5\%}$	$\%_{1\%}$	$\%_{5\%}$	$\%_{1\%}$	$\bar{\pi}_+$	$\%_{5\%}$	$\%_{1\%}$	$\%_{5\%}$	$\%_{1\%}$	$\%_{5\%}$	$\%_{1\%}$	$\bar{\pi}_+$
Simulation design without jumps and noise																					
2 sec	7.2	1.9	7.1	2.1	5.5	1.6	0.50	5.5	1.5	5.1	0.6	5.4	1.2	0.50	4.7	0.7	4.7	1.0	4.5	0.9	0.50
3 sec	10.6	2.6	8.6	1.8	8.1	2.3	0.50	7.0	1.7	6.3	1.8	6.8	1.5	0.50	6.6	1.3	5.5	1.6	5.3	1.3	0.50
5 sec	15.8	4.7	11.8	3.3	10.7	2.9	0.50	9.7	2.2	8.5	1.9	8.2	1.8	0.50	7.2	1.4	6.4	1.4	7.0	1.0	0.50
Simulation design with jumps, but without noise																					
2 sec	12.9	3.3	10.0	2.1	11.0	3.1	0.50	10.2	2.3	7.7	2.0	10.5	2.3	0.50	7.8	1.2	6.2	1.2	7.2	1.9	0.50
3 sec	17.3	5.6	12.9	2.9	14.6	4.2	0.50	12.2	2.9	8.9	2.4	10.6	3.1	0.50	10.1	2.4	7.7	1.9	7.7	1.9	0.50
5 sec	23.6	7.8	16.2	4.3	15.9	3.8	0.50	12.6	3.9	10.3	3.0	9.5	2.4	0.50	9.8	2.4	7.3	1.6	9.3	2.4	0.50
Simulation design without jumps, but with i.i.d. noise																					
2 sec	100	100	0.0	0.0	100	100	0.22	100	100	0.0	0.0	100	100	0.17	100	100	0.0	0.0	100	100	0.13
3 sec	100	100	0.0	0.0	100	100	0.33	100	100	0.0	0.0	100	100	0.29	100	100	0.0	0.0	100	100	0.27
5 sec	63.2	38.2	0.0	0.0	100	99.8	0.42	64.3	38.3	0.0	0.0	99.9	99.9	0.40	67.3	44.9	0.0	0.0	99.9	99.5	0.38
Simulation design with jumps and i.i.d. noise																					
2 sec	100	100	0.0	0.0	100	100	0.25	100	100	0.0	0.0	100	100	0.20	100	100	0.0	0.0	100	100	0.17
3 sec	100	99.8	0.0	0.0	100	100	0.35	100	100	0.0	0.0	100	100	0.32	100	100	0.0	0.0	100	100	0.30
5 sec	63.5	34.7	0.0	0.0	99.9	99.1	0.43	57.3	29.0	0.0	0.0	99.9	99.0	0.41	57.7	30.5	0.0	0.0	99.8	98.0	0.40
	K = 2379							K = 1586							K = 1159						

Table 1: Significance of first-order return autocorrelations for local intervals with simulated returns of length $T =$ 10, 15, and 20 minutes. Columns $\%_{5\%}$ and $\%_{1\%}$ report the percentage rejections for null hypothesis at the 5% and 1% significance level, respectively. Columns marked by (I) refer to the test based on all simulated intervals, while the columns labeled (I_+) and (I_-) refer to results from intervals with only positive or only negative autocorrelations, respectively. Columns $\bar{\pi}_+$ provide the fraction of intervals with a positive return autocorrelation in the full sample of K simulated intervals (K is given in the bottom of the table). The tests are conducted for 1000 simulations, each containing 61 trading days to mirror the empirical test design from Section 2.1 in the main manuscript.

not indicate persistence or anti-persistence in the autocorrelation regimes.

In summary, our tests possess reasonable statistical properties. The size distortions differ by an order of magnitude from the test outcomes in Sections 2.1 and 2.2 in the main manuscript, suggesting that the empirical evidence on the variability, significance and persistence of local autocorrelation regimes is not spurious, but rather driven by structural features of the price formation process.

2 Estimation

Define Y as the vector of observed returns, $Y := (r_1, \ldots, r_T)$ and $\theta = (\sigma_*^2, \sigma_\varepsilon^2, \alpha, \gamma)'$ as the vector of parameters to be estimated. By assuming that ε_i and ε_i^* are i.i.d. jointly (for the local interval) Gaussian with variance σ_ε^2 and σ_*^2, the log likelihood function is (up to the constant $T\ln(2\pi)/2$) given by the log density of the corresponding T-dimensional zero-mean normal

Δ	T = 10 min					T = 15 min					T = 20 min				
	$\%_{2.5}$	$\%_{97.5}$	$\%_{0.5}$	$\%_{99.5}$	$\overline{\pi}_+$	$\%_{2.5}$	$\%_{97.5}$	$\%_{0.5}$	$\%_{99.5}$	$\overline{\pi}_+$	$\%_{2.5}$	$\%_{97.5}$	$\%_{0.5}$	$\%_{99.5}$	$\overline{\pi}_+$
	Simulation design without jumps and noise														
2 sec	3.2	1.9	0.9	0.3	0.50	1.1	2.5	0.5	0.3	0.50	2.8	1.5	0.6	0.2	0.50
3 sec	2.7	3.0	0.4	0.3	0.50	2.3	2.2	0.4	0.3	0.50	2.7	2.3	0.1	0.4	0.50
5 sec	3.5	2.6	0.6	0.3	0.50	2.7	2.4	0.5	0.6	0.50	1.8	3.9	0.2	1.2	0.50
	Simulation design with jumps, but without noise														
2 sec	2.6	2.6	0.3	0.7	0.50	2.5	1.2	0.3	0.1	0.50	3.3	2.5	0.6	0.2	0.50
3 sec	3.0	1.2	0.2	0.2	0.50	2.6	1.7	0.7	0.2	0.50	2.1	3.1	0.4	0.8	0.50
5 sec	2.5	2.2	0.6	0.1	0.50	2.5	2.5	0.5	0.3	0.50	2.5	1.7	0.3	0.1	0.50
	Simulation design without jumps, but with i.i.d. noise														
2 sec	2.2	2.4	0.1	0.4	0.22	3.4	1.2	1.2	0.3	0.17	4.3	1.0	1.0	0.2	0.13
3 sec	2.8	2.7	0.4	0.6	0.33	2.0	2.4	0.2	0.3	0.29	3.4	1.8	0.4	0.7	0.27
5 sec	3.7	2.2	0.6	0.7	0.42	2.6	3.3	0.4	0.5	0.40	1.7	1.3	0.3	0.1	0.38
	Simulation design with jumps and i.i.d. noise														
2 sec	3.0	2.2	0.3	0.5	0.25	3.1	1.8	0.5	0.4	0.20	3.8	1.8	0.9	0.3	0.17
3 sec	1.8	1.6	0.3	0.8	0.35	2.6	1.6	0.5	0.6	0.32	2.5	1.8	0.8	0.1	0.30
5 sec	2.6	2.6	0.8	0.5	0.43	2.6	2.6	0.7	0.3	0.41	3.1	1.9	0.7	0.1	0.40
	K = 2379					K = 1586					K = 1159				

Table 2: The table reports on Wald–Wolfowitz runs tests applied for local intervals with simulated returns of length $T = 10$, 15, and 20 minutes. The null hypothesis asserts that the occurrence of positive vs. negative return autocorrelation regimes follows an independent random sequence. We test the null hypothesis at 5% and 1% significance levels. The rejection frequency for the null hypothesis is given in the columns labeled $\%_{2.5}$, $\%_{0.5}$ (left tail rejections) and $\%_{97.5}$, $\%_{99.5}$ (right tail rejections). $\overline{\pi}_+$ indicates the fraction of intervals with positive return autocorrelations in the full sample of K simulated intervals (K is reported in the bottom of the table). The tests are conducted for 1000 simulations, each spanning 61 trading days to mirror the test design in Section 2.2 in the main manuscript.

distribution with covariance matrix Σ,

$$\ell(Y,\theta) \;=\; -\ln|(\Sigma)|/2 \;-\; Y'\,\Sigma^{-1}\,Y/2, \tag{1}$$

where the diagonal and off-diagonal elements of Σ are given, respectively, by,

$$\Sigma^{ii} = \sigma_*^2(1 + 2/(2-\alpha)F_\alpha(\gamma,\lambda)), \tag{2}$$

$$\Sigma^{ij} = -\sigma_*^2 F_\alpha(\gamma,\lambda)\psi(|i-j|-1)\,. \tag{3}$$

Under the standard regularity conditions for ML inference, we have for the ML estimates $\hat{\theta}$,

$$T^{1/2}(\hat{\theta}-\theta) \xrightarrow{d} N(0, \mathcal{I}_1^{-1}),$$

with $\mathcal{I}_1 = T^{-1}\mathcal{I} = -T^{-1}\mathbb{E}[\partial^2\ell(Y,\theta)/\partial\theta\partial\theta']$. Define DC as the $mn \times q$ matrix $\partial vec(C)/\partial\theta'$, where C is a $m \times n$ matrix depending on a $q \times 1$ parameter vector θ. Then, $\partial\ell(Y,\theta)/\partial\theta'$ and

	Persistence of positive regimes (H_0: $p_{11} = p$)									Persistence of negative regimes (H_0: $p_{00} = 1 - p$)								
	10 min			15 min			20 min			10 min			15 min			20 min		
Δ	$\%_{2.5}$	$\%_{97.5}$	$\overline{\pi}_+$	$\%_{2.5}$	$\%_{97.5}$	$\overline{\pi}_+$	$\%_{2.5}$	$\%_{97.5}$	$\overline{\pi}_+$	$\%_{2.5}$	$\%_{97.5}$	$\overline{\pi}_-$	$\%_{2.5}$	$\%_{97.5}$	$\overline{\pi}_-$	$\%_{2.5}$	$\%_{97.5}$	$\overline{\pi}_-$
	Simulation design without jumps and noise																	
2 sec	2.2	3.1	0.50	2.7	1.2	0.50	1.8	2.6	0.50	2.3	2.9	0.50	3.0	1.0	0.50	1.9	2.1	0.50
3 sec	3.1	2.4	0.50	2.5	2.3	0.50	3.0	2.2	0.50	3.2	2.3	0.50	2.7	2.0	0.50	3.4	2.3	0.50
5 sec	3.0	3.4	0.50	2.6	2.6	0.50	4.4	1.4	0.50	3.6	3.1	0.50	3.3	2.4	0.50	4.6	1.2	0.50
	Simulation design with jumps, but without noise																	
2 sec	3.1	2.0	0.50	1.8	2.6	0.50	3.0	2.9	0.50	3.2	2.1	0.50	2.1	1.8	0.50	3.6	2.4	0.50
3 sec	1.5	2.3	0.50	2.2	2.7	0.50	3.3	1.9	0.50	1.8	1.9	0.50	2.2	2.2	0.50	3.7	1.4	0.50
5 sec	2.7	2.5	0.50	2.9	2.3	0.50	1.9	2.6	0.50	3.2	2.1	0.50	3.0	2.2	0.50	2.7	1.8	0.50
	Simulation design without jumps, but with i.i.d. noise																	
2 sec	14.7	14.9	0.22	17.2	18.9	0.17	20.3	23.8	0.13	0.0	0.0	0.78	0.0	0.0	0.83	0.0	0.0	0.87
3 sec	8.7	9.1	0.33	9.9	9.2	0.29	13.4	11.5	0.27	0.5	0.1	0.67	0.0	0.0	0.71	0.2	0.1	0.73
5 sec	4.7	5.7	0.42	6.9	5.2	0.40	5.2	5.3	0.38	1.4	0.8	0.58	1.6	0.4	0.60	0.3	0.1	0.62
	Simulation design with jumps and i.i.d. noise																	
2 sec	13.9	13.6	0.25	14.7	14.2	0.20	16.8	17.3	0.17	0.1	0.0	0.75	0.0	0.0	0.80	0.0	0.1	0.83
3 sec	7.7	5.1	0.35	9.4	8.8	0.32	11.3	9.9	0.30	0.9	0.1	0.65	0.3	0.0	0.68	0.1	0.3	0.70
5 sec	5.2	3.8	0.43	5.7	4.7	0.41	4.4	4.9	0.40	1.4	1.0	0.57	1.2	0.9	0.59	0.8	0.8	0.60
	K = 2379			K = 1586			K = 1159			K = 2379			K = 1586			K = 1159		

Table 3: The table reports the number of rejections for the null hypothesis of independent transitions between autocorrelation regimes across simulated local intervals. On the left, we test for persistence in positive autocorrelation regimes under H_0: $p_{11} = p$, and on the right, we test for persistence in regimes with negative autocorrelation under H_0: $p_{00} = 1 - p$. The fraction of rejections of the null hypothesis at the 2.5% and 97.5% level is given in the columns labeled $\%_{2.5}$ and $\%_{97.5}$. Columns $\overline{\pi}_+$ and $\overline{\pi}_-$ indicate the fraction of intervals with positive and negative return autocorrelations in the total sample of K simulated intervals, with K reported at the bottom of the table. The tests are conducted for 1000 simulations, each containing 61 trading days to mirror the test design in Section 2.2 in the main manuscript.

$\partial^2 \ell(Y,\theta)/\partial\theta\partial\theta'$ can be computed as,

$$\partial \ell(Y,\theta)/\partial\theta' = -\frac{1}{2}\left(vec'[\Sigma^{-1}(I_T - yy'\Sigma^{-1})]D\Sigma\right) \tag{4}$$

$$\begin{aligned}\partial^2 \ell(Y,\theta)/\partial\theta\partial\theta' = -&\frac{1}{2}\left\{-D\Sigma'((I_T - \Sigma^{-1}yy')\Sigma^{-1} \otimes \Sigma^{-1})D\Sigma\right.\\ &\left.+D\Sigma'(\Sigma^{-1} \otimes \Sigma^{-1}yy'\Sigma^{-1})D\Sigma\right\},\end{aligned} \tag{5}$$

where $\otimes$ denotes the Kronecker product. Then, $\mathbb{E}[\partial^2 \ell(Y,\theta)/\partial\theta\partial\theta']$ is given by,

$$\mathbb{E}[\partial^2 \ell(Y,\theta)/\partial\theta\partial\theta'] = -\frac{1}{2}\left\{D\Sigma'(\Sigma^{-1} \otimes \Sigma^{-1})D\Sigma\right\}. \tag{6}$$

The computation of $D\Sigma$ is straightforwardly obtained based on the derivatives of equation (2) and (3) with respect to θ. ML inference provides a standard $\sqrt{T}$ convergence rate for fixed Δ and $T \to \infty$.

The computation of $\ell(Y,\theta)$ is challenging because expressions for Σ^{-1} generally are not available in closed form. However, if $\alpha = 1$, Σ reduces to a tri-diagonal matrix, as $\Sigma^{ij} = 0$ for $|i-j| > 1$. In this scenario, Σ^{-1} may be computed using results by Usmani (1994). If $\alpha = 1$ and $\gamma = 1$, the autocovariance structure of r_i is governed by an MA(1) process, and convenient expressions for Σ^{-1} are readily available, see Hamilton (1994). Ait-Sahalia et al. (2005) exploit this formulation to derive a tractable approximation to $\mathbb{E}[\partial^2 \ell(Y,\theta)/\partial\theta\partial\theta']$ for $T \to \infty$.

In the general case, such simplifications are not applicable. Σ attains a special Toeplitz form for which the inverse of Σ is not available in closed form. Then, it is more convenient to specify the log likelihood through a prediction error decomposition, see, e.g., Harvey (1989),

$$\ell(Y,\theta) \;=\; -\frac{1}{2}\sum_{i=1}^{T}\ln(s_i) \;-\; \frac{1}{2}\sum_{i=1}^{T}\frac{\nu_i^2}{s_i^2}, \tag{7}$$

where ν_i denote (optimal) linear predictions of r_i given past returns $(r_{i-1},\ldots,r_1)$, and s_i^2 denotes the conditional variance $s_i^2 = \mathbb{E}[\nu_i^2|r_{i-1},\ldots,r_1]$.

Given the normality for ε_i and ε_i^*, $\ell(Y,\theta)$ can be readily computed by the Kalman filter based on a reformulation of the model in terms of a linear state-space system. Denote X_i as a state vector at i with $X_i = (\mu_i, \mu_{i-1}, \widetilde{\varepsilon}_i)'$. Then, the return dynamics takes the form,

$$\begin{aligned} r_i &= F X_i, \\ X_i &= G X_{i-1} + w_i, \end{aligned}$$

with $F = (0, -\alpha, 1)$, and,

$$G := \begin{pmatrix} (1-\alpha) & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The error term vector is given by $w_i = (\varepsilon_i^{\mu}, 0, \widetilde{\varepsilon}_i)$ with covariance matrix,

$$\Sigma_w = \begin{pmatrix} (\gamma-1)^2\sigma_*^2 + \sigma_\varepsilon^2 & 0 & \sigma_*^2[\gamma(\gamma-1)+\lambda] \\ 0 & 0 & 0 \\ \sigma_*^2[\gamma(\gamma-1)+\lambda] & 0 & \gamma^2\sigma_*^2 + \sigma_\varepsilon^2 \end{pmatrix}.$$

To capture the covariance structure between ε_i^{μ} and ε_i, we treat ε_i and μ_{i-1} as separate latent components of X_i. Accordingly, the system contains three state equations.

The Kalman filter (Kalman (1960), Kalman (1963)) generates optimal forecasts of the latent variables X_i, given observations up to time $i-1$, and thus optimal predictions $r_{i,i-1}$ minimizing the mean-squared error s_i, see Hamilton (1994). Given the linear Gaussian state space model, the Kalman filter therefore enables us to compute the exact log-likelihood function yielding (asymptotically) efficient inference. In particular, given the state space formulation in Section 2, we have,

$$\partial\ell(Y,\theta)/\partial\theta' = -\frac{1}{2}\sum_{i=1}^{T}\left\{(s_i^{-2}(1-\nu_i^2 s_i^{-2}))Ds_i^2 + 2\nu_i s_i^{-2}D\nu_i\right\} \tag{8}$$

$$\begin{aligned}\partial^2\ell(Y,\theta)/\partial\theta\partial\theta' \;=\; -\frac{1}{2}\sum_{i=1}^{T}&\left\{-(Ds_i^2)'((1-\nu_i^2/s_i^2)s_i^{-4})Ds_i^2 - 2(Ds_i^2)'s_i^{-4}\nu_i D\nu_i\right.\\ &+(Ds_i^2)'s_i^{-6}\nu_i^2 Ds_i^2 + 2(\nu_i s_i^{-2}\otimes I_3)\partial^2\nu_i/\partial\theta\partial\theta'\\ &\left.-2\,(\nu_i s_i^{-2}\otimes(D\nu_i)'s_i^{-2})Ds_i^2 + 2(D\nu_i)'s_i^{-2}D\nu_i\right\},\end{aligned} \tag{9}$$

yielding

$$\mathbb{E}[\partial^2\ell(Y,\theta)/\partial\theta\partial\theta'] = -\mathbb{E}[2(D\nu_i)'s_i^{-2}D\nu_i - (Ds_i^2)'s_i^{-4}Ds_i^2]. \tag{10}$$

Up to initializations Ds_i^2 and $D\nu_i^2$ can then be computed based on the Kalman filter recursions (e.g., Kalman (1960), Kalman (1963))

$$\nu_i = r_i - FX_{i|i-1} \tag{11}$$

$$X_{i|i-1} = GX_{i-1|i-2} + K_{i-1}\nu_{i-1} \tag{12}$$

$$K_i = GP_iF's_i^{-2} \tag{13}$$

$$P_i = GP_{i-1}L'_{i-1} + \Sigma_w \tag{14}$$

$$s_i^2 = FP_iF' \tag{15}$$

$$L_i = G - K_iF \tag{16}$$

with derivatives

$$D\nu_i = -X_{i|i-1}DF - FDX_{i|i-1} \tag{17}$$

$$Ds_i^2 = (F \otimes F)DP_i + 2(FP_i)DF \tag{18}$$

$$DK_i = (s_i^{-2}FP_i)DG + (s_i^{-2}F \otimes G)DP_i$$
$$+ (s_i^{-2} \otimes GP_i)DF - (s_i^{-2} \otimes GP_iF's_i^{-2})Ds_i^2 \tag{19}$$

$$DX_{i|i-1} = GDX_{i-1|i-2} + (X_{i-1|i-2} \otimes I_3)DG + K_{i-1}D\nu_{i-1} + (\nu_{i-1} \otimes I_3)DK_{i-1} \tag{20}$$

$$DL_i = DG - (F' \otimes I_3)DK_i - (I_3 \otimes K_i)DF \tag{21}$$

$$DP_i = (L_{i-1} \otimes G)DP_{i-1} + (L_{i-1}P_{i-1} \otimes I_3)DG$$
$$+ (I_3 \otimes GP_{i-1})\mathcal{K}_{33}DL_{i-1} + 2N_3(\Sigma_w^{1/2} \otimes I_3)D\Sigma_w^{1/2}, \tag{22}$$

where $\Sigma_w = \Sigma_w^{1/2}(\Sigma_w^{1/2})'$. Moreover, $N_m = \frac{1}{2}(I_{m^2} + \mathcal{K}_{mm})$ and $\mathcal{K}_{mn}$ is defined as commutation matrix satisfying $\mathcal{K}_{mn}vec(C) = vec(C')$ with C denoting a $m \times n$ matrix.

If the errors ε_i^* and ε_i (and state vector X_i) are *not* normally distributed, equation (7) admits the interpretation of a *quasi* maximum likelihood function. Under non-normality, the linear predictions of the state variables X_i (implied by the Kalman filter algorithm) are not optimal among all prediction functions, but remain the best *linear* ones. This argument renders the (linear) Kalman filter applicable in a quasi maximum likelihood setting, as discussed in Gourieroux et al. (1984). A formal proof of the consistency of the Kalman-filter based (Q)ML estimator in a non-Gaussian linear state space system is given by Schlemm and Stelzer (2012).

Alternatively, the system could be estimated directly from the unconditional moment restrictions implied by the model following the standard GMM procedure of Hansen (1982).

3 Proof of Lemma 2

We first establish a few initial covariance relations.
From Section 3.2.1 in the main manuscript, recall that log-return may be written, $r_i = -\alpha\,\mu_{i-1} + (\varepsilon_i^* + \varepsilon_i^\mu)$. Hence,

$$\mathbb{C}ov(r_i\,, r_{i-h}) \;=\; \alpha^2\,\mathbb{C}ov(\mu_i\,, \mu_{i-h}) \;-\; \alpha\,\mathbb{C}ov(\mu_i\,, \varepsilon_{i-h+1}^\mu + \varepsilon_{i-h+1}^*).$$

The AR(1) representation Eq. (12) in the main manuscript implies that μ_i takes the form,

$$\mu_i \;=\; \mu_{i-h} \;+\; \sum_{k=0}^{h-1}(1-\alpha)^k\,\varepsilon_{i-k}^\mu\,, \qquad \text{for any integer } h \geq 1.$$

It follows readily that,

$$\mathbb{C}ov(\mu_i\,,\mu_{i-h}\,)\;=\;(1-\alpha)^{\,h}\,\mathbb{V}(\varepsilon^{\mu}_{i-k})\;=\;\frac{(1-\alpha)^{\,h}}{\alpha\,(2-\alpha)}\,[\,(\gamma-1)^2+\lambda\,]\,\sigma^2_*,\quad \text{for any integer } h\geq 1.$$

The remaining covariance term equals,

$$\mathbb{C}ov(\mu_i\,,\varepsilon^{\mu}_{i-h+1}+\varepsilon^{*}_{i})\;=\;(1-\alpha)^{\,h-1}\,[\,(\gamma-1)\,\gamma)\,+\,\lambda\,]\,\sigma^2_*,\quad \text{for } h\geq 1.$$

Substituting into the return covariance result above, using the definition for $\psi(h-1)$, and rearranging terms yields,

$$\mathbb{C}ov(r_i\,,r_{i-h}\,)\;=\;\psi(h-1)\,[\,(1-\alpha)\,[\,(\gamma-1)^2\,+\,\lambda\,]\,-\,(2-\alpha)\,[\,(\gamma-1)^2\,+\,\gamma\,+\,\lambda\,-\,1\,]\,]\,\sigma^2_*.$$

Straightforward computations now verify equation (20) in the main manuscript of Lemma 2,

$$\mathbb{C}ov(r_i\,,r_{i-h}\,)\;=\;\psi(h-1)\,[\,-\,(\gamma^2\,+\,\lambda)\,-\,\alpha\,(1\,-\,\gamma)\,+\,1\,]\,\sigma^2_*.$$

3.1 Additional Tables and Figures

Parameter	Mean	Std.	Skew.	Ex.Kurt.	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.95}$
Liquidity group 1 (20 most liquid assets from NASDAQ 100)									
α	0.902	0.095	-1.501	12.637	0.772	0.852	0.904	0.957	1.040
σ_*^2 $(\times 10^{-8})$	2.332	4.426	6.408	58.953	0.204	0.520	1.052	2.286	8.385
$F_\alpha(\gamma, \lambda)$	-0.009	0.138	3.266	31.820	-0.178	-0.081	-0.018	0.049	0.166
corr(r_{t+1}, r_t)	0.014	0.088	-0.201	1.470	-0.133	-0.042	0.015	0.070	0.156
γ_{max}	0.988	0.107	0.930	6.658	0.832	0.925	0.984	1.046	1.151
λ_{max}	0.295	0.120	8.459	100.714	0.201	0.248	0.282	0.320	0.391
Liquidity group 2									
α	0.907	0.101	-1.321	12.594	0.769	0.856	0.910	0.964	1.052
σ_*^2 $(\times 10^{-8})$	2.163	4.598	6.834	64.933	0.156	0.404	0.847	2.015	8.018
$F_\alpha(\gamma, \lambda)$	0.002	0.154	3.424	29.002	-0.180	-0.074	-0.009	0.059	0.190
corr(r_{t+1}, r_t)	0.006	0.093	-0.273	2.552	-0.148	-0.050	0.007	0.064	0.157
γ_{max}	0.997	0.116	1.094	7.302	0.830	0.931	0.992	1.055	1.172
λ_{max}	0.304	0.139	7.604	76.942	0.200	0.252	0.287	0.326	0.406
Liquidity group 3									
α	0.915	0.098	-0.883	12.493	0.777	0.865	0.918	0.969	1.055
σ_*^2 $(\times 10^{-8})$	1.388	3.459	10.257	141.462	0.151	0.344	0.618	1.230	4.359
$F_\alpha(\gamma, \lambda)$	0.012	0.152	3.691	31.522	-0.166	-0.063	0.000	0.066	0.193
corr(r_{t+1}, r_t)	-0.001	0.092	-0.382	4.229	-0.150	-0.056	-0.000	0.055	0.145
γ_{max}	1.006	0.114	1.347	9.665	0.844	0.941	1.000	1.061	1.174
λ_{max}	0.309	0.138	7.540	75.339	0.206	0.257	0.292	0.330	0.408
Liquidity group 4									
α	0.920	0.100	-1.067	11.113	0.775	0.871	0.921	0.975	1.063
σ_*^2 $(\times 10^{-8})$	1.251	3.423	11.224	164.292	0.122	0.294	0.547	1.097	3.735
$F_\alpha(\gamma, \lambda)$	0.023	0.166	3.712	28.051	-0.164	-0.055	0.005	0.075	0.214
corr(r_{t+1}, r_t)	-0.008	0.095	-0.236	3.108	-0.163	-0.063	-0.004	0.047	0.144
γ_{max}	1.015	0.120	1.301	7.749	0.845	0.949	1.005	1.069	1.193
λ_{max}	0.318	0.157	6.952	61.399	0.207	0.262	0.295	0.336	0.421
Liquidity group 5 (20 least liquid assets from NASDAQ 100)									
α	0.923	0.092	-0.869	12.505	0.784	0.902	0.920	0.960	1.061
σ_*^2 $(\times 10^{-8})$	1.477	3.485	9.652	129.026	0.122	0.339	0.668	1.352	4.866
$F_\alpha(\gamma, \lambda)$	0.029	0.168	4.445	34.487	-0.146	-0.017	0.000	0.058	0.215
corr(r_{t+1}, r_t)	-0.012	0.090	-0.243	6.199	-0.165	-0.048	-0.000	0.015	0.130
γ_{max}	1.019	0.117	1.799	11.659	0.862	0.984	1.000	1.053	1.193
λ_{max}	0.321	0.164	7.055	61.352	0.214	0.282	0.292	0.325	0.421

Table 4: The table reports summary statistics for the model parameter estimates (estimated under under $\gamma = 0$) obtained from T = 10 min intervals with intra-interval sampling at Δ= 2 sec frequency. The statistics is based on the estimates from all intra-daily intervals over the first 61 trading days of 2014. The results are reported separately for five groups of stocks sorted by the average daily mid-quote revisions. Intervals where parameter estimates of σ_*^2 and λ_{max} exceed their 99.5th percentiles are excluded from the sample.

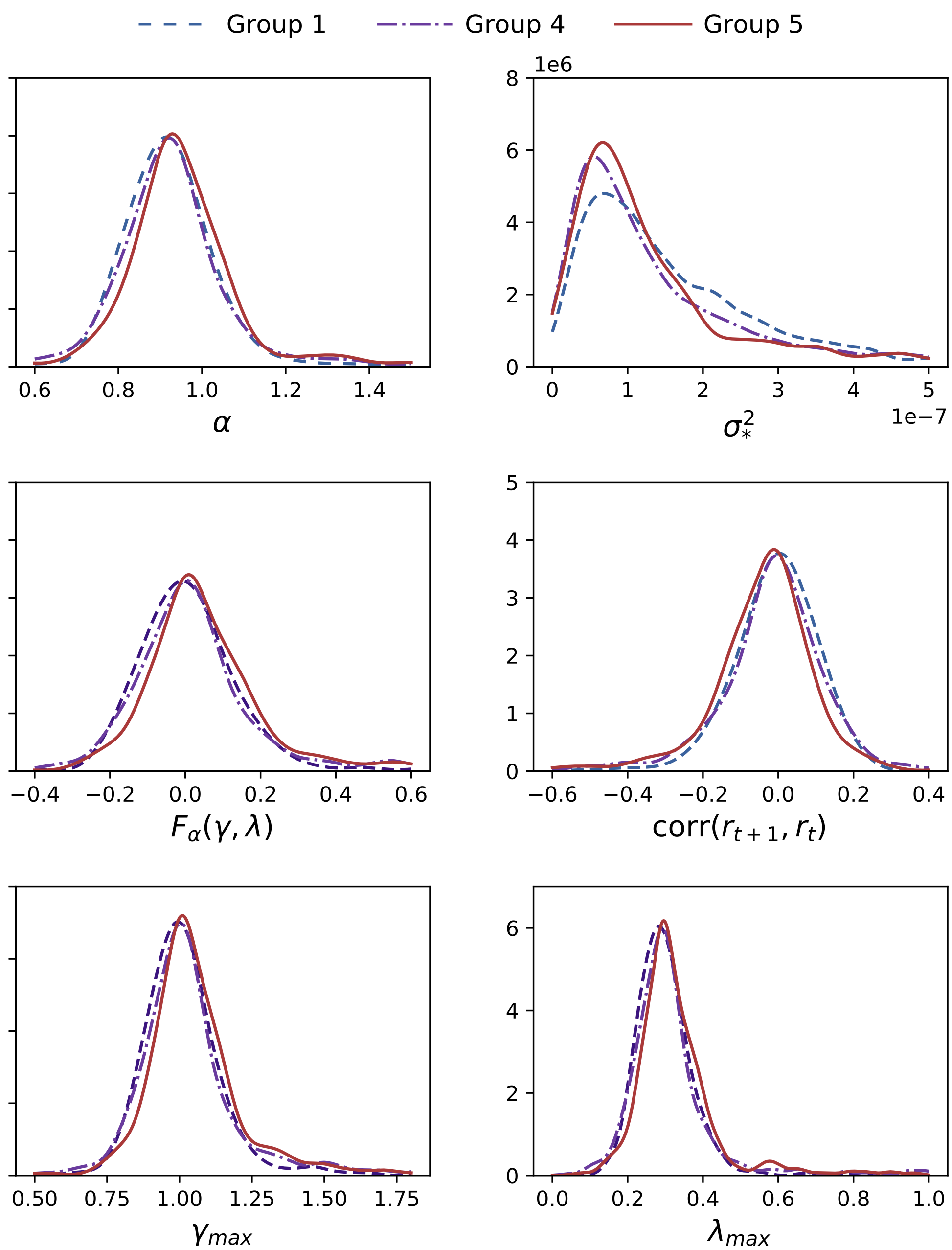

Figure 1: The figure depicts empirical distributions of the model parameter estimates (estimated under restriction $\gamma = 0$) obtained from T = 10 min intervals between 9:30 and 9:40 ET (first ten minutes after market opening) with intra-interval sampling at Δ= 2 sec frequency. Results are reported for the stocks from 1st, 4th and 5th groups of stocks sorted by the average number of daily mid-quote revisions (with 20 stocks in each group) for the first 61 trading days in 2014.

Parameter	Mean	Std.	Skew.	Ex.Kurt.	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.95}$
	Liquidity group 1 (20 most liquid assets from NASDAQ 100)								
α	0.917	0.133	-0.135	10.946	0.765	0.853	0.914	0.975	1.093
σ_*^2 $(\times 10^{-8})$	16.687	13.445	1.527	2.385	2.765	6.969	12.481	22.281	43.548
$F_\alpha(\gamma, \lambda)$	0.023	0.196	3.507	26.337	-0.181	-0.077	0.001	0.078	0.258
corr(r_{t+1}, r_t)	-0.007	0.115	-1.317	6.587	-0.190	-0.065	-0.001	0.067	0.158
γ_{max}	1.013	0.145	1.731	9.534	0.828	0.928	1.001	1.072	1.233
λ_{max}	0.320	0.176	6.054	47.309	0.201	0.251	0.293	0.338	0.449
	Liquidity group 2								
α	0.925	0.170	1.436	10.860	0.744	0.848	0.915	0.982	1.122
σ_*^2 $(\times 10^{-8})$	18.774	15.512	1.403	1.484	3.227	7.424	13.626	24.893	51.677
$F_\alpha(\gamma, \lambda)$	0.042	0.253	3.186	15.924	-0.197	-0.079	0.003	0.090	0.445
corr(r_{t+1}, r_t)	-0.016	0.152	-2.075	9.814	-0.226	-0.075	-0.002	0.069	0.174
γ_{max}	1.027	0.188	2.260	10.127	0.813	0.926	1.002	1.082	1.358
λ_{max}	0.338	0.230	4.825	27.025	0.191	0.249	0.293	0.343	0.589
	Liquidity group 3								
α	0.942	0.185	2.477	11.235	0.758	0.857	0.919	0.986	1.249
σ_*^2 $(\times 10^{-8})$	15.742	14.464	1.790	2.849	2.798	6.149	10.733	18.949	50.559
$F_\alpha(\gamma, \lambda)$	0.059	0.293	3.739	18.339	-0.181	-0.070	0.003	0.091	0.556
corr(r_{t+1}, r_t)	-0.028	0.176	-2.399	9.867	-0.287	-0.075	-0.003	0.061	0.163
γ_{max}	1.041	0.220	2.924	12.721	0.826	0.935	1.003	1.083	1.395
λ_{max}	0.347	0.251	4.660	24.831	0.197	0.253	0.293	0.344	0.690
	Liquidity group 4								
α	0.931	0.182	1.550	10.277	0.731	0.854	0.919	0.979	1.187
σ_*^2 $(\times 10^{-8})$	15.899	15.410	1.726	2.494	2.198	5.551	10.206	20.353	53.455
$F_\alpha(\gamma, \lambda)$	0.063	0.296	3.000	12.367	-0.205	-0.073	0.006	0.090	0.555
corr(r_{t+1}, r_t)	-0.024	0.167	-1.882	8.702	-0.275	-0.075	-0.005	0.063	0.185
γ_{max}	1.040	0.212	2.079	8.168	0.804	0.932	1.006	1.082	1.475
λ_{max}	0.357	0.275	4.258	19.816	0.184	0.251	0.296	0.343	0.793
	Liquidity group 5 (20 least liquid assets from NASDAQ 100)								
α	0.958	0.171	1.864	9.320	0.758	0.883	0.935	1.012	1.257
σ_*^2 $(\times 10^{-8})$	14.704	14.093	1.921	3.450	2.004	5.789	9.805	17.316	47.607
$F_\alpha(\gamma, \lambda)$	0.081	0.252	3.488	19.957	-0.167	-0.035	0.030	0.128	0.531
corr(r_{t+1}, r_t)	-0.048	0.157	-1.947	8.429	-0.314	-0.105	-0.026	0.030	0.147
γ_{max}	1.062	0.192	2.444	11.760	0.843	0.967	1.028	1.117	1.399
λ_{max}	0.359	0.226	4.682	27.520	0.205	0.271	0.309	0.366	0.656

Table 5: The table reports summary statistics for the model parameter estimates (estimated under under $\gamma = 0$) obtained from T = 10 min intervals with intra-interval sampling at Δ= 2 sec frequency. The statistics is based on the estimates from all intra-daily intervals over the first 61 trading days of 2014. The statistics is based on the estimates from intra-daily intervals between 9:30 and 9:40 ET (first ten minutes after market opening) over the first 61 trading days of 2014. The results are reported separately for five groups of stocks sorted by the average daily mid-quote revisions. Intervals where parameter estimates of σ_*^2 and λ_{max} exceed their 99.5th percentiles are excluded from the sample.

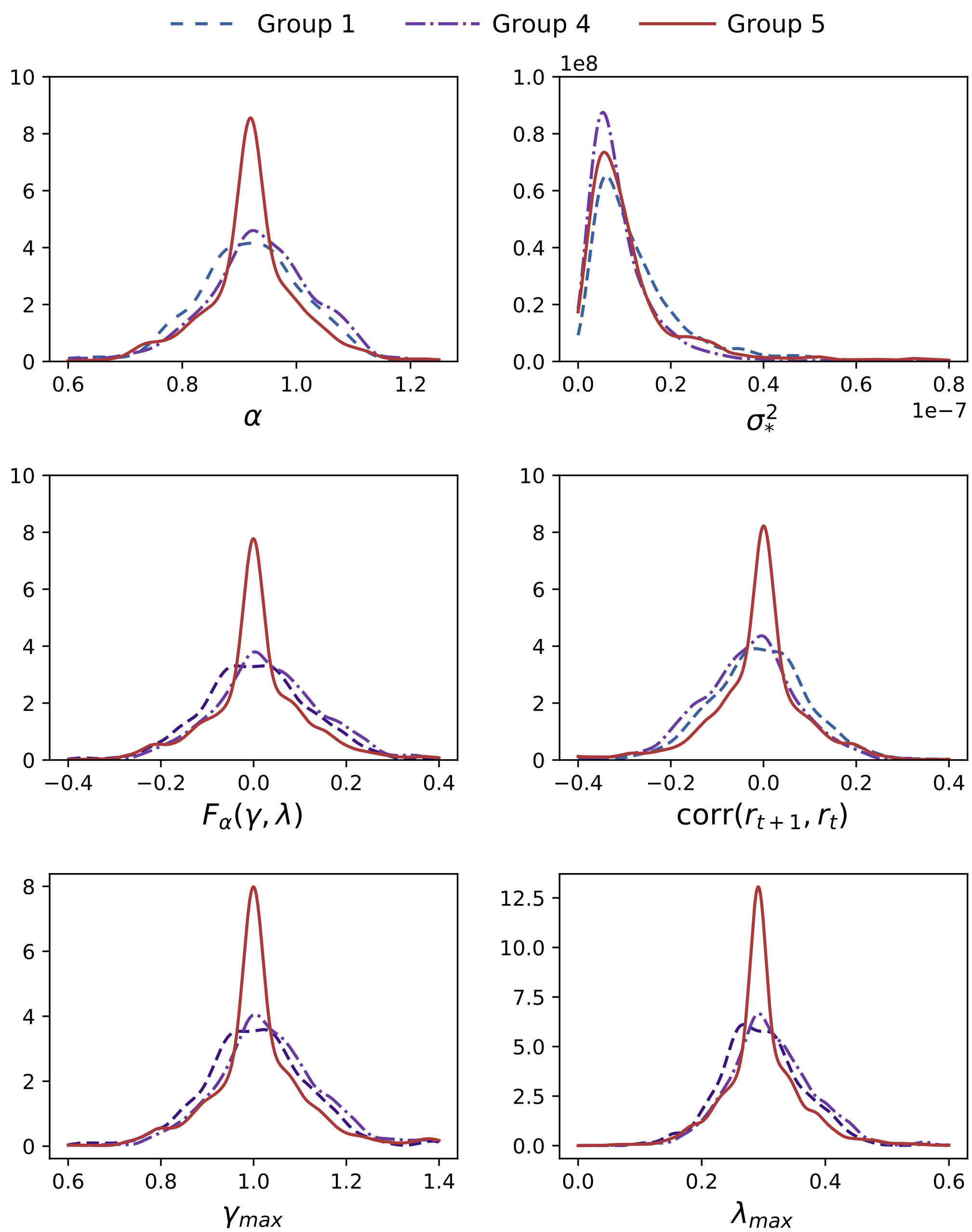

Figure 2: The figure depicts empirical distributions of the model parameter estimates (estimated under restriction $\gamma = 0$) obtained from T = 10 min intervals between 15:50 and 16:00 ET (last ten minutes of trading) with intra-interval sampling at Δ= 2 sec frequency. Results are reported for the stocks from 1st, 4th and 5th groups of stocks sorted by the average number of daily mid-quote revisions (with 20 stocks in each group) for the first 61 trading days in 2014.

Parameter	Mean	Std.	Skew.	Ex.Kurt.	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.95}$
Liquidity group 1 (20 most liquid assets from NASDAQ 100)									
α	0.912	0.130	-1.010	11.286	0.752	0.856	0.919	0.980	1.074
$\sigma_*^2\ (\times 10^{-8})$	1.415	1.562	5.035	41.219	0.310	0.578	0.996	1.653	3.748
$F_\alpha(\gamma, \lambda)$	0.038	0.239	3.654	20.261	-0.184	-0.067	0.007	0.088	0.256
corr(r_{t+1}, r_t)	-0.009	0.111	-0.681	5.468	-0.178	-0.073	-0.006	0.058	0.162
γ_{max}	1.021	0.155	1.735	8.910	0.827	0.938	1.006	1.081	1.232
λ_{max}	0.338	0.241	5.471	33.308	0.198	0.256	0.297	0.344	0.446
Liquidity group 2									
α	0.922	0.123	-1.455	9.684	0.753	0.863	0.927	0.987	1.085
$\sigma_*^2\ (\times 10^{-8})$	1.608	2.326	6.304	64.191	0.238	0.496	0.896	1.755	5.318
$F_\alpha(\gamma, \lambda)$	0.038	0.206	3.065	19.889	-0.178	-0.062	0.020	0.095	0.264
corr(r_{t+1}, r_t)	-0.015	0.107	0.159	1.584	-0.188	-0.079	-0.016	0.053	0.157
γ_{max}	1.025	0.142	0.738	5.780	0.833	0.942	1.018	1.087	1.241
λ_{max}	0.332	0.199	5.842	40.306	0.200	0.258	0.303	0.348	0.447
Liquidity group 3									
α	0.939	0.116	-0.819	6.782	0.778	0.884	0.944	1.005	1.097
$\sigma_*^2\ (\times 10^{-8})$	1.045	1.309	7.117	76.149	0.232	0.483	0.743	1.171	2.625
$F_\alpha(\gamma, \lambda)$	0.072	0.217	3.495	18.523	-0.134	-0.033	0.039	0.121	0.338
corr(r_{t+1}, r_t)	-0.037	0.103	-0.334	1.985	-0.206	-0.101	-0.033	0.029	0.117
γ_{max}	1.052	0.142	1.460	5.548	0.874	0.969	1.036	1.111	1.286
λ_{max}	0.357	0.226	5.167	30.733	0.220	0.273	0.315	0.362	0.558
Liquidity group 4									
α	0.932	0.121	-0.565	6.768	0.753	0.878	0.935	0.998	1.093
$\sigma_*^2\ (\times 10^{-8})$	0.977	1.147	7.388	85.064	0.226	0.453	0.693	1.146	2.411
$F_\alpha(\gamma, \lambda)$	0.071	0.234	3.255	15.092	-0.156	-0.037	0.029	0.117	0.367
corr(r_{t+1}, r_t)	-0.032	0.110	-0.291	3.274	-0.200	-0.097	-0.024	0.032	0.142
γ_{max}	1.048	0.152	1.401	4.809	0.851	0.966	1.026	1.107	1.317
λ_{max}	0.360	0.244	4.728	24.834	0.207	0.272	0.309	0.362	0.565
Liquidity group 5 (20 least liquid assets from NASDAQ 100)									
α	0.922	0.114	-0.487	10.431	0.756	0.890	0.920	0.963	1.078
$\sigma_*^2\ (\times 10^{-8})$	1.237	1.468	4.286	28.755	0.209	0.488	0.812	1.350	3.586
$F_\alpha(\gamma, \lambda)$	0.035	0.211	4.275	29.208	-0.173	-0.028	0.000	0.065	0.269
corr(r_{t+1}, r_t)	-0.012	0.105	-0.678	4.006	-0.192	-0.055	-0.000	0.024	0.151
γ_{max}	1.022	0.142	1.876	9.733	0.837	0.974	1.000	1.060	1.240
λ_{max}	0.329	0.205	6.253	45.569	0.203	0.276	0.292	0.328	0.455

Table 6: The table reports summary statistics for the model parameter estimates (estimated under under $\gamma = 0$) obtained from T = 10 min intervals with intra-interval sampling at Δ= 2 sec frequency. The statistics is based on the estimates from all intra-daily intervals over the first 61 trading days of 2014. The statistics is based on the estimates from intra-daily intervals between 15:50 and 16:00 ET (last ten minutes of trading) over the first 61 trading days of 2014. The results are reported separately for five groups of stocks sorted by the average daily mid-quote revisions. Intervals where parameter estimates of σ_*^2 and λ_{max} exceed their 99.5th percentiles are excluded from the sample.

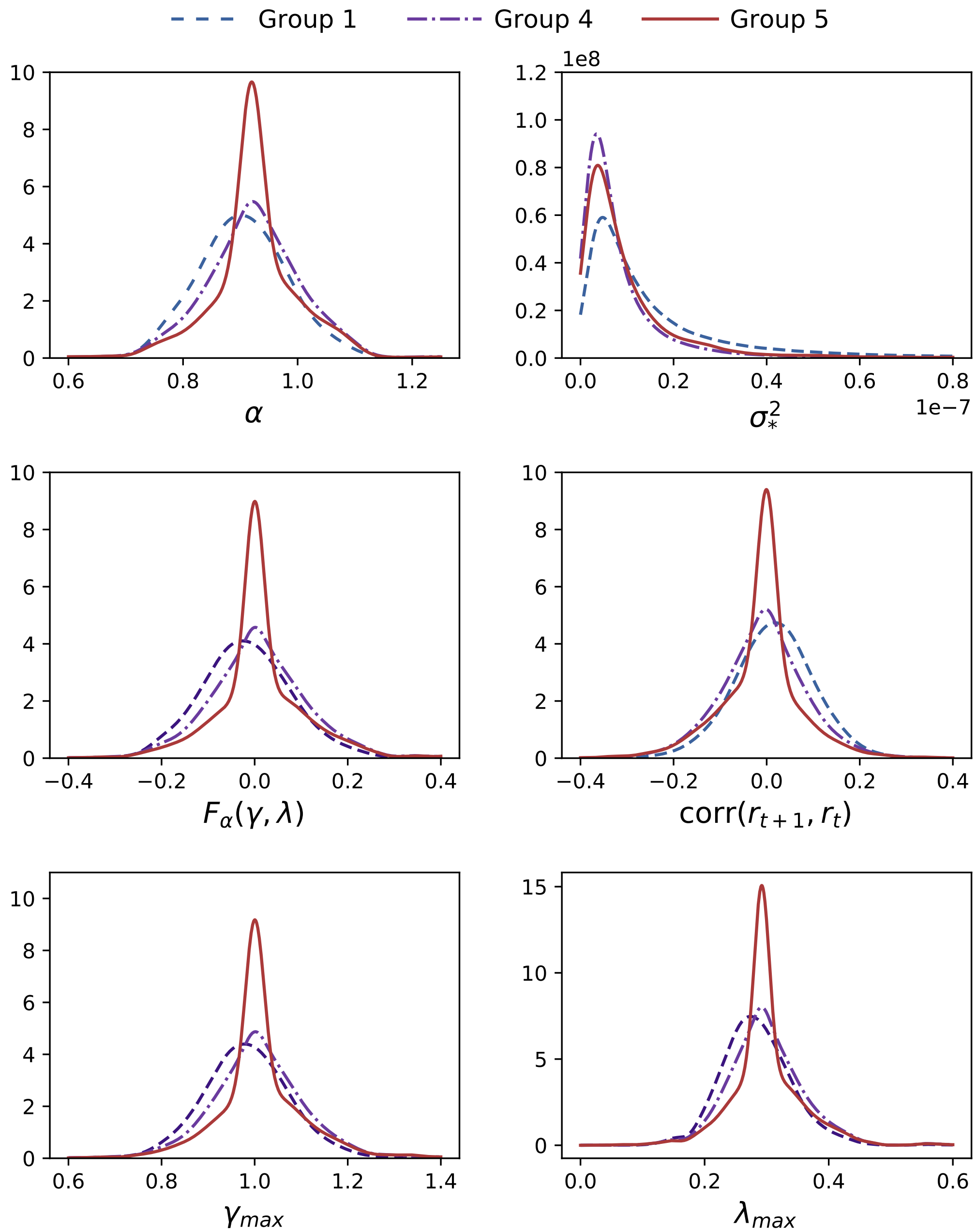

Figure 3: The figure depicts empirical distributions of the model parameter estimates (estimated under restriction $\gamma = 0$) obtained from T = 10 min intervals between 9:50 and 15:40 ET (excluding first and last 20 minutes of a trading session) with intra-interval sampling at Δ= 2 sec frequency. Results are reported for the stocks from 1st, 4th and 5th groups of stocks sorted by the average number of daily mid-quote revisions (with 20 stocks in each group) for the first 61 trading days in 2014.

Parameter	Mean	Std.	Skew.	Ex.Kurt.	$Q_{0.05}$	$Q_{0.25}$	$Q_{0.50}$	$Q_{0.75}$	$Q_{0.95}$
	Liquidity group 1 (20 most liquid assets from NASDAQ 100)								
α	0.901	0.093	-1.665	12.387	0.773	0.851	0.903	0.955	1.037
σ_*^2 ($\times 10^{-8}$)	1.845	2.842	6.410	72.207	0.197	0.499	0.993	2.054	6.148
$F_\alpha(\gamma, \lambda)$	-0.012	0.132	2.982	29.679	-0.179	-0.082	-0.019	0.047	0.160
corr(r_{t+1}, r_t)	0.015	0.087	-0.081	0.598	-0.129	-0.040	0.016	0.071	0.156
γ_{max}	0.986	0.104	0.748	5.383	0.831	0.924	0.982	1.043	1.146
λ_{max}	0.293	0.113	8.596	107.238	0.200	0.248	0.281	0.319	0.388
	Liquidity group 2								
α	0.906	0.098	-1.637	11.713	0.770	0.856	0.909	0.962	1.049
σ_*^2 ($\times 10^{-8}$)	1.679	2.888	6.504	71.626	0.152	0.388	0.793	1.781	5.965
$F_\alpha(\gamma, \lambda)$	-0.000	0.148	3.273	28.591	-0.180	-0.075	-0.010	0.058	0.184
corr(r_{t+1}, r_t)	0.008	0.091	-0.064	0.781	-0.144	-0.048	0.008	0.065	0.157
γ_{max}	0.995	0.112	0.899	5.841	0.830	0.930	0.991	1.053	1.166
λ_{max}	0.302	0.133	7.783	82.328	0.200	0.251	0.286	0.325	0.403
	Liquidity group 3								
α	0.914	0.094	-1.419	10.254	0.777	0.865	0.917	0.968	1.052
σ_*^2 ($\times 10^{-8}$)	1.011	1.724	13.343	337.298	0.147	0.332	0.587	1.108	3.082
$F_\alpha(\gamma, \lambda)$	0.009	0.144	3.347	28.886	-0.167	-0.064	-0.001	0.064	0.187
corr(r_{t+1}, r_t)	0.000	0.089	0.013	0.982	-0.147	-0.054	0.000	0.055	0.146
γ_{max}	1.003	0.109	0.911	5.824	0.843	0.940	1.000	1.059	1.169
λ_{max}	0.306	0.130	7.755	81.795	0.206	0.257	0.292	0.329	0.404
	Liquidity group 4								
α	0.919	0.097	-1.410	9.660	0.776	0.871	0.920	0.974	1.061
σ_*^2 ($\times 10^{-8}$)	0.885	1.498	14.685	438.713	0.116	0.282	0.515	0.985	2.710
$F_\alpha(\gamma, \lambda)$	0.021	0.160	3.670	28.733	-0.163	-0.055	0.004	0.074	0.208
corr(r_{t+1}, r_t)	-0.007	0.092	0.002	1.117	-0.160	-0.062	-0.004	0.047	0.143
γ_{max}	1.013	0.116	1.129	6.475	0.846	0.949	1.004	1.068	1.188
λ_{max}	0.316	0.151	7.177	66.409	0.208	0.262	0.295	0.335	0.418
	Liquidity group 5 (20 least liquid assets from NASDAQ 100)								
α	0.922	0.089	-1.332	10.733	0.786	0.903	0.920	0.958	1.059
σ_*^2 ($\times 10^{-8}$)	1.109	1.916	10.889	226.273	0.118	0.324	0.625	1.222	3.361
$F_\alpha(\gamma, \lambda)$	0.027	0.163	4.443	34.739	-0.145	-0.015	0.000	0.055	0.209
corr(r_{t+1}, r_t)	-0.011	0.087	0.061	4.449	-0.162	-0.046	-0.000	0.013	0.129
γ_{max}	1.018	0.114	1.641	10.345	0.863	0.986	1.000	1.051	1.189
λ_{max}	0.320	0.161	7.187	63.698	0.215	0.283	0.292	0.324	0.421

Table 7: The table reports summary statistics for the model parameter estimates (estimated under under $\gamma = 0$) obtained from T = 10 min intervals with intra-interval sampling at Δ= 2 sec frequency. The statistics is based on the estimates from all intra-daily intervals over the first 61 trading days of 2014. The statistics is based on the estimates from intra-daily intervals between 9:50 and 15:40 ET (excluding first and last 20 minutes of a trading session) over the first 61 trading days of 2014. The results are reported separately for five groups of stocks sorted by the average daily mid-quote revisions. Intervals where parameter estimates of σ_*^2 and λ_{max} exceed their 99.5th percentiles are excluded from the sample.

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