Emergent Geometry and Gravity from Matrix Models

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Motivation

- expect quantum structure of space-time at Planck scale

  due to $\boxed{\text{Gravity} \leftrightarrow \text{Quantum Mechanics}}$

- fine-tuning problems (cosm. const. etc.)
- ”dark matter, dark energy” ... ??

$\Rightarrow$ perhaps gravity is modified ?

pre-geometric theory of gravity:

Matrix Models $\rightarrow$ noncommutative space-time & gravity

Outline:

- geometry from matrix models:
  - NC branes
  - effective geometry
  - dynamics
  - examples
- gauge theory point of view
- quantization
- curvature, etc.

review: H.S., arXiv:1003.4134
D. Blaschke, H. S. arXiv:1003.4132, arXiv:1005.0499

Matrix Models

candidate for quantum theory of fundamental interactions

$$S = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} + \bar{\Psi}\Gamma^a[X_a, \Psi]\right)$$

$$X^a \in Mat(\infty, \mathbb{C}), \qquad a = 1, ..., 10$$

(IKKT Model 1996)

- no geometrical pre-requisites, extremely simple
- $\left\{ \begin{array}{l} \text{NC space-time} \\ \text{metric (=gravity)} \end{array} \right\}$ emerge
- $\left\{ \begin{array}{l} \text{nonabelian gauge fields} \\ \text{gravitons} \end{array} \right\}$ ... fluctuations of NC space
- well-behaved under quantization
  new perspectives for dark energy / dark matter

Ishibashi, Kawai, Kitazawa and Tsuchiya 1996, ff

Rivelles 2002, Yang 2006, H.S. 2007 ff, ...

Space-time & geometry from matrix models:

e.o.m.: $\delta S = 0 \;\Rightarrow\; [X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

solutions:

1. $[X^a, X^b] = i\theta^{ab}\,\mathbf{1}$, rank $\theta^{ab} = 2n$
   separate $X^a = (X^\mu, \Phi^i)$, $\mu = 1, ..., 2n$

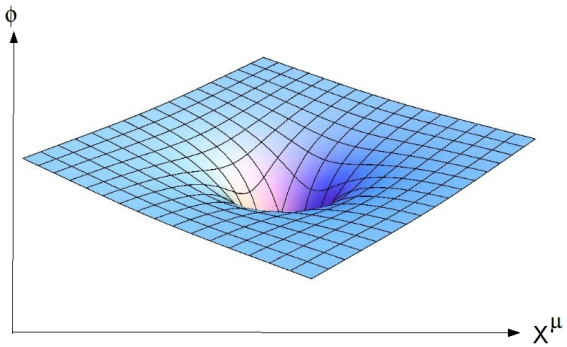
$$\begin{aligned} [X^\mu, X^\nu] &= i\theta^{\mu\nu}\,\mathbf{1} \\ \Phi^i &= 0 \end{aligned}$$

...“quantum plane” $\mathbb{R}^{2n}_\theta$

2. $[X^a, X^b] = i\theta^{ab}(X) \sim i\theta^{ab}(x)$ ...generic quantum space

$\rightarrow$ space-time as 3+1-dimensional brane solution $\mathcal{M}^4 \subset \mathbb{R}^{10}$

$$\begin{aligned} X^a &= (X^\mu, \Phi^i), \qquad \mu = 1, ..., 4; \qquad X^\mu \sim x^\mu \\ \Phi^i &= \Phi^i(X^\mu) \end{aligned}$$



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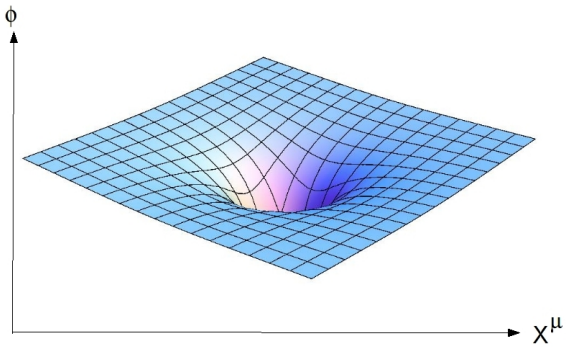


1) Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$

$$\begin{aligned} [X^\mu, X^\nu] &= i\theta^{\mu\nu}\,\mathbf{1}, \qquad \mu,\nu = 1,...,4 \\ \phi^i &= 0 \end{aligned}$$

... Heisenberg algebra, interpreted as space of functions on $\mathbb{R}^4_\theta$

uncertainty relations $\Delta x^\mu \Delta x^\nu \geq |\theta^{\mu\nu}|$

relation with classical $\mathbb{R}^4$:

$$\mathcal{C}(\mathbb{R}^4) \ni \phi(x) = \int d^4k\, e^{ik_\mu x^\mu} \;\leftrightarrow\; \int d^4k\, e^{ik_\mu X^\mu} =: \Phi(X) \in Mat(\infty,\mathbb{C})$$

note:

$X^\mu \in Mat(\infty,\mathbb{C})$ ... quantized coordinate functions on $\mathbb{R}^4_\theta$

$\Phi(X^\mu) \in Mat(\infty,\mathbb{C})$ ... general function on $\mathbb{R}^4_\theta$

$[X^\mu, \Phi] =: i\theta^{\mu\nu}\partial_\nu \Phi \sim i\theta^{\mu\nu}\partial_\nu \phi(x) \;\rightarrow$ NC field theory

2) Noncommutative spaces and Poisson structure

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dimensional manifold with Poisson structure

Its quantization $\mathcal{M}_\theta$ is NC algebra such that

$$\begin{aligned} \mathcal{I}: \ \mathcal{C}(\mathcal{M}) &\to \mathcal{A} \cong Mat(\infty, \mathbb{C}) \\ f(x) &\mapsto \hat{f}(X) \\ x^i &\mapsto X^i, \qquad e^{ikx} \mapsto e^{ikX} \end{aligned}$$

such that $\quad [\hat{f}(X), \hat{g}(X)] = \mathcal{I}(i\{f(x), g(x)\}) + O(\theta^2)$

("nice") $\Phi \in Mat(\infty, \mathbb{C}) \quad \leftrightarrow \quad$ quantized function on $\mathcal{M}$

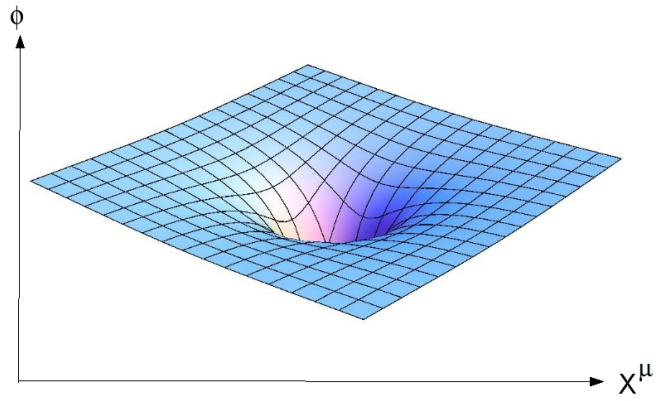
<u>furthermore:</u>

$$\begin{aligned} (2\pi)^2 \, Tr(\phi(X)) &\sim \int d^4x \, \rho(x) \, \phi(x) \\ \rho(x) &= \mathrm{Pfaff}\,(\theta^{-1}_{\mu\nu}) \ \ldots \quad \text{symplectic volume} \end{aligned}$$

(cf. Bohr-Sommerfeld quantization)

Interpretation of X^a in matrix model:

$X^a = (X^\mu, \phi^i(X^\mu)) : \quad \mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ ...(quantized) embedding function



Effective geometry of NC brane:

consider scalar field coupled to Matrix Model ("test particle")

use $[X^\mu, \varphi] \sim i\theta^{\mu\nu}(x)\partial_\nu\varphi \quad \Rightarrow \quad \theta^{\mu\nu} = \{x^\mu, x^\nu\}$

$$\begin{aligned} S[\varphi] &= Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} \qquad (U(\mathcal{H}) \text{ gauge inv.!}) \\ &\sim \int d^4x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\varphi\partial_\nu\varphi \end{aligned}$$

$$\boxed{\begin{aligned} G^{\mu\nu}(x) &= e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\;g_{\mu'\nu'}(x) \quad \text{effective metric (cf. open string m.)} \\ g_{\mu\nu}(x) &= \partial_\mu x^a\partial_\nu x^b\eta_{ab} \quad \text{induced metric on } \mathcal{M}^4_\theta \text{ (cf. closed string m.)} \end{aligned}}$$

$$e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}, \qquad |G_{\mu\nu}| = |g_{\mu\nu}| \qquad \text{for } \dim(\mathcal{M}) = 4$$

φ couples to metric $G^{\mu\nu}(x)$, determined by $\theta^{\mu\nu}(x)$ & embedding $\phi^i(x)$

same for gauge fields, fermions

... quantized Poisson manifold with metric $(\mathcal{M}, \theta^{\mu\nu}(x), G_{\mu\nu}(x))$

so: all matter couples to dynamical metric $G_{\mu\nu}$ $\Rightarrow$ effective gravity

<u>however</u>: metric is not fundamental d.o.f.
rather: matrices X^a resp. $(\phi^i, \theta^{\mu\nu})$ resp. $(\phi^i, F_{\mu\nu})$
$\Rightarrow$ *dynamics* of gravity NOT given by Einstein equations

turns out to be different from GR (long distances!)
may be close enough to observation (?)

<u>note:</u> $D = 10$ just enough to describe most general $g_{\mu\nu}(x)$ (locally)

result:

$(\mathcal{M}, \omega)$ symplectic manifold, $\omega = \frac{1}{2}\theta^{-1}_{\mu\nu} dx^\mu \wedge dx^\nu$
$x^a : \mathcal{M} \hookrightarrow \mathbb{R}^D$ ... embedding in $\mathbb{R}^D$
induced metric $g_{\mu\nu}$ and $G^{\mu\nu}$ as above. Then:

$$\begin{aligned} \{x^a, \{x^b, \varphi\}\}\eta_{ab} &= e^\sigma \Box_G \varphi \\ \nabla^\mu_G (e^\sigma \theta^{-1}_{\mu\nu}) &= G_{\nu\rho}\, \theta^{\rho\mu} \left(e^{-\sigma} \partial_\mu \eta + \partial_\mu x^a \Box_G x^b \eta_{ab} \right) \end{aligned}$$

for $\varphi \in \mathcal{C}^\infty(\mathcal{M})$, ∇_G ... Levi-Civita, $\Box_G$ ... Laplace- Op. w.r.t. $G_{\mu\nu}$, and

$$\eta(x) := \frac{1}{4} e^\sigma\, G^{\mu\nu} g_{\mu\nu}.$$

(H.S., 2008)

in particular:

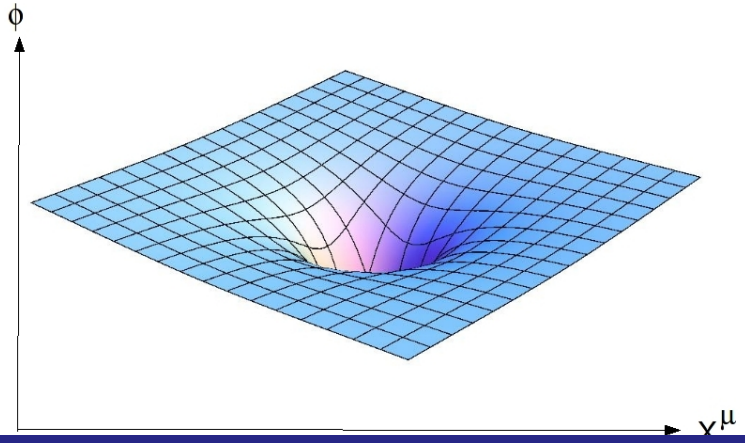
matrix e.o.m: $[X^a, [X^b, X^{a'}]]\eta_{aa'} = 0 \quad \Longleftrightarrow$

$$\boxed{\begin{aligned} \Delta_G \Phi^i &= 0, \quad \Delta_G x^\mu = 0 \\ \nabla^\mu (e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma}\, G_{\rho\nu} \theta^{\rho\mu} \partial_\mu \eta \\ \eta &= \tfrac{1}{4} e^\sigma\, G^{\mu\nu} g_{\mu\nu} \end{aligned}}$$

... covariant formulation in semi-classical limit

in particular:

$\mathcal{M}^4 \hookrightarrow \mathbb{R}^D$ is harmonic embedding (w.r.t. $G_{\mu\nu}$)
minimal surface



dynamics of NC structure $\theta^{\mu\nu}$:

$$S_{YM} = -Tr[X^a, X^b][X^a, X^b] \sim \int d^4x \sqrt{g} e^{-\sigma} \eta$$

Euclidean case: at $p \in \mathcal{M}$, diagonalize $g_{\mu\nu} = (1,1,1,1)$ using $SO(4)$ $\rightarrow$ standard form

$$\theta^{\mu\nu} = \theta \begin{pmatrix} 0 & -\alpha & 0 & 0 \\ \alpha & 0 & 0 & 0 \\ 0 & & 0 & \pm\alpha^{-1} \\ 0 & 0 & \mp\alpha^{-1} & 0 \end{pmatrix} .$$

effective metric $G^{\mu\nu} = (\alpha^2, \alpha^2, \alpha^{-2}, \alpha^{-2})$.
Note

$$\begin{aligned} \tfrac{1}{4} G^{\mu\nu} g_{\mu\nu} &= e^{-\sigma}\eta = \tfrac{1}{2}(\alpha^2 + \alpha^{-2}) \geq 1 \\ \star\omega &= \pm\omega \Leftrightarrow e^{-\sigma}\eta = 1 \Leftrightarrow G_{\mu\nu} = g_{\mu\nu} \Leftrightarrow S_{YM} \text{ minimal} \end{aligned}$$

minimum of S_{YM} $\Leftrightarrow$ $\theta^{\mu\nu}$ (A)SD $\Leftrightarrow$ $G_{\mu\nu} = g_{\mu\nu}$.

special class of solutions:

$$\begin{aligned} g_{\mu\nu} &= G_{\mu\nu}, \\ \Delta_G \phi^i &= 0 \\ \nabla^\mu \theta^{-1}_{\mu\nu} &= 0 \end{aligned}$$

holds for (anti)self-dual symplectic structure $\theta^{-1}_{\mu\nu}$,

$$\begin{aligned} \star(\theta^{-1}) &= \pm\theta^{-1} && \text{Euclidean} \\ \star(\theta^{-1}) &= \pm i\theta^{-1} && \text{Minkowski (Wick rotation } X^0 \to it \text{)} \end{aligned}$$

then

$$S_{MM} \sim Tr[X^a, X^b][X^{a'}, X^{\beta'}] = \int d^4x \sqrt{|g_{\mu\nu}|}$$

... same structure as vacuum energy, “brane tension”.

Dynamics of emergent NC gravity

effective action

$$S = \int d^4x \sqrt{|g|}\,(-2\Lambda^4 + \Lambda_4^2 R) + S_{\text{matter}}$$

leads to

$$\begin{aligned} \delta S &= \int d^4x \sqrt{|g|}\,\delta g_{\mu\nu}(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}) \\ &= -2\int \delta\phi^i \partial_\mu(\sqrt{|g|}\,(-\Lambda^4 g^{\mu\nu} + 8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}))\partial_\nu \phi^i \end{aligned}$$

since $g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \phi^i \partial_\nu \phi^i$

1. "Einstein branch"

$$\Lambda^4 g^{\mu\nu} + \Lambda_4^2 \mathcal{G}^{\mu\nu} = 8\pi T^{\mu\nu}$$

2. "harmonic branch"

$$\Lambda^4 \Box_g \phi = (8\pi T^{\mu\nu} - \Lambda_4^2 \mathcal{G}^{\mu\nu}) \nabla_\mu \partial_\nu \phi$$

prototype: flat space $\mathbb{R}^4_\theta \subset \mathbb{R}^{10}$, even for $\Lambda \gg 0$!

1) harmonic branch

... (NC) minimal surfaces $\mathcal{M} \subset \mathbb{R}^D$, deformed by matter

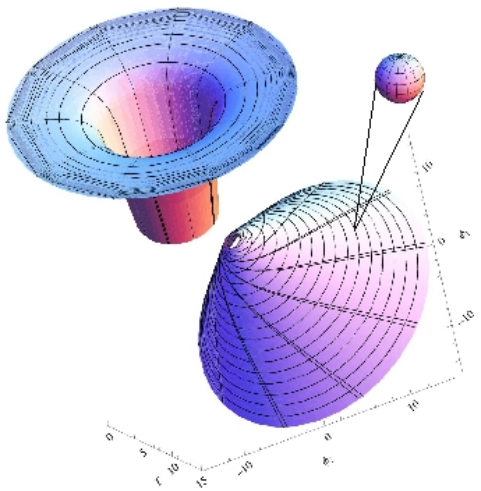
- prototype: flat space $\mathbb{R}^4_\theta \subset \mathbb{R}^{10}$
  insensitive to vacuum energy (minimal surface) !
- interesting "near-realistic " cosmological solution
  (FRW, big bounce) D. Klammer, H.S. arXiv:0903.0986, PRL 102
  compatible with type Ia supernovae without fine-tuning
- matter $\rightarrow$ deformed "gravity bags", Newtonian gravity
  H.S:, arXiv:0909.4621
  (but post-newtonian corrections probably not acceptable)

2) Einstein branch

example: Schwarzschild geometry (Blaschke, H.S. arXiv:1005:0499)

embedding $\mathcal{M} \subset \mathbb{R}^7$, asymptotically flat (harmonic), $e^\sigma \to const$

$$x^a = \begin{pmatrix} t \\ r\cos\sin \\ r\sin\sin \\ r\cos \\ \omega\sqrt{\frac{r_c}{r}}\cos\left(\omega(t+r)\right) \\ \omega\sqrt{\frac{r_c}{r}}\sin\left(\omega(t+r)\right) \\ \omega\sqrt{\frac{r_c}{r}} \end{pmatrix},$$



with $\eta_{ab} = (-,+,+,+,+,+,-)$.

central singularity: embedding $\hookrightarrow \infty$

(presumably modified e.g. via fuzzy sphere ...)

with complexified SD symplectic form

$\star\theta^{-1} = i\theta^{-1}, \qquad \theta^{-1} \to const$ for $r \to \infty$

alternative interpretation of M.M: NC gauge theory

parametrize matrices as fluctuations around $\mathbb{R}^4_\theta$:

$$X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu, \qquad \bar{X}^\mu \text{...Moyal-Weyl}$$

$$\begin{aligned}[X^\mu, X^\nu] &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'}\left(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]\right) + i\bar{\theta}^{\mu\nu} \\ &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'} F_{\mu'\nu'} + i\bar{\theta}^{\mu\nu}\end{aligned}$$

$F_{\mu\nu}(x)$ ... $\mathfrak{u}(1)$ field strength

action:

$$S_{YM} \sim \int d^4x (F_{\mu\nu} + i\bar{\theta}^{-1}_{\mu\nu})(F_{\mu'\nu'} + i\bar{\theta}^{-1}_{\mu'\nu'})\bar{G}^{\mu\mu'}\bar{G}^{\nu\nu'}$$

... NC $U(1)$ gauge theory on $\mathbb{R}^4_\theta$

<u>however:</u>

- $U(1)$ sector does not decouple from $SU(n)$ sector, ...
- one-loop: UV/IR mixing, except in $\mathcal{N} = 4$ SUSY case: finite (!?)

.... understood in interpretation in terms of emergent gravity.

Quantization

Quantization of matrix model:

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]} \;=\; e^{-S_{eff}}$$

2 interpretations:

1. NC SYM on $\mathbb{R}^4_\theta$: UV/IR mixing ($U(1)$ sector only!)
   except for IKKT model ($\mathcal{N}=4$ SUSY, $D=10$): perturb. finite !(?)
2. $U(1)$ absorbed in $\theta^{\mu\nu}(x)$ $\rightarrow$ gravity, induced E-H. action

$$S_{eff} \sim \int d^4x \sqrt{|G|} \, (\Lambda^4 + c\Lambda_4^2 R[G] + ...)$$

($R[G]$ due to UV/IR mixing in NC gauge theory)

- explanation for UV/IR mixing & $U(1)$ entanglement
- $D=10$ required for quantization (maximal SUSY)

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$su(n)$ gauge fields: same model, new vacuum

$$Y^a = \begin{pmatrix} Y^\mu \\ Y^i \end{pmatrix} = \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$

include fluctuations:

$$Y^a = (1 + \mathcal{A}^\rho \partial_\rho) \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n + \Phi^i \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{A}^\mu &= -\theta^{\mu\nu} A_{\nu,\alpha} \otimes \lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{su}(n) \\ \Phi^i &= \Phi^i_\alpha \otimes \lambda^\alpha \end{aligned}$$

$\Rightarrow$ effective action:

$$\boxed{S_{YM} = \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'}\, tr\, F_{\mu\nu} F_{\mu'\nu'} + 2 \int \eta(x)\, tr\, F \wedge F}$$

(H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009))

... $\mathfrak{su}(n)$ Yang-Mills coupled to metric $G^{\mu\nu}(x)$

higher-order terms, curvature

$$
\begin{aligned}
H^{ab} &:= \tfrac{1}{2}[[X^a, X^c], [X^b, X_c]]_+ \\
T^{ab} &:= H^{ab} - \tfrac{1}{4}\eta^{ab} H, \quad H := H^{ab}\eta_{ab} = [X^c, X^d][X_c, X_d]\,, \\
\Box X &:= [X^b, [X_b, X]]
\end{aligned}
$$

result:

for 4-dim. $\mathcal{M} \subset \mathbb{R}^D$ with $g_{\mu\nu} = G_{\mu\nu}$:

$$
Tr\left(2T^{ab}\Box X_a \Box X_b - T^{ab}\Box H_{ab}\right) \sim \frac{2}{(2\pi)^2}\int d^4x\sqrt{g}\, e^{2\sigma} R
$$

$$
Tr([[X^a, X^c], [X_c, X^b]][X_a, X_b] - 2\Box X^a \Box X^a)
$$

$$
\sim \frac{1}{(2\pi)^2}\int d^4x\sqrt{g}\, e^{\sigma}\left(\tfrac{1}{2}e^{-\sigma}\theta^{\mu\eta}\theta^{\rho\alpha}R_{\mu\eta\rho\alpha} - 2R + \partial^\mu\sigma\partial_\mu\sigma\right)
$$

(Blaschke, H.S. arXiv:1003.4132)

(cf. Arnlind, Hoppe, Huisken arXiv:1001.2223)

$\Rightarrow$ Einstein-Hilbert- type action for gravity as matrix model
pre-geometric version of (quantum?) gravity, background indep.!

proof: (assume $g = G$)

$$\begin{aligned} H^{ab} &= \tfrac{1}{2}[[X^a, X^c], [X^b, X_c]]_+ \sim -e^\sigma G^{\mu\nu}\partial_\mu x^a \partial_\nu x^b \overset{g=G}{=} e^\sigma \mathcal{P}_T^{ab}, \\ T^{ab} &= H^{ab} - \tfrac{1}{4}\eta^{ab} H \sim e^\sigma \mathcal{P}_N^{ab} \end{aligned}$$

$\mathcal{P}_N, \mathcal{P}_T$ ... projector on normal / tangential bundle vof $\mathcal{M} \subset \mathbb{R}^D$. note

$$\begin{aligned} R_{\nu\mu\lambda\kappa} &= \mathcal{P}_N^{ab}\left(-\partial_\kappa\partial_\nu x_a \partial_\lambda\partial_\mu x_b + \partial_\kappa\partial_\mu x_a \partial_\nu\partial_\lambda x_b\right) \\ &= -\nabla_\kappa\nabla_\nu x^a \nabla_\lambda\nabla_\mu x_a + \nabla_\kappa\nabla_\mu x^a \nabla_\nu\nabla_\lambda x_a \end{aligned}$$

(i.e. Gauss-Codazzi theorem) and

$$\begin{aligned} T^{bc}[X^a, [X_a, T_{bc}]] &\sim e^{2\sigma}\mathcal{P}_N^{bc}\nabla_\mu\nabla^\mu(e^\sigma\eta_{bc} - e^\sigma\partial^\nu x_b \partial_\nu x_c)) \\ &= e^{2\sigma}\Big((D-4)\Box e^\sigma - 2P_N^{bc}(e^\sigma\nabla^\mu\partial^\nu x_b \nabla_\mu\partial_\nu x_c)\Big) \\ &= e^{2\sigma}\Big((D-4)\Box e^\sigma - 2e^\sigma\nabla^\mu\partial^\nu x^a \nabla_\mu\partial_\nu x_a\Big) \end{aligned}$$

further comments:

- generalization for $g \neq G$:
  still find M.M. terms with purely tensorial meaning $(\mathcal{M}, G_{\mu\nu}, \theta^{\mu\nu})$
  obtain

$$\begin{aligned} S_{10} &= \int \sqrt{g} e^{2\sigma} R[G] + \nabla_G g \nabla_G g + \ldots \\ &\approx \int \sqrt{g} e^{2\sigma} (R[g] + 3R^{\mu\nu} h_{\mu\nu}), \\ h_{\mu\nu} &= -e^{\bar\sigma} (\bar\theta^{-1} g F)_{\mu\nu} - e^{\bar\sigma} (F g \bar\theta^{-1})_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} (\bar\theta F) \\ S_4 = S_{YM} &= \int (\sqrt{g} + (F - \star_g F) \star_g (F - \star_g F)) \\ \theta^{-1}_{\mu\nu} &= \bar\theta^{-1}_{\mu\nu} + F_{\mu\nu}, \qquad \star\bar\theta^{-1} = \pm\bar\theta^{-1} \end{aligned}$$

  ... work in progress (with D. Blaschke)

- probably (!?!) $F_{\mu\nu}$ resp. $\theta^{-1}_{\mu\nu}$ should be integrated out
- $\exists$ "extrinsic" terms $Tr \Box x^a \Box x^a$, depend on embedding $\mathcal{M} \subset \mathbb{R}^D$

  (minimal surfaces preferred ...)

fermions

natural (only?) action

$$\begin{aligned} S[\Psi] &= \mathrm{Tr}\,\overline{\Psi}\gamma_a[X^a,\Psi] \\ &\sim \int d^4x\,\rho(x)\,\overline{\Psi} i\gamma_\mu\theta^{\mu\nu}(x)\partial_\nu\Psi, \qquad \{\gamma_\mu,\gamma_\nu\} = 2G_{\mu\nu} \end{aligned}$$

<u>note</u>:

- naturally SUSY $\rightarrow$ IKKT model
- couple to $G_{\mu\nu}$, but non-standard spin connection (submanifold!)
- quantization induces E-H action plus additional terms

$$\begin{aligned} \Gamma_\Psi &= \tfrac{1}{4\pi^2}\int d^4x\sqrt{|g|}\Big(2\Lambda^4 + \Lambda^2\Big(-\tfrac{1}{3}R[g] + \tfrac{1}{4}\partial_\mu\sigma\partial^\mu\sigma \\ &\quad + \tfrac{1}{8}e^{-\sigma}R[g]_{\mu\nu\rho\sigma}\theta^{\mu\nu}\theta^{\rho\sigma} + \tfrac{1}{4}(\Box_g x^a)(\Box_g x^b)\eta_{ab}\Big) + \mathcal{O}(\log\Lambda)\Big). \end{aligned}$$

- precise matching with UV/IR mixing (checked in $D=4$)

(D. Klammer, H.S., arXiv:0901.2322, arXiv:0909.5298)

Summary and Conclusion

- matrix-model $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$

  dynamical NC spaces $\leftrightarrow$ emergent gravity & gauge thy
- *not* same as G.R., long-distance corrections (extrinsic geometry)
- intriguing cosmological solutions,
  physics of vacuum energy different from GR
- suitable for quantizing gravity !
  (IKKT model, $N = 4$ SUSY in $D = 4$)
- ... more work is needed !

Deformations of Moyal-Weyl plane: gravitons

dynamical X^μ $\Rightarrow$ dynamical $(\theta^{\mu\nu}(x),\, G^{\mu\nu}(x))$

parametrize fluctuations

$$X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu$$

$$\begin{aligned} i\theta^{\mu\nu}(x) &\sim [X^\mu, X^\nu] \\ &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'}\left(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]\right) + i\bar{\theta}^{\mu\nu} \\ &= i\bar{\theta}^{\mu\mu'}\bar{\theta}^{\nu\nu'} F_{\mu'\nu'} + i\bar{\theta}^{\mu\nu} \\ G^{\mu\nu}(x) &= \bar{\eta}^{\mu\nu} - h^{\mu\nu} \qquad (+O(F^2)) \end{aligned}$$

$F_{\mu\nu}(x)$ ... $\mathfrak{u}(1)$ field strength

therefore

$$\boxed{h_{\mu\nu} = \bar{\eta}_{\nu\nu'}\bar{\theta}^{\nu'\rho} F_{\rho\mu} + \bar{\eta}_{\mu\mu'}\bar{\theta}^{\mu'\eta} F_{\eta\nu} - \tfrac{1}{2}\bar{\eta}_{\mu\nu}\left(\bar{\theta}^{\rho\eta} F_{\rho\eta}\right)}$$

... linearized metric fluctuation (cf. Rivelles [hep-th/0212262])

e.o.m for fluctuations of Moyal-Weyl plane (linearized):

$$\begin{aligned} [X^\mu,[X^\nu,X^{\mu'}]]\eta_{\mu\mu'} &= 0 \\ \Rightarrow\ \partial^\mu F_{\mu\nu} &= 0 \\ \Rightarrow\ R_{\mu\nu}[G] &= 0 \quad (\partial^\mu h_{\mu\nu} = 0 \ldots \text{harm. gauge}) \end{aligned}$$

cf. Rivelles [hep-th/0212262]

while $R_{\mu\nu\rho\eta} \neq 0$ ... nonvanishing curvature

$\Rightarrow$ on-shell d.o.f. of gravitational waves on Minkowski space

i.e.: trace-$U(1)$ photons on $\mathbb{R}^4_\theta$ are actually gravitons

NC $U(1)$ does not decouple, couples like graviton

Relation with string theory: branes in background B field

brane $\mathcal{M} \subset \mathbb{R}^{10}$ in B-field background:
DBI action: $S_{DBI} \sim \int d^4x \sqrt{\det(g_{\mu\nu} + (B_{\mu\nu} + F_{\mu\nu}))}$

where $g_{\mu\nu}$ ... closed-string metric (pull-back from bulk)
$G^{\mu\nu} \sim B^{\mu\mu'} B^{\nu\nu'} g_{\mu'\nu'}$ ... open-string metric on $\mathcal{M}$
(∂F neglected...)

here:

- NO 10-D bulk ! fields only live on brane
- $U(1)$ field strength F absorbed in

$$\theta^{-1}_{\mu\nu}(x) = B_{\mu\nu} + F_{\mu\nu}$$

(splitting is unphysical)
eff. metric for nonabelian gauge fields etc. on $\mathcal{M}$:

$$G^{\mu\nu} \sim \theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x) g_{\mu'\nu'}$$

Cosmological solution

D. Klammer, H. S., PRL 102 (2009)

assume: vacuum energy $\Lambda^4 \gg$ energy density ρ

$\Rightarrow$ look for harmonic embedding $\Delta x^a = 0$ of FRW metric

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + \sinh^2(\chi)d\Omega^2),$$

Ansatz

$$x^a(t,\chi,\theta,\varphi) = \begin{pmatrix} a(t)\begin{pmatrix}\cos\psi(t)\\ \sin\psi(t)\end{pmatrix} \otimes \begin{pmatrix}\sinh(\chi)\sin\theta\cos\varphi\\ \sinh(\chi)\sin\theta\sin\varphi\\ \sinh(\chi)\cos\theta\\ \cosh(\chi)\end{pmatrix} \\ 0 \\ x_c(t) \end{pmatrix} \in \mathbb{R}^{10}$$

(cf. B. Nielsen, JGP **4**, (1987))

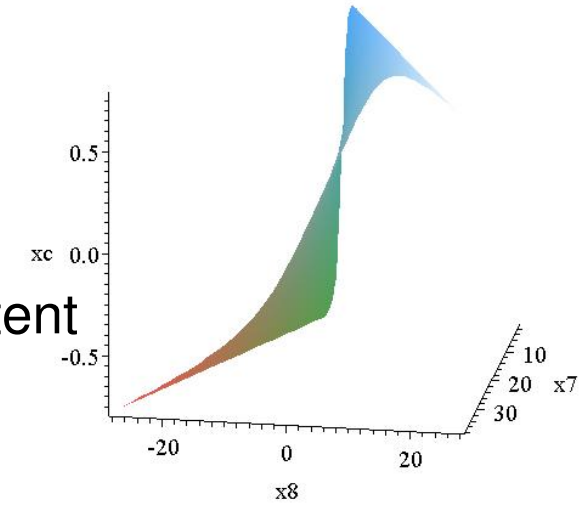
Evolution $a(t), \Psi(t), x_c(t)$ determined by $\Delta x^a = 0$

solution of M.M + leading term $\int d^4x\sqrt{G}\Lambda^4$ in Γ_{1-loop}

harmonic embedding $\Delta_g x^a = 0$ leads to

analog of Friedmann equations

$$\begin{aligned} H^2 = \frac{\dot{a}^2}{a^2} &= -b^2 a^{-10} + d^2 a^{-8} - \frac{k}{a^2}. \\ \frac{\ddot{a}}{a} &= -3d^2 a^{-8} + 4b^2 a^{-10}. \end{aligned}$$



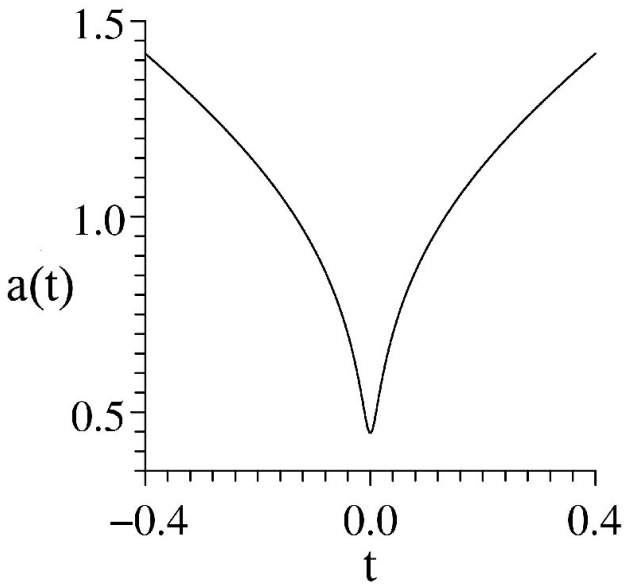
largely independent of detailed matter/energy content as long as $\Lambda^4 \gg \rho$

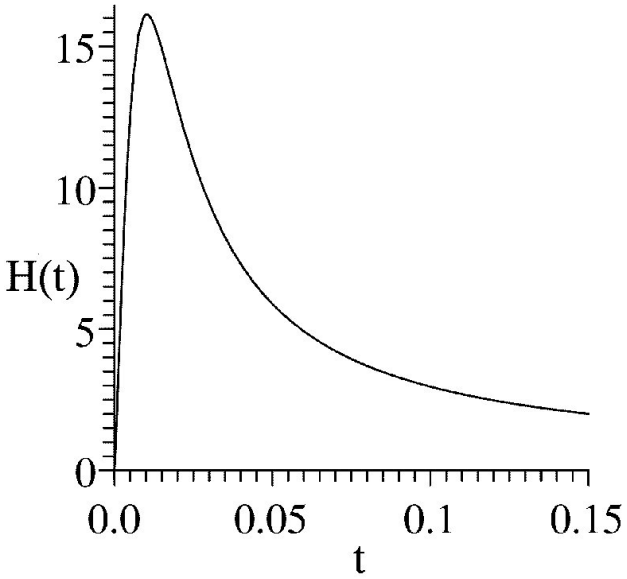
$k = -1$ (negative spatial curvature) most interesting

Implications:

1) early universe:

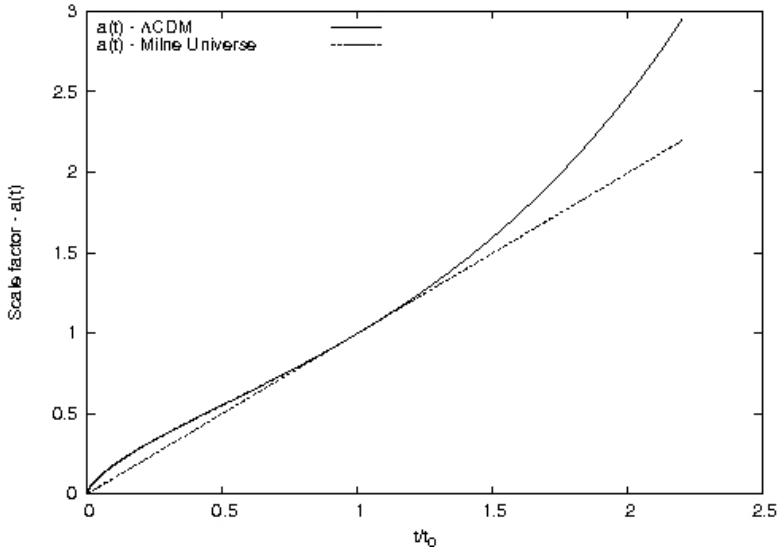
- big bounce: $\dot{a} = 0$ for $a = a_{min} \sim b^{1/4}$
  ($\exists$ bound for energy density ρ vs. vacuum energy Λ^4)
- inflation-like phase $a(t) \sim t^2$, ends at $a(t_{\text{exit}}) = \sqrt{\frac{4}{3}\frac{b}{d}}$
  geometric mechanism (no scalar field required),
  no fine-tuning





2) late evolution (now): $\dot{a} \to 1$

approaches Milne-like universe ($k = -1$, spatial curvature),



in remarkably good agreement with observation
(age $13.8 \cdot 10^9\ yr$, type Ia supernovae)
different physics for early universe (recombination etc.)

A. Benoit-Levy and G. Chardin, [arXiv:0903.2446]

CMB acoustic peak argued to be at correct scale (?)

no fine-tuning of cosm. const., no need for dark energy !