Chiral low-energy physics from squashed branes in deformed $\mathcal{N} = 4$ SYM

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H.S., J. Zahn arXiv:1409.1440, H.S. arXiv:1504.05703

Motivation

- simplicity:
  re-consider softly broken $SU(N)$ $\mathcal{N} = 4$ SYM
  cubic potential $\Rightarrow$ remarkable new vacua, extended SSM
  chiral low energy physics possible for suitable Higgs VEV
- Higgs mechanism $\Rightarrow$ dynamical generation of extra dimensions
  $$\mathbb{R}^4 \times \mathcal{K}_N$$
  Madore, Myers, Arkani-Hamed etal, HS-Zoupanos-Chatzistavrakidis, Aschieri, Manousselis, O'Connor etal, Polchinski-Strassler, Andrews-Dorey,...
- aspects of string theory (intersecting branes, KK modes ...) realized in 4-D gauge theory

outline

- $SU(N)$ $\mathcal{N} = 4$ SYM with cubic (flux) terms
- new vacuum solutions with $SU(3)$ structure
- geometric interpretation:
  self-intersecting fuzzy branes in extra dimensions
  - zero modes & chiral fermions
  - KK modes
- extended SSM from stacks of squashed branes
  mirror particles

$\mathcal{N} = 4$ SYM and squashed fuzzy branes

starting point: $SU(N)$ $\mathcal{N} = 4$ SUSY

$$\boxed{\begin{aligned} S &= \int d^4x \frac{1}{4g^2} tr\Big(- F^{\mu\nu}F_{\mu\nu} - 2D^\mu\Phi^a D_\mu\Phi_a + [\Phi^a,\Phi^b][\Phi_a,\Phi_b]\Big) \\ &\qquad + tr\Big(\bar\Psi\gamma^\mu(i\partial_\mu + [A_\mu,.])\Psi + \bar\Psi\Gamma^a[\Phi_a,\Psi]\Big) \end{aligned}}$$

- $N = 1$ SYM in 10 D dim. red. to 4D
- 6 scalar fields Φ^a, global $SO(6)_R$
- $(\gamma^\mu, \Gamma^a) = \Gamma^A$ ... 10D Clifford generators, $\Psi \to$ 4 Weyl fermions
- most symmetric 4D gauge theory, UV finite

$\mathcal{N}=4$ SYM beautiful but too "round" for physics

more structure:

- spontaneous symmetry breaking (SSB): still no "interesting" (chiral) low-energy physics
- add soft susy breaking terms to potential

scalar potential: $\mathcal{V}[\Phi] = \big(V_4[\Phi] + V_{\rm soft}[\Phi]\big)$,

$$\boxed{\begin{aligned} V_4[\Phi] &= -\tfrac{1}{4}\mathrm{tr}[\Phi_a,\Phi_b][\Phi^a,\Phi^b], \\ V_{\rm soft}[\Phi] &= \mathrm{tr}\big(im\, f_{abc}\Phi^a\Phi^b\Phi^c + M^2_{ab}\Phi^a\Phi^b\big) \end{aligned}}$$

f_{abc} ... tot. antisymm., soft SUSY breaking

- known: $f_{abc} \sim \epsilon_{abc}$: $\rightarrow$ fuzzy sphere solutions $\Phi^a \sim J^a$
- new: $f_{abc} \sim$ truncated $su(3)$ structure constants

  $\rightarrow$ squashed coadjoint $SU(3)$ orbits

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dim.less fields $\Phi_a = mY_a$, label by $\mathfrak{su}(3)$ roots

$$\begin{aligned} Y_1^{\pm} &= \frac{1}{\sqrt{2}}(Y_4 \pm iY_5) \equiv Y_{\pm\alpha_1}, \\ Y_2^{\pm} &= \frac{1}{\sqrt{2}}(Y_6 \mp iY_7) \equiv Y_{\pm\alpha_2}, \\ Y_3^{\pm} &= \frac{1}{\sqrt{2}}(Y_1 \mp iY_2) \equiv Y_{\pm\alpha_3} \end{aligned}$$

let $f_{abc} = c_{abc}|_{a,b,c\neq 3,8}$ ... structure constants of $\mathfrak{su}(3)$ without Cartan

observe

$$\mathrm{Tr}\big(if_{abc} Y^a Y^b Y^c\big) \sim \mathrm{Tr}\big(\varepsilon_{ijk} Y_i^+ Y_j^+ Y_k^+ + h.c.\big)$$

breaks $SO(6)_R$ to $SU(3)_R$

eom:

$$\begin{aligned} 0 &= \big(\Box_4 + m^2 \Box_Y\big) Y_i^+ + 4m^2 \varepsilon_{ijk} Y_j^- Y_k^- \\ \Box_Y &\equiv [Y^a, [Y_a, .]] \end{aligned}$$

squashed $SU(3)$ brane solutions $\mathcal{C}_N[\mu]$

$$Y_{\pm\alpha_i} = r_i\, \pi_\mu(T_{\pm\alpha_i})$$

... root generators of $\mathfrak{su}(3)$, for any rep. $\mathcal{H}_\mu$

symmetries: background $Y_i^\pm = T_i^\pm$

- breaks $SU(3)_R \nrightarrow U(1) \times U(1)$ up to gauge trafo

  residual symmetry: $U(1)_{K_1} \times U(1)_{K_2}$ generated by

$$K_i := \underbrace{2\tau_i}_{\in SU(3)_R} - \underbrace{[H_{\alpha_i}, .]}_{\text{gauge}}$$

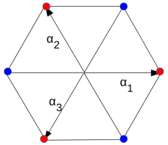
- $SU(N)$ gauge symmetry completely broken (Higgs)

  $\rightarrow$ massive KK modes!

organization:

via $SU(3)_Y \subset SU(N)$ generated by $Y_i^\pm \sim T_{\pm\alpha_i}$ (not symmetry!)

scalar fields transform as $\begin{cases} (3) & \text{under } SU(3)_R \\ (8) - 2 & \text{under } SU(3)_Y \end{cases}$



fluctuation & KK modes

determine fluctuation modes & masses on new vacua

all fields (scalar, gauge fields, fermions) take values in

$$\mathfrak{u}(N) = Mat(\mathcal{H}_\mu) \cong \mathcal{H}_\mu \otimes \mathcal{H}_\mu^* = \oplus_\Lambda \mathcal{H}_\Lambda \quad \text{of} \quad SU(3)_Y$$

expand into $SU(3)_Y$ harmonics:

gauge fields:

$$A_\mu(x) = \sum A_\mu^{(M,\Lambda)}(x) \underbrace{\hat{Y}_M^\Lambda}_{SU(3)_Y \text{modes}}$$

scalar fields:

$$\Phi_a = mY^a + \varphi^a(x)$$
$$\varphi_a(x) = \sum \varphi_a^{(M,\Lambda)}(x) \hat{Y}_M^\Lambda$$

fermions ... (similar)

massive gauge bosons as KK modes

Higgs effect: gauge modes $A_\mu(x) = \sum A_\mu^{(M,\Lambda)}(x)\,\hat{Y}_M^\Lambda$ acquire mass

$$\int \mathrm{tr}(D_\mu Y_a)^\dagger D_\mu Y_a = \int \mathrm{tr}(\partial_\mu Y_a^\dagger \partial_\mu Y_a + \sum_{\Lambda,M} m^2_{\Lambda,M}\, A^\dagger_{\mu,(\Lambda M)} A^\mu_{(\Lambda,M)}) + S_{int}$$

given by

$$\Box_Y \hat{Y}_M^\Lambda \equiv [Y^a,[Y_a,\hat{Y}_M^\Lambda]] = m^2_{(\Lambda M)}\,\hat{Y}_M^\Lambda$$

$\Rightarrow$ tower of massive KK modes $A_{\mu,(\Lambda M)}(x)$, mass $m^2_{(\Lambda M)} > 0$

(no massless gauge modes)

geometric interpretation of Higgs effect

massive gauge modes = KK modes on squashed brane $\mathcal{C}_N[\mu]$,

$\rightarrow$ effect. gauge theory on $\mathbb{R}^4 \times \mathcal{C}_N[\mu]$

similar for scalar fields & fermions

internal geometry: squashed fuzzy coadjoint orbits

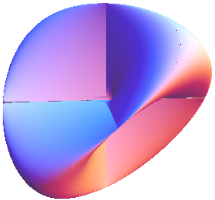
claim: $Y_a = \pi_\mu(T_a)$ ... quantized coadjoint $SU(3)$ orbit,
projected along Cartan directions

$$\begin{array}{cccc} \mathcal{C}[\mu] \hookrightarrow \mathbb{R}^8 & \stackrel{\Pi}{\to} & \mathbb{R}^6 \\ (y^a)_{a=1,\ldots,8} & \mapsto & (y^a)_{a=1,2,4,5,6,7} \end{array}$$

4- or 6-dimensional variety in $\mathbb{R}^6$, self-intersecting

H.S., J. Zahn arxiv:1409.1440

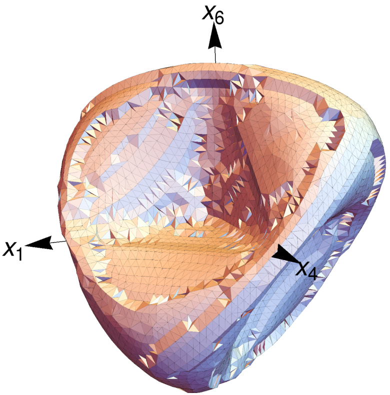
4-dim branes = squashed $\mathbb{C}P^2$:



triple self-intersection at origin

$\Rightarrow$ zero-modes: strings linking sheets at origin

computer measurement of $Y^a = \pi_\mu(T^a)$:



squashed $\mathcal{C}_N[\mu]$ for $\mu = (20, 0)$.

L. Schneiderbauer, H.S. (unpublished)

scalar fluctuation modes

scalar fluctuations: $\phi_\alpha = mY_\alpha + \varphi_\alpha$

quadratic potential for φ_α:

$$
\begin{aligned}
V_2[\varphi] &= \mathrm{tr}\varphi^\alpha \left(\Box_Y + 2\not{D}_{\mathrm{diag}} - 2\not{D}_{\mathrm{mix}}\right)\varphi_\alpha, \\
(\not{D}_{\mathrm{mix}}\varphi)_i^+ &= -\varepsilon_{ikj}[Y_k^-, \varphi_j^-], \qquad \tau\not{D}_{\mathrm{mix}} = -\not{D}_{\mathrm{mix}}\tau
\end{aligned}
$$

can show:

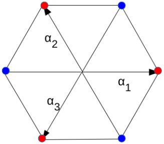
- no negative modes (!!)
- $\exists$ zero modes: regular & exceptional

H.S., J. Zahn arxiv:1409.1440

- regular zero modes: corresponding to $\not{D}_{\rm mix}\phi_\alpha^{(0)} = 0$

  6 zero modes for each $\mathcal{H}_\Lambda \subset Mat(\mathcal{H}_\mu)$

  (corresp. to 6 extremal charges of $U(1)_{K_i}$ in $\mathcal{H}_\Lambda \otimes (8)$)



  geometric interpretation:

  string between coincident sheets of $\mathcal{C}[\mu]$ at origin

  e.g. $\varphi_\Lambda = |\Omega\mu\rangle\langle\mu|$, $|\mu\rangle$ ... coherent states

- exceptional zero modes:

  in particular: 6 Goldstone bosons (from $SU(3)/_{U(1)\times U(1)}$)

(verified numerically)

fermions

Dirac operator on squashed $\mathcal{C}_{\mathcal{N}}[\mu]$:

$$\not{D}_{(int)}\Psi \quad = 2\sum_{i=1}^{3}\Big(\gamma_i[Y_i^+, .] + \gamma_i^\dagger[Y_1^-, .]\Big)$$

$\{\gamma_i, \gamma_j^\dagger\} = \delta_{ij}$ ... $SO(6)$ Clifford algebra

zero modes:

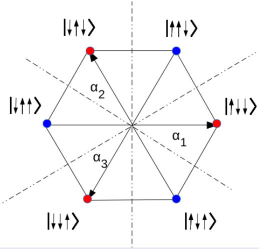
$$\not{D}_{(int)}\Psi_\Lambda = 0$$

- have distinct chirality determined by quantum numbers Λ (!)
- in one-to-one correspondence to regular zero modes

  (fermionic strings connecting branes)
- except for exceptional zero modes (SUSY broken)

chiral fermions

for each $\mathcal{H}_\Lambda \subset Mat(\mathcal{H}_\mu)$:



3 generations of Weyl/ Majorana spinors on $\mathbb{R}^4$ (Weyl rotations $\frac{2\pi}{3}$)

chirality $\gamma_5 \Psi_{\Lambda'} = (+-)\Psi_{\Lambda'}$ determined by $U(1)_{K_i}$ charges

$$\Psi_{-\Lambda'} = C\Psi_{\Lambda'}$$

towards interesting physics:

stacks of squashed $\mathcal{D}_i = \mathcal{C}[\mu_i]$ branes (=reducible rep.'s of $\mathfrak{su}(3)$)

$$Y^a = \begin{pmatrix} Y^a_{\mu_1} & \\ & Y^a_{\mu_2} \end{pmatrix} \quad \cong \quad \begin{pmatrix} \mathcal{D}_1 & \\ & \mathcal{D}_2 \end{pmatrix}$$

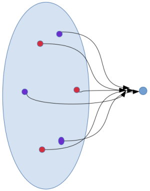
off-diagonal fermions

$$\Psi = \begin{pmatrix} 0 & \Psi_{12} \\ \Psi_{21} & 0 \end{pmatrix}$$

zero-modes in bi-fundamental $\mathcal{H}_1 \otimes \mathcal{H}_2^*$ of $U(N_1) \times U(N_2)$

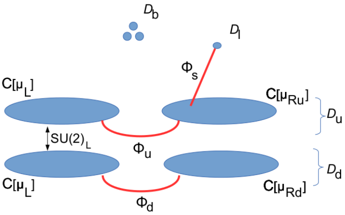
e.g. quarks = links $\mathcal{C}[\mu]$ with 3 "baryonic" point-branes:

quarks: $\Psi_{12} = |\mu_i\rangle_1 \langle 0_j|_2 \in \mathcal{H}_\mu \otimes \mathbb{C}^3$



3+3 chiral zero-modes attached to $\mathcal{C}[\mu]$

A standard-model-like brane configuration



gauge symmetry $U(2)_L \times U(1)_{Ru} \times U(1)_{Rd} \times U(1)_l \times U(3)_c$

assume Higgs $\phi_u, \phi_d, \phi_S \neq 0$ connecting branes (zero modes!)

unbroken symm: $SU(3)_c \times U(1)_Q \times U(1)_B$

$$Q = \frac{1}{2}(\mathbb{1}_{Ru} + \mathbb{1}_{Lu} - \mathbb{1}_{Rd} - \mathbb{1}_{Ld} + \mathbb{1}_l - \frac{1}{3}\mathbb{1}_b)$$

mass $O(\phi)$: $SU(2)_L \times U(1)_Y$

"anomalous" at low energy ($\to$ massive): $U(1)_5,\ U(1)_B$

fermions with SM quantum numbers arise as zero modes linking branes

$$\Psi = \begin{pmatrix} *_2 & H_d & H_u & l_L & Q_L \\ & * & e' & e_R & d_R \\ & & * & \nu_R & u_R \\ & & & * & u' \\ & & & & *_3 \end{pmatrix},$$

$$(Q,Y) = \begin{pmatrix} * & \begin{pmatrix}(1,1)\\(0,1)\end{pmatrix} & \begin{pmatrix}(0,-1)\\(-1,-1)\end{pmatrix} & \begin{pmatrix}(0,-1)\\(-1,-1)\end{pmatrix} & \begin{pmatrix}(\frac{2}{3},\frac{1}{3})\\(-\frac{1}{3},\frac{1}{3})\end{pmatrix} \\ & * & (-1,-2) & (-1,-2) & (-\frac{1}{3},-\frac{2}{3}) \\ & & * & (0,0) & (\frac{2}{3},\frac{4}{3}) \\ & & & * & (\frac{2}{3},\frac{4}{3}) \\ & & & & * \end{pmatrix}$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},$$

- 3 generations ($\mathbb{Z}_3$ Weyl rotations!)
- all standard model fermions, 2 Higgs doublets, + superpartners
- extra fermions $\nu_R, u', e', \lambda \sim U(1)_L$ etc., no exotic charges

chirality: index =0 in $\mathcal{N} = 4$ SYM !

but:

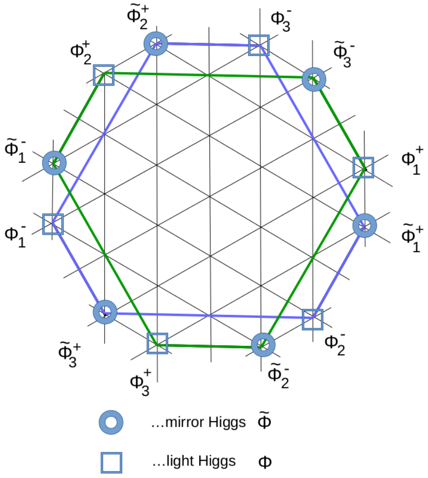
- rich Higgs sector (=zero modes)
  cubic potential $\rightarrow$ SSB
  H.S. arXiv:1504.05703
- $\exists$ suitable Higgs VEV's such that

$$\Psi = \left\{ \begin{array}{lll} \text{light fermions,} & \text{chirality of S.M.} & \text{light} \\ \text{mirror fermions,} & \text{opposite chirality} & \text{heavy} \end{array} \right\}$$

  protected by unbroken $U(1)_{K_i}$
- mirror fermions couple to mirror Higgs,
  $\rightarrow$ acquire larger masses
- ν_R (light & mirror)

can obtain extended chiral S.M. at low energy,
+ heavier mirror sector

separation of Higgs components into light & mirror:



separated by cubic potential

summary

- $\exists$ rich class of vacuum solutions of deformed $SU(N)$ $\mathcal{N} = 4$ SYM
  $\rightarrow$ self-intersecting extra dim $\mathbb{R}^4 \times \mathcal{C}[\mu]$
- chiral low-energy physics possible, + mirror sector at high scale
- can be surprisingly similar to standard model in broken phase
- open issues:
  elaborate rich Higgs sector (zero modes)
  sufficient hierarchy for mirror sector?
  quantum corrections

(soft) SUSY breaking:

action is *almost* $\mathcal{N} = 1^*$ deformation of $\mathcal{N} = 4$ SYM:
consider superpotential

$$W = \frac{\sqrt{2}}{g}\mathrm{tr}([\Phi_1^+, \Phi_2^+]\Phi_3^- - m\Phi_3^-\Phi_3^-) \qquad ...\mathcal{N} = 1^*$$

(declare $\Phi_1^+\Phi_2^+, \Phi_3^-$ as holomorphic coords)

$\rightarrow$ effective potential

$$\begin{aligned} V(\Phi) &= -\tfrac{1}{4g^2}\mathrm{tr}[\Phi^\alpha, \Phi^\beta][\Phi_\alpha, \Phi_\beta] \\ &+4\tfrac{1}{g^2}\mathrm{tr}\big(-m[\Phi_1^+, \Phi_2^+]\Phi_3^+ - m[\Phi_2^-, \Phi_1^-]\Phi_3^- + 2m^2\Phi_3^+\Phi_3^-\big). \end{aligned}$$

$\equiv$ present potential,
however, mass $M_3^2 = 2m^2$ too large for squashed $SU(3)$ brane solutions