An extended standard model and its Higgs geometry from the IKKT model

Harold Steinacker

Department of Physics, University of Vienna

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H.S., J. Zahn, arXiv:1401.2020

Motivation

- need quantum theory of fundamental interactions incl. gravity
  Higgs ? cosmological constant? dark ... (whatever)?
- too many string theory compactifications, “vacua”
  $\rightarrow$ lack of predictivity
- here: matrix model approach:
  constructive, predictive, non-perturbative

  $\rightarrow$ S.M. ?

Matrix Models as fundamental theory?

- 1996: BFSS model, IKTT model proposed as
  non-perturbative definition of M-theory / IIB string theory
  focus on IKKT: Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[X] = Tr\Big([X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} + \bar\Psi\Gamma_a[X^a, \Psi]\Big)$$

$$X^a = X^{a\dagger} \in Mat(N, \mathbb{C}), \qquad a = 0, ..., 9$$

- describes (dynamical, non-commutative) branes in $\mathbb{R}^{10}$
- simple, constructive, predictive
  "predicts" $3+1$ dim !! Kim, Nishimura, Tsuchiya 2012
- IKKT as starting point, extract resulting physics
  here: towards the standard model

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basic objects: branes $(\to$ brane-worlds)

e.o.m.: $\delta S = 0 \;\Rightarrow\; [X_a, [X^a, X^b]] = 0$

basic solutions:

- flat $D(2n-1)$ branes $\mathbb{R}^{2n}_\theta$ embedded in $\mathbb{R}^{10}$

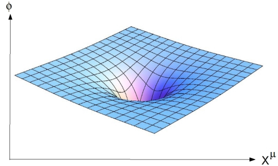
$$X^a = \begin{pmatrix} X^\mu \\ 0 \end{pmatrix}$$

$$[X^\mu, X^\nu] = i\theta^{\mu\nu}\,\mathbf{1}$$

"quantum plane"

- generic (curved) $D(2n-1)$ branes

$$X^a = \begin{pmatrix} X^\mu \\ \phi^i(X^\mu) \end{pmatrix}$$



non-commutative, B-field $\theta^{ab} \;\to\; \theta^{ab}(x)$

- *D*-branes in IIB string theory recovered

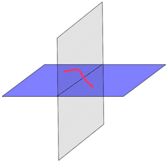
  interaction in SUGRA recovered at one loop

  (IKKT, BFSS, Kabat-Taylor,...)

- multiple branes: $X^a = \begin{pmatrix} X^a_{(1)} & 0 \\ 0 & X^a_{(2)} \end{pmatrix}$

- intersecting branes, stacks

$$X^a = \begin{pmatrix} X^a_{(11)} & \phi_{(12)} \\ \phi_{(21)} & X^a_{(22)} \end{pmatrix}$$

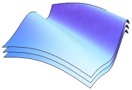


Aoki, Banks, Chepelev, Fischler, Iso, Kawai, Kitazawa, Kimura, Nair, Raamsdonk, Randjibar-Daemi, Shenker, Susskind, Zarembo, ...

stack of coincident branes $\to$ $su(N)$ gauge theory

background

$$X^a = \begin{pmatrix} \bar{X}^\mu \otimes \mathbf{1}_N \\ \bar{\phi}^i \otimes \mathbf{1}_N \end{pmatrix}$$



include fluctuations:

$$X^a = \begin{pmatrix} \bar{X}^\mu \otimes \mathbf{1}_N + \mathcal{A}^\mu \\ \bar{\phi}^i \otimes \mathbf{1}_N + \Phi^i \end{pmatrix}$$

write $\mathcal{A}^\mu = \theta^{\mu\nu} A_\nu$, note $[\bar{X}^\mu, f] \sim i\theta^{\mu\nu}\partial_\nu f$

$$\begin{aligned} [X^\mu, X^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'}(\partial_{\mu'}A_{\nu'} - \partial_{\nu'}A_{\mu'} + [A_{\mu'}, A_{\nu'}]) \\ &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} F_{\mu'\nu'} \qquad \text{field strength} \end{aligned}$$

$\Rightarrow$ effective action on $\mathbb{R}^4_\theta$:

$$
\begin{aligned}
S &= \Lambda_0^4\, Tr\Big([X^a,X^b][X_a,X_b] + \bar\Psi\Gamma_a[X^a,\Psi]\Big) \\
&= \int d^4x\,\sqrt{G}\, tr_N\Big(\tfrac{1}{4g^2}(\mathcal{F}\mathcal{F})_G + \tfrac{1}{2}G^{\mu\nu}D_\mu\Phi^i D_\nu\Phi_i - \tfrac{1}{4}g^2[\Phi^i,\Phi^j][\Phi_i,\Phi_j] \\
&\qquad + \bar\psi\tilde\gamma^\mu(i\partial_\mu + [\mathcal{A}_\mu,.])\psi + g\bar\psi\Gamma^i[\Phi_i,\psi]\Big)
\end{aligned}
$$

where

$$
\begin{aligned}
G^{\mu\nu} &= \rho\theta^{\mu\nu'}\theta^{\nu\nu'}g_{\mu'\nu'}, \qquad \rho = \sqrt{|\theta^{-1}|} \\
\tilde\gamma^\mu &= \rho^{1/2}\,\theta^{\nu\mu}\gamma_\nu, \\
\frac{1}{4g^2} &= \frac{\Lambda_0^4}{(2\pi)^2}\rho^{-1}
\end{aligned}
$$

IKKT on stack of branes $\rightarrow$ $SU(n)$ $\mathcal{N}=4$ SYM coupled to $G^{\mu\nu}$

holds also for curved branes, $U(1)_{\rm tr} \rightarrow$ dynamical $G^{\mu\nu}$

H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009) this talk:

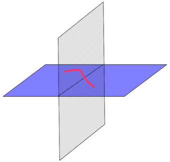
... use language of $SU(N)$ $\mathcal{N}=4$ SYM for sufficiently large N

(ignore gravity, quantum space-time)

- find explicit brane solution which breaks
  $SU(N) \rightsquigarrow SU(3)_c \times U(1)_Q \times U(1)_B$
- resembles S.M. at low energies:
  - correct matter content of S.M. (2 generations ...) + ν_R
    coupled to $SU(3)_c \times SU(2)_L \times U(1)_Y$
  - electroweak SSB $SU(2)_L \times U(1)_Y \rightsquigarrow U(1)_Q$ via
    two Higgs doublets,
    intrinsic part of geometry (minimal fuzzy spheres),
    essential for chiral nature of fermions
- mirror fermions at intermediate energies (above m_W),
  decay to S.M. fermions via new heavy gauge bosons
- singlet Higgs breaks $SU(2)_R$, may induce Majorana mass for ν_R
- gauginos, towers of massive KK modes
  ultimately completing $\mathcal{N}=4$ SUSY

chiral fermions on intersecting NC branes

A. Chatzistavrakidis, H.S., G. Zoupanos (2011)



$$\Phi^i = \begin{pmatrix} \Phi^i_{(1)} & \\ & \Phi^i_{(2)} \end{pmatrix}, \qquad \Psi = \begin{pmatrix} & \Psi_{(12)} \\ \Psi_{(21)} & \end{pmatrix}$$

M.M. Dirac operator on $\mathbb{R}^2_\theta \cap \mathbb{R}^2_\theta$

$$\begin{aligned} \not{D}_{\rm int}\Psi_{(12)} &= \Gamma_i[\Phi^i,\Psi_{(12)}] = \Gamma_i(\Phi^i_{(1)}\Psi_{(12)} - \Psi_{(12)}\Phi^i_{(2)}) \\ &= \not{D}_{(1)}\Psi_{(12)} - \not{D}_{(2)}\Psi_{(12)} \end{aligned}$$

use oscillator basis for (noncommutative!) branes

$$\begin{aligned} a &= \Phi^4 - i\Phi^5, \qquad b = \Phi^6 - i\Phi^7, \\ \alpha &= \tfrac{1}{2}(\Gamma^4 + i\Gamma^5), \quad \beta = \tfrac{1}{2}(\Gamma^6 + i\Gamma^7) \\ \not{D}_{(1)}\Psi &= (\alpha a^\dagger + \alpha^\dagger a)\Psi \\ \not{D}_{(2)}\Psi &= \beta\Psi b^\dagger + \beta^\dagger\Psi b \end{aligned}$$

$$\not{D}_{\rm int}\Psi_{(12)} = 0 \quad \Leftrightarrow \quad \Psi_{(12)} = |0,\downarrow\rangle_{(1)}\langle 0,\uparrow|_{(2)}$$

chiral zero mode in $\mathbb{R}^2 \times \mathbb{R}^2$

localized at intersection (coherent state)

towards the standard model

- consider intersecting branes $\mathbb{R}^4 \times \mathcal{K}_i \subset \mathbb{R}^{10}$

  $\mathcal{K}_i$... fuzzy spaces (=quantized compact spaces)

  e.g. $S^2_N,\ T^2_N,\ S^2_N \times S^2_N$ ...

  $\rightarrow$ chiral fermions localized at $\mathcal{K}_i \cap \mathcal{K}_j$, propagate on $\mathbb{R}^4$

- stacks of n_i branes $\rightarrow$ $SU(n_i)$ gauge fields

  fermions $\Psi_{(12)}$ in $(n_1) \otimes (\bar{n}_2)$

standard model fields embedded in adjoint of $SU(N)$:

A. Chatzistavrakidis, H.S., G. Zoupanos (2011)

$$\Psi = \begin{pmatrix} 0_2 & 0 & 0 & l_L & Q_L \\ & 0 & \multicolumn{2}{c}{\begin{pmatrix} 0 & e_R \\ 0 & \nu_R \end{pmatrix}} & Q_R \\ & & & 0 & 0 \\ & & & & 0_3 \end{pmatrix},$$

where

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \qquad Q_R = \begin{pmatrix} d_R \\ u_R \end{pmatrix}$$

realized by 5 types of branes:

- $\mathcal{D}_l$ ”leptonic“ brane
- $3 \otimes \mathcal{D}_B$ “baryonic” branes $\rightarrow$ $SU(3)_c$
- $2 \otimes \mathcal{D}_w$ “electroweak“ branes $\rightarrow$ $SU(2)_L$
- $1 + 1$ ”right-handed“ branes $\mathcal{D}_a, \mathcal{D}_b$

(cf. intersecting brane models in string theory)

gauge fields: adjoint action $Q\Psi_{(12)} = [t_Q, \Psi_{(12)}]$ etc.,
e.g.

$$t_Q = \frac{1}{2}\begin{pmatrix} \sigma_3 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -\frac{1}{3} \end{pmatrix}, \qquad t_Y = \begin{pmatrix} 0_2 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & -\frac{1}{3} \end{pmatrix}.$$

problems:

- compact $\mathcal{K}_i \cap \mathcal{K}_j \;\to\;$ pairs of intersections (zero index of $\not{D}_{\rm int}$)
  $\to\;$ additional chiral fermions besides S.M. fermions
- need EW Higgs:

$$\Phi^a_{(H)} = \begin{pmatrix} 0_2 & H_d & H_u & 0 & 0 \\ & 0 & 0 & 0 & 0 \\ & & 0 & S & 0 \\ & & & 0 & 0 \\ & & & & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \phi_u & 0 & 0 \\ 0 & 0 & \phi_d & 0 & 0 & 0 \\ 0 & \phi_d^\dagger & 0 & 0 & 0 & 0 \\ {\phi_u}^\dagger & 0 & 0 & 0 & S & 0 \\ 0 & 0 & 0 & S^\dagger & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

S ... sterile Higgs

solution:

Higgs connects 2 patches into one brane
... intrinsic part of fuzzy internal geometry

other proposal: warped extra dimensions
Nishimura, Tsuchiya 2013, Aoki Nishimura Tsuchiya 2014

deconstructing compact branes

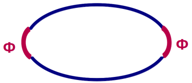
consider block-config. with off-diagonal Higgs

$$\Phi^i = \begin{pmatrix} \Phi^1_{(1)} & 0 \\ 0 & \Phi^1_{(2)} \end{pmatrix} + \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix}$$

acting irreducibly on

$$\mathcal{H} = \mathcal{H}_{(1)} \oplus \mathcal{H}_{(2)}$$

joining two branes $\mathcal{D}_{(1)}$ and $\mathcal{D}_{(2)}$



Higgs as glue for compact branes from patches

example: fuzzy sphere S^2_N by glueing two disks with Higgs

Example: the fuzzy sphere S^2_N

fuzzy sphere S^2_N : (Madore, Hoppe)

$$\boxed{\begin{aligned} [X^i, X^j] &= R i \varepsilon^{ijk} X^k \ , \\ X^i X^i &= \tfrac{R^2}{4}(N^2 - 1) \end{aligned}}$$

... quantization of S^2, N quantum cells

Poisson structure $\{x^i, x^j\} = \frac{2R}{N}\, \varepsilon^{ijk}\, x^k$

$$\boxed{X^i := R J^i_{(N)} \quad \ldots \quad N-\text{dim irrep of } \mathfrak{su}(2) \text{on } \mathbb{C}^N}$$

minimal fuzzy ellipsoid: $N = 2$

$$X_4 \pm i X_5 = \phi \sigma_\pm = \begin{pmatrix} 0 & \phi \\ \phi^\dagger & 0 \end{pmatrix}, \qquad X_6 = r \sigma_3 = \begin{pmatrix} r & \\ & -r \end{pmatrix}$$

Example: the fuzzy sphere S^2_N

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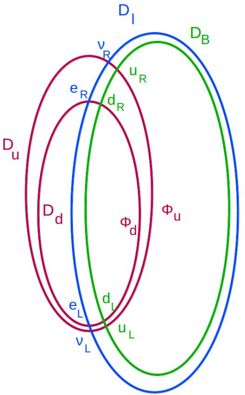
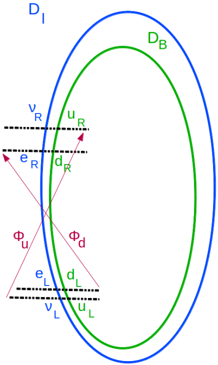
join $\begin{array}{ccc} \mathcal{D}_a \cup \mathcal{D}_w & \to & \mathcal{D}_u \\ \mathcal{D}_b \cup \mathcal{D}_{w'} & \to & \mathcal{D}_d \end{array}$ ... deconstructed minimal ellipsoids

$\mathcal{D}_u, \mathcal{D}_d$ intersecting with $\mathcal{D}_l, \mathcal{D}_B$ $\approx$ standard model





- obtain precisely chiral matter content of standard model + ν_R (one generation)
- $SU(2)_L$ at coinciding ”south poles“ of S^2_2 in $\mathcal{D}_u, \mathcal{D}_d$ broken by Higgs ϕ

intersecting brane solutions

branes interact (1-loop $\to \approx$ SUGRA, typically attraction)

$\to$ deform model by $SO(6)$-invariant potential

$$S \to S - \rho \int d^4x\, V_{\rm quant},$$

$$V_{\rm quant} = f(tr_N \sum_{i=4}^{9} X_i X^i) \stackrel{e.g.}{=} -m^2 tr(X_i X^i) + \lambda (tr X_i X^i)^2$$

(note: $\mathbb{R}^4_\theta$ has scale Λ_{NC} !)

<u>e.o.m.</u>

$$\Box X^i = -(2\pi g \rho^{-\frac{1}{2}} f')\, X^i, \qquad \Box = [X^j, [X_j, .]]$$

(similarly for branes rotating in $45, 67, 89$ directions,

cf. IR regularization Kim Nishimura (2012))

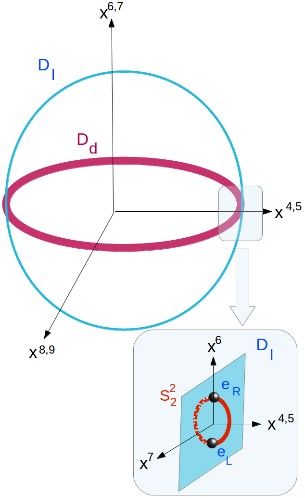
intersecting brane solutions

$\mathcal{D}_u \sim \mathcal{D}_d$:

$$\Phi^a_{(u)} = \begin{pmatrix} R'_u L_3 + \phi_u \sigma'_1 \\ \phi_u \sigma'_2 \\ r_u \sigma'_3 \\ 0 \\ R_u L_1 \\ R_u L_2 \end{pmatrix} \cong S^2_{N_u} \tilde{\times} S^2_2$$

$\mathcal{D}_l \sim \mathcal{D}_B$:

$$\Phi^a_{(l)} = \begin{pmatrix} R'_l K_3 \\ 0 \\ R_l K_1 \\ R_l K_2 \\ 0 \\ 0 \end{pmatrix} \cong S^2_{N_l}$$



solution of eom if

$$R_u = R'_u = R_l = R'_l = r_u = \phi_u = \sqrt{-\pi g \rho^{-1/2} f'}$$

intersections:

assume $N \gg 1$:
2 intersection regions (2 generations!) near $\pm N_l R_2'(1,0,0,0,0,0)$

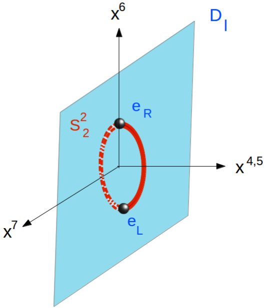
flat limit $S^2_N \to \mathbb{R}^2_\theta$ near intersections:

replace $\mathcal{D}_u \to \mathbb{R}^2_\theta \tilde{\times} S^2$ and $\mathcal{D}_l \to \mathbb{R}^2_\theta$

fermions on intersection $S^2_2 \cap \mathbb{R}^2_\theta$:

$$\Phi^i = \begin{pmatrix} \Phi^i_{(1)} & \\ & \Phi^i_{(2)} \end{pmatrix}, \quad \Psi = \begin{pmatrix} & \Psi_{(12)} \\ \Psi_{(21)} & \end{pmatrix}$$

with $\Phi^i_{(1)} = \begin{pmatrix} \phi\sigma_1 \\ \phi\sigma_2 \\ r\sigma_3 \\ 0 \end{pmatrix}, \quad \Phi^i_{(2)} = \begin{pmatrix} 0 \\ 0 \\ y^6 \\ y^7 \end{pmatrix}$



1. no Higgs $\phi = 0$: 2 points at $x^6 = \pm r$
   pair of zero modes, both chiralities, at each location
2. switch on Higgs $\phi \neq 0$:
   one chiral zero mode localized at each intersection $x^6 = \pm r$
   (coherent states)
   $$e_R = |+,\downarrow\rangle_{(1)}\langle +r,\uparrow|_{(2)}, \quad e_L = |-,\uparrow\rangle_{(1)}\langle -r,\uparrow|_{(2)}$$
   massive mirror fermion at each intersection, mass $m \sim \phi$
   $$\tilde{e}_L = |+,\uparrow\rangle_{(1)}\langle +r,\uparrow|_{(2)}, \quad \tilde{e}_R = |-,\downarrow\rangle_{(1)}\langle -r,\uparrow|_{(2)}$$

on $S^2 \times \mathbb{R}^2 \cap \mathbb{R}^2$:

- exact chiral zero modes, e.g. $\Psi_{(12)} = |+0,\uparrow\downarrow\rangle_{(1)}\langle +r,\downarrow|_{(2)}$
- massive mirror fermions, e.g. $\tilde{\Psi}_{(12)} = |+0,\downarrow\downarrow\rangle_{(1)}\langle +r,\downarrow|_{(2)}$,
  mass $m \sim \phi$
  (opposite chirality on S^2_2, same localization)

on $S^2_N \tilde{\times} S^2_2 \cap S^2_N$:

expect pairs of near-zero eigenmodes of $\not{D}_{\rm int}$
consisting of nearly-localized chiral states

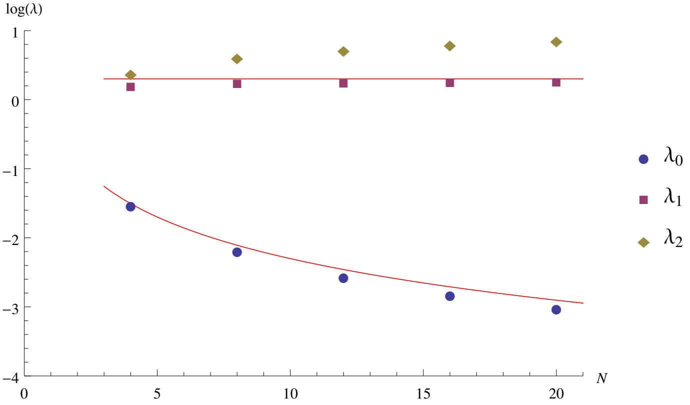
gives Yukawa coupling

$$tr_N \Psi^*_{-R}\gamma_0\gamma_5 \not{D}_{\rm int}\Psi_{+L} \approx \phi f_\psi$$

spin non-alignment at intersections $\rightarrow$ expect

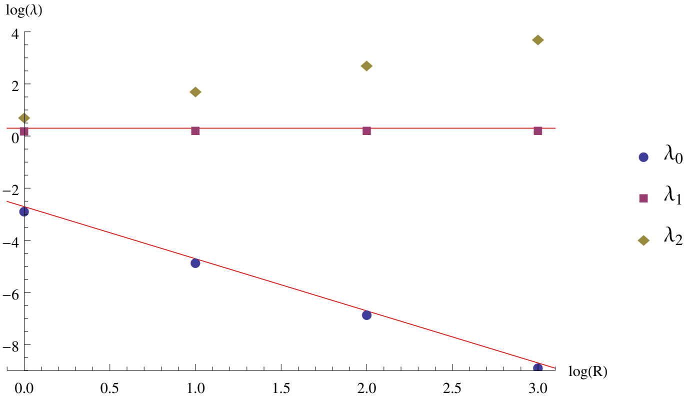
$$f_\psi \approx 4\frac{r^2}{N_l^2 R_l^2}$$

numerical results: lowest eigenvalues of $\not{D}_{\rm int}$ (= Yukawas)
for $N_i = N, R_i = 1, r = \phi = 1$:

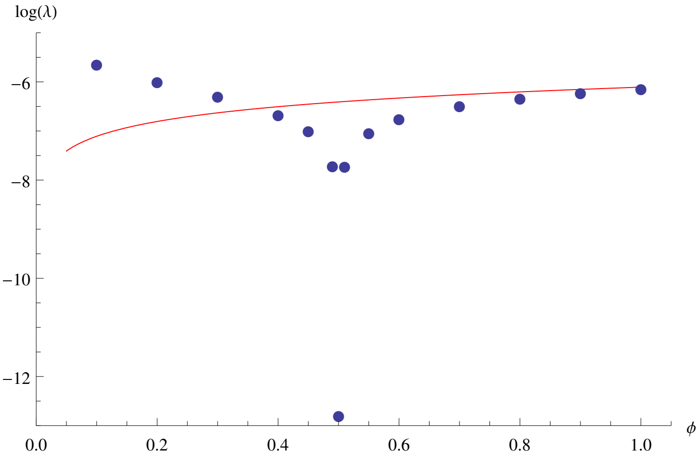


approximate localization also verified

lowest eigenvalues for $N_i = 16, R_i = R, r = \phi = 1$:



coupling to "Higgs" ϕ: expected linear;
lowest eigenvalue as a function of ϕ, for $N_i = 8, R_i = r = 1$



singlet Higgs S

links $\mathcal{D}_u$ with $\mathcal{D}_l$ at ν_R

$$H^a_{(S)} = h^a S \; + h.c., \qquad S = \sum_n |p_n +\rangle_u \langle q_n|_l$$

rotating in 8-9 plane, $h^a = h(e^8 + ie^9)e^{i\omega_S t}$

solves linearized eom for suitable ω_S

- breaks $SU(2)_R$ and $U(4) \to SU(3)_c \times U(1)_B$ (and $U(1)_{B-L}$)
  (not needed for other geometries)
- might induce Majorana mass for ν_R

$$\int d^4x \, tr_N(\nu_R^T \gamma^0 S^\dagger \nu_R S^\dagger).$$

SSB, low-energy action

2 stage SSB:

- 4 coincident branes $\mathcal{D}_B \cong \mathcal{D}_l$, 2 coincident branes $\mathcal{D}_u \cong \mathcal{D}_d$
  $\to$ unbroken $U(4) \times U(2)$
- switch on S
  $\to$ $SU(3)_c \times U(1)_Q \times U(1)_B \times U(1)_{\mathrm{tr}}$

note: electric charge generator

$$t_Q = t_3 + \frac{1}{2} t_Y = \frac{1}{2}(\mathbf{1}_u - \mathbf{1}_d + \mathbf{1}_l - \frac{1}{3}\mathbf{1}_B)$$

guaranteed to be unbroken

gauge bosons:

from fluctuations $X^\mu + \mathcal{A}^\mu$ of space-time matrices

$$\mathcal{A} = g_W(W_- t_+ + W_+ t_- + W_3 t_3) + \frac{1}{2} g' B\, t_Y + g_5 B_5\, t_5 + g_S A_\alpha t_\alpha + ...$$

identify gauge couplings

$$g_w = \frac{g}{\sqrt{N_u}}, \quad g_S = \frac{g}{\sqrt{N_l}}, \quad \frac{1}{2} g' = \frac{g}{\sqrt{8 N_u + \frac{8}{3} N_l}},$$

EW symmetry breaking:

recall $\Phi^+ := \Phi^4 + i\Phi^5 = \begin{pmatrix} R'_u L_3 & 0 & 0 & \phi_u \mathbb{1} & & \\ 0 & R'_d L_3 & \phi_d \mathbb{1} & 0 & & \\ 0 & 0 & R'_d L_3 & 0 & & \\ 0 & 0 & 0 & R'_u L_3 & & \\ & & & & R'_2 K_3 & \\ & & & & & R'_2 K_3 \mathbb{1}_3 \end{pmatrix}$

kinetic term for two Higgs doublets $H_d = \begin{pmatrix} 0 \\ \phi_d \end{pmatrix}, \quad H_u = \begin{pmatrix} \phi_u \\ 0 \end{pmatrix}$

$$\begin{aligned} S[\phi] &= -\tfrac{1}{2}\int d^4x\, G^{\mu\nu}\, tr_N\Big((D_\mu H_d)^\dagger D_\nu H_d + (D_\mu H_u)^\dagger D_\nu H_u\Big) \\ &= -\int d^4x\, tr_N\Big(\tfrac{1}{4}\phi^2 g_w^2 (W_1^2 + W_2^2) + \tfrac{1}{4}\phi^2 (g_w^2 + g'^2) Z^2 + \phi^2 g_5^2 B_5^2\Big) \end{aligned}$$

gives W mass

$$m_W^2 = \frac{1}{2} g^2 \phi^2$$

Weinberg angle: $\sin^2\theta_W = \frac{1}{2+\frac{2N_l}{3N_u}}$

for $N_u = N_l$: $g_S = g_W, \quad \sin^2\theta_W = 3/8$

fermion masses

Yukawa coupling

$$\int d^4x\, g tr_N \bar{\psi}\Gamma^a[\Phi_a, .]\psi = 2\int d^4x\, g f_\psi \phi \bar{\psi}_{12}\psi_{12}$$

gives

$$m_\psi = g\phi f_\psi$$

- f_ψ arbitrarily small for S.M. fermions $\equiv$ would-be zero modes
- $\tilde{f}_\psi = 1$ for lowest mirror fermions, e.g. $\tilde{e}_R$
  $\Rightarrow \quad \tilde{m}_\psi \approx \sqrt{2} m_W$
  however: tree-level masses at high energies
  KK modes on $\mathcal{D}_{B,l}$ couple to chiral fermions, not to W, Z bosons
  integrate out $\Rightarrow$ significant running of Yukawas

e.g. $\alpha \int d^4x\, g tr_N \bar{\psi}\Gamma^a[\Phi_a, .]\psi, \quad \alpha > 1 \quad \Rightarrow$ mirror fermions heavier

Higgs mass ?

two doublets $H_d = \begin{pmatrix} 0 \\ \phi_d \end{pmatrix}, \quad H_u = \begin{pmatrix} \phi_u \\ 0 \end{pmatrix}$, (cf. MSSM)

$\tan\beta = \frac{\phi_u}{\phi_d} = 1$ at tree level

assume $\phi_{u,d}$ is physical Higgs fluctuation !?

$$m_\phi^2 = 2g^2\phi^2\left(1 + 2\pi^2 f''\right) \quad (\approx 4m_W^2)$$

(... just illustration !)

extra gauge bosons:

C_μ ... link mirror fermions to S.M. fermions, e.g. $\tilde{e}_L \leftrightarrow \nu_L, e_L$

(extend $SU(2)_L \times SU(2)_R$ to $SU(4)$)

extra $U(1)$'s:

unbroken gauge group $SU(3)_c \times U(1)_Q \times U(1)_B \times U(1)_{\rm tr}$

anomalous $U(1)_B$ gauge bosons
expected to acquire (Stückelberg) mass

general predictions:

- mirror fermions at "intermediate" (??) energies
- right-handed neutrinos
- towers of massive KK modes, completing $\mathcal{N} = 4$ spectrum
  no "grand desert"
- requires non-standard ("emergent") gravity mechanism

more specific (BG-dependent):

- two Higgs doublets (physical fluct. ?)
- mirror fermons approx. degenerate

issues:

- stabilization of compact branes ?
  - quantum effects $\rightarrow$ SUGRA in extra dimensions
    $\rightarrow$ (attractive) force between branes
  - rotation in extra dimensions
- low scale of mirror fermions?

  quantum corrections will modify Yukawas

  e.g: $\int \alpha\, \bar\psi \not{D}_{\rm int} \psi, \qquad \alpha > 1 \;\; \rightarrow$ mirror fermions above m_W
- generations? multiple intersections of branes

  e.g.: $\mathcal{D}_u = S^2_N \tilde\oplus S^2_2 \rightarrow (S^2_{N_1} \oplus S^2_{N_2} \oplus S^2_{N_3}) \tilde\oplus S^2_2$

no obstacle in principle to get S.M. from IKKT at low energy

the simplest possible model might actually work !?

explicit brane background:

$$\Phi^4 + i\Phi^5 = \begin{pmatrix} R'_u L_3 & 0 & 0 & \phi\mathbb{1} & & \\ 0 & R'_d L_3 & \phi\mathbb{1} & 0 & & \\ 0 & 0 & R'_d L_3 & 0 & & \\ 0 & 0 & 0 & R'_u L_3 & & \\ & & & & R'_2 K_3 & \\ & & & & & R'_2 K_3 \mathbb{1}_3 \end{pmatrix},$$

$$\Phi^6 + i\Phi^7 = \begin{pmatrix} -\frac{r}{2}\mathbb{1} & & & & & \\ & -\frac{r}{2}\mathbb{1} & & & & \\ & & \frac{r}{2}\mathbb{1} & & & \\ & & & \frac{r}{2}\mathbb{1} & & \\ & & & & RK_+ & \\ & & & & & RK_+ \mathbb{1}_3 \end{pmatrix},$$

$$\Phi^8 + i\Phi^9 = \begin{pmatrix} RL_+ & & & & & \\ & RL_+ & & & & \\ & & RL_+ & & & \\ & & & RL_+ & 2he^{i\omega_s t} S & \\ & & & & 0 & \\ & & & & & 0_3 \end{pmatrix}.$$