Quantized cosmological spacetimes and higher spin in the IKKT model

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Matrix Models ... natural framework for fundamental theory

- pre-geometric, constructive
- dynamical “quantum” (NC) spaces, gauge theory
- stringy features
  - max. SUSY \(\rightarrow\) inherit good behavior of critical string (UV)
- avoid string compactifications
  - \(\rightarrow\) need different mechanism for gravity & chirality
- IKKT: allows to describe “beginning of time”!
outline:

- matrix models & matrix geometry
- 4D covariant quantum spaces: fuzzy $S^4_N, H^4_n$
- cosmological space-times: $\mathcal{M}^{3,1}$ & BB!
- fluctuations $\rightarrow$ higher spin gauge theory
- metric, vielbein; gravity?

HS, arXiv:1606.00769
M. Sperling, HS arXiv:1707.00885
M. Sperling, HS arXiv:1806.05907
The IKKT model

IKKT or IIB model

\[ S[X, \psi] = -\text{Tr} \left( [X^a, X^b][X^{a'}, X^{b'}] \eta_{aa'} \eta_{bb'} + \bar{\psi} \gamma_a [X^a, \psi] \right) \]

\[ X^a = X^{a\dagger} \in \text{Mat}(N, \mathbb{C}), \quad a = 0, \ldots, 9, \quad N \text{ large} \]

gauge symmetry \( X^a \rightarrow UX^a U^{-1}, \ SO(9, 1), \ \text{SUSY} \)

proposed as non-perturbative definition of IIB string theory

- quantized Schild action for IIB superstring
- reduction of \( 10D \) SYM to point, \( N \) large
- add \( m^2 X^a X_a \) to set scale, IR regularization

\[ Z = \int dX d\psi \, e^{iS[X]} \]

Kim, Nishimura, Tsuchiya arXiv:1108.1540 ff
different points of view:

- **classical solutions** = “branes” justified by max. SUSY (cf. critical string thy)

generically NC geometry, “matrix geometry”

fluctuations $\rightarrow$ field theory, 3+1D physics, dynamical geometry
UV/IR mixing $\rightarrow$ IKKT model $\rightarrow$ unique 4D NC gauge theory

**hypothesis**

space-time = (near-) classical solution of IIB model

10 bulk physics:
sugra arises in M.M. from quantum effects (loops)

“holographic”

Kabat-Taylor, IKKT,...

cf. HS arXiv:1606.00646
“matrix geometry” (≈ NC geometry):

- \( S_E \sim Tr[\{X^a, X^b\}]^2 \) ⇒ config’s with small \([X^a, X^b] \neq 0\) dominate

  i.e. “almost-commutative” configurations

- ∃ quasi-coherent states \(|x\rangle\), minimize \(\sum_a \langle x|\Delta X^2_a |x\rangle\)

\[ X^a \approx \text{diag.}, \quad \text{spectrum} =: \mathcal{M} \subset \mathbb{R}^{10} \]

\[ \langle x| X^a |x'\rangle \approx \delta(x - x')x^a, \quad x \in \mathcal{M} \]

- hypothesis: classical solutions dominate

“condensation” of matrices, geometry

NC branes embedded in target space \(\mathbb{R}^{10}\)

\[ X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10} \]

cf. Q.M: replace functions \(x^a \rightsquigarrow\) matrices / observables \(X^a\)
typical examples: quantized Poisson manifolds

- Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$:
  \[
  [X^a, X^b] = i\theta^{ab} 1
  \]
  quantized symplectic space $(\mathbb{R}^4, \omega)$
  admits translations, no rotation invariance

- fuzzy 2-sphere $S^2_N$
  \[
  X_1^2 + X_2^2 + X_3^2 = R^2_N, \quad [X_i, X_j] = i\epsilon_{ijk} X_k
  \]
  fully covariant under $SO(3)$ (Hoppe; Madore)

generically:
fluctuations $\rightarrow$ NC gauge theory, & dynamical geometry
issues for NC spaces / field theory:

- quantization $\rightarrow$ UV / IR mixing
  $\leftrightarrow$ max. SUSY model: IKKT, BFSS, BMN

- Lorentz / $SO(4)$ covariance in 4D?

- obstacle: NC spaces: $[X^\mu, X^\nu] =: i\theta^{\mu\nu} \neq 0$ breaks Lorentz invariance

- $\exists$ fully covariant fuzzy four-sphere $S^4_N$
  Grosse-Klimcik-Presnajder 1996; Castelino-Lee-Taylor; Ramgoolam; Kimura; Abe
  Hasebe; Medina-O’Connor; Karabali-Nair; Zhang-Hu 2001 (QHE!) ...

price to pay: “internal structure” $\rightarrow$ higher spin theory
covariant fuzzy four-sphere $S^4_N$

5 hermitian matrices $X_a, \ a = 1, \ldots, 5$ acting on $\mathcal{H}_N$

$$\sum_a X_a^2 = R^2$$

covariance: $X_a \in \text{End}(\mathcal{H}_N)$ transform as vectors of $SO(5)$

$$[\mathcal{M}_{ab}, X_c] = i(\delta_{ac} X_b - \delta_{bc} X_a),$$
$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\delta_{ac} \mathcal{M}_{bd} - \delta_{ad} \mathcal{M}_{bc} - \delta_{bc} \mathcal{M}_{ad} + \delta_{bd} \mathcal{M}_{ac}).$$

$\mathcal{M}_{ab} ... so(5)$ generators acting on $\mathcal{H}_N$
covariant fuzzy four-sphere $S^4_N$

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$\mathcal{M}_{ab}$ ... so(5) generators acting on $\mathcal{H}_N$

oscillator construction:

$$X_a = \psi^\dagger \gamma_a \psi,$$
$$\mathcal{M}^{ab} = \psi^\dagger \Sigma^{ab} \psi$$

acting on $\mathcal{H}_N = \psi^\dagger_{\alpha_1} \ldots \psi^\dagger_{\alpha_N} |0\rangle \cong (\mathbb{C}^4)^{\otimes S^N} \cong (0, N)_{5\otimes(5)}$

Grosse-Klimcik-Presnajder 1996; ...

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relations:

\[ X_a X_a = R^2 \sim \frac{1}{4} r^2 N^2 \]

\[ [X^a, X^b] = i r^2 \mathcal{M}^{ab} =: i \Theta^{ab} \]

\[ \epsilon^{abcde} X_a X_b X_c X_d X_e = (N + 2) R^2 r^3 \text{ (volume quantiz.)} \]

geometry from coherent states \( |p\rangle \):

\[ \{p_a = \langle p | X_a | p \rangle \} = S^4 \]

closer inspection:

degeneracy of coherent states, “internal” \( S^2 \) fiber

cf. Karczmarek, Yeh, arXiv:1506.07188
semiclassical picture: hidden bundle structure

\[ \mathbb{C}P^3 \ni \psi \\downarrow \quad S^4 \ni x^a = \psi + \Gamma^a \psi \]

Ho-Ramgoolam, Medina-O’Connor, Abe, ...

fuzzy case:
oscillator construction \([\psi, \psi^\dagger] = \delta \rightarrow \) functions on fuzzy \(\mathbb{C}P^3_N\)

fuzzy \(S^4_N\) is really fuzzy \(\mathbb{C}P^3_N\), hidden extra dimensions \(S^2\)!

Poisson tensor

\[ \theta^{\mu\nu}(x, \xi) \sim -i[X^\mu, X^\nu] \]

local \(SO(4)_x\) rotates fiber \(\xi \in S^2\)

averaging over fiber \(\rightarrow [\theta^{\mu\nu}(x, \xi)]_0 = 0\), local \(SO(4)\) preserved!

... 4D “covariant” quantum space
fields and harmonics on $S^4_N$

"functions" on $S^4_N$:

\[ \text{End}(\mathcal{H}_N) \cong \bigoplus_{s=0}^{N} C^s \]

\[ C^s = \bigoplus_{n=0}^{N} (n, 2s) \]

$(n, 0)$ modes = scalar functions on $S^4$:

\[ \phi(X) = \phi_{a_1...a_n} X^{a_1}...X^{a_n} \]

$(n, 2)$ modes = selfdual 2-forms on $S^4$

\[ \phi_{bc}(X) \theta^{bc} = \phi_{a_1...a_n b; c} X^{a_1}...X^{a_n} \theta^{bc} \]

\[ \text{End}(\mathcal{H}) \cong \text{fields on } S^4 \text{ taking values in } \mathfrak{h}_5 = \bigoplus \]

higher spin modes = would-be KK modes on $S^2$

(local $SO(4)$ acts on $S^2$ fiber)
relation with spin $s$ fields: one-to-one map

$$C^s \cong T^* \otimes S^s S^4$$

$$\phi(s) = \phi_{b_1 \ldots b_s; c_1 \ldots c_s} (x) \theta^{b_1 c_1} \ldots \theta^{b_s c_s} \quad \mapsto \quad \phi^{(s)}_{c_1 \ldots c_s} (x) = \phi_{b_1 \ldots b_s; c_1 \ldots c_s} x^{b_1} \ldots x^{b_s}$$

{\{x^{c_1}, \ldots, x^{c_s}, \phi^{(s)}_{c_1 \ldots c_s} (x)\}} \mapsto {\{x^{c_1}, \ldots, x^{c_s}, \phi_{c_1 \ldots c_s} (x)\}} \leftarrow \phi^{(s)}_{c_1 \ldots c_s} (x)

... "symbol" of $\phi \in C^s$

M. Sperling & HS, arXiv:1707.00885

$C^s \cong$ symm., traceless, tang., div.-free rank $s$ tensor field on $S^4$

$$\phi_{c_1 \ldots c_s} (x) x^{c_i} = 0,$$

$$\phi_{c_1 \ldots c_s} (x) g^{c_1 c_2} = 0,$$

$$\partial^{c_i} \phi_{c_1 \ldots c_s} (x) = 0.$$
Poisson calculus: (semi-classical limit)  

\[ \mathbb{C}P^3 = \text{symplectic manifold, } \{x^a, x^b\} = \theta^{ab} \]

\[ \bar{\partial}^a \phi := -\frac{1}{r^2 R^2} \theta^{ab} \{x_b, \phi\}, \quad \{x^a, \cdot\} = \theta^{ab} \bar{\partial}_b \]

satisfy

\[ \bar{\partial}^a x^c = P_{\ T}^{ac} = g^{ac} - \frac{1}{R^2} x^a x^c \]

matrix Laplacian:

\[ \Box = [x^a, [x_a, \cdot]] \sim -\{x^a, \{x_a, \cdot\}\} = -r^2 R^2 \bar{\partial}^a \bar{\partial}_a \]

covariant derivative:

\[ \nabla = P_T \circ \bar{\partial}, \quad \nabla \theta^{ab} = 0 \]

curvature

\[ \mathcal{R}_{ab} := \mathcal{R}[\bar{\partial}_a, \bar{\partial}_b] = [\nabla_a, \nabla_b] - \nabla_{[\bar{\partial}_a, \bar{\partial}_b]} \]

... Levi-Civita connection on \( S^4 \)
local description: pick north pole $p \in S^4$

$\rightarrow$ tangential & radial generators

$$X^a = \begin{pmatrix} X^\mu \\ X^5 \end{pmatrix}, \quad \mu = 1, \ldots, 4$$
tangential coords at $p$

separate $SO(5)$ into $SO(4)$ & translations

$$M^{ab} = \begin{pmatrix} M^{\mu\nu} & P^\mu \\ -P^\mu & 0 \end{pmatrix}$$

where $P^\mu = M^{\mu5}$

Poisson algebra $\{P_\mu, X^\nu\} \approx \delta^\nu_\mu$ locally
local form of spin $s$ harmonics: e.g. spin 2:

$$\phi^{(2)} = \phi_{\mu\nu}(x) P^\mu P^\nu + \omega_{\mu;\alpha\beta}(x) P^\mu M^{\alpha\beta} + \Omega_{\alpha\beta;\mu\nu}(x) M^{\alpha\beta} M^{\mu\nu}$$

recall $\text{End}(\mathcal{H}) = \bigoplus C^s$, $C^s \cong \text{rank } s$ tensor fields $\phi_{a_1...a_s}(x)$

unique irrep $(n, 2s) \in \text{End}(\mathcal{H}) \Rightarrow$ constraints!

$$\omega_{\mu;\alpha\beta} \propto \partial_\alpha \phi_{\mu\beta} - \partial_\beta \phi_{\mu\alpha}$$

$$\Omega_{\alpha\beta;\mu\nu} \propto \mathcal{R}_{\alpha\beta\mu\nu}[\phi]$$

... linearized spin connection and curvature determined by $\phi_{\mu\nu}$
similarly:

**cosmological quantum space-times** $\mathcal{M}^{3,1}_n$:

- exactly homogeneous & isotropic
- finite density of microstates
- mechanism for Big Bang
- starting point: fuzzy hyperboloid $H^4_n$
Euclidean fuzzy hyperboloid $H^4_n$

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) .$$

$\eta^{ab} = \text{diag}(-1, 1, 1, 1, 1, -1)$

choose “short” discrete unitary irreps $\mathcal{H}_n$ (“minireps”, doubletons) special properties:

- irreps under $\mathfrak{so}(4, 1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\text{spec}(\mathcal{M}^{05}) = \{E_0, E_0 + 1, \ldots\}, \quad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace of $\mathcal{M}^{05}$ is $n+1$-dim. irrep of $SU(2)_L$: fuzzy $S^2_n$
**fuzzy hyperboloid** $H^4_n$

**def.**

$$X^a := r \mathcal{M}^a_5, \quad a = 0, \ldots, 4$$

$$[X^a, X^b] = i r^2 \mathcal{M}^{ab} =: i \Theta^{ab}$$

5 hermitian generators $X^a = (X^a)^\dagger$ satisfy

$$\eta_{ab}X^a X^b = X^i X^i - X^0 X^0 = -R^2 1, \quad R^2 = r^2 (n^2 - 4)$$

one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4,1)$

**note:** induced metric: Euclidean $AdS^4$
oscillator construction: 4 bosonic oscillators $[\psi_\alpha, \bar{\psi}^\beta] = \delta_\alpha^\beta$

$H_n = \text{suitable irrep in Fock space}$

Then

$$ M_{ab} = \bar{\psi} \Sigma_{ab} \psi, \quad \gamma_0 = \text{diag}(1, 1, -1, -1) $$

$$ X^a = r \bar{\psi} \gamma^a \psi $$

$H^4_n = \text{quantized } \mathbb{C}P^{1,2} = S^2 \text{ bundle over } H^4, \text{ selfdual } \theta^{\mu\nu}$

analogous to $S^4_N$, finite density of microstates
fuzzy "functions" on $H^4_n$:

$$\text{End}(H_n) \cong \bigoplus_{s=0}^{n} C^s = \int_{\mathbb{C}P^{1,2}} d\mu f(m) \langle m \rangle \langle m \rangle$$

= fields on $H^4$ taking values in $\mathfrak{h}_s = \bigoplus_s M^{a_1b_1} \ldots M^{a_sb_s}$

spin $s$ sectors $C^s$ selected by spin Casimir

$$S^2 = \sum_{a<b \leq 4} \left[ [M^{ab}, [M_{ab}, \cdot]], + r^{-2} [X_a, [X^a, \cdot]] \right].$$

can show:

$$S^2|_{C^s} = 2s(s+1), \quad s = 0, 1, \ldots, n$$

M. Sperling & H.S. 1806.05907
open FRW universe from $H^4_n$

$Y^\mu := X^\mu$, for $\mu = 0, 1, 2, 3$ (drop $X^4$ !)

$\mathcal{M}_n^{3,1}$ = projected $H^4_n$ embedded in $\mathbb{R}^{1,3}$ via projection

$Y^\mu \sim y^\mu : \mathbb{C}P^{1,2} \rightarrow H^4 \xrightarrow{\Pi} \mathbb{R}^{1,3}$.

satisfies

$[Y^\mu, [Y^\mu, Y^\nu]] = ir^2[Y^\mu, M^{\mu\nu}]$ (no sum)

$= r^2 \begin{cases} 
Y^\nu, & \nu \neq \mu \neq 0 \\
-Y^\nu, & \nu \neq \mu = 0 \\
0, & \nu = \mu
\end{cases}$

hence

$\Box Y^\mu = [Y^\nu, [Y^\nu, Y^\mu]] = 3r^2 Y^\mu$.

... solution of IKKT with $m^2 = 3r^2$.  

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properties:

- $SO(3, 1)$ manifest $\Rightarrow$ foliation into $SO(3, 1)$-invariant space-like 3-hyperboloids $H^3_t$

- double-covered FRW space-time with hyperbolic ($k = -1$) spatial geometries

\[
ds^2 = dt^2 - a(t)^2 d\Sigma^2,
\]

$d\Sigma^2$ ... $SO(3, 1)$-invariant metric on space-like $H^3$
metric properties

reference point \( p \in H^4 \subset \mathbb{R}^{1,4} \)

\[ p^a = R(\cosh(\eta), \sinh(\eta), 0, 0, 0) \]

induced metric:

\[ g_{\mu\nu} = (-1, 1, 1, 1) = \eta_{\mu\nu}, \quad \mu, \nu = 0, 1, 2, 3 \quad \text{(Minkowski!)} \]

→ Milne metric:

\[ ds_g^2 = -dt^2 + t^2 d\Sigma^2 \]

however: induced metric \( \neq \) effective (“open string”) metric
**Effective metric** (for scalar fields)

encoded in \( \Box_Y = [Y_\mu, [Y^\mu, .]] \sim \frac{1}{\sqrt{|G|}} \partial_\mu (\sqrt{|G|} G^{\mu \nu} \partial_\nu .): \)

\[
G^{\mu \nu} = \alpha \gamma^{\mu \nu}, \quad \alpha = \sqrt{\frac{\theta^{\mu \nu}}{\gamma^{\mu \nu}}},
\gamma^{\mu \nu} = g_{\mu' \nu'} [\theta^{\mu' \mu} \theta^{\nu' \nu}]_{S^2}
\]

where \([.]_{S^2} \) ... averaging over the internal \( S^2 \).

\[
\gamma^{\mu \nu} = \frac{\Delta^4}{4} \text{diag}(c_0(\eta), c(\eta), c(\eta), c(\eta))
\]

at \( p \), where

\[
c(\eta) = 1 - \frac{1}{3} \cosh^2(\eta)
\]

\[
c_0(\eta) = \cosh^2(\eta) - 1 \geq 0
\]

**Signature change at \( c(\eta) = 0 \)**

\( \cosh^2(\eta_0) = 3 \) ...Big Bang!

Euclidean for \( \eta < \eta_0 \), Minkowski \( (+ - - -) \) for \( \eta > \eta_0 \)
conformal factor \[ \alpha = \sqrt{\frac{\theta_{\mu\nu}}{\gamma_{\mu\nu}}} = \frac{4}{\Delta^4} |c(\eta)|^{-\frac{3}{2}} \]

from \( SO(4, 2) \)-inv. (Kirillov-Kostant) symplectic \( \omega \) on \( \mathbb{C}P^{1,2} \)

\[ \rightarrow \text{ effective metric at } p \]

\[ G_{\mu\nu} = \text{diag}\left( \frac{|c(\eta)|^\frac{3}{2}}{c_0(\eta)}, -|c(\eta)|^{\frac{1}{2}}, -|c(\eta)|^{\frac{1}{2}}, -|c(\eta)|^{\frac{1}{2}} \right) \]

FLRW metric and scale factor (after BB)

\[ ds_G^2 = dt^2 - a^2(t) d\Sigma^2 \]

late times:

linear coasting cosmology

\[ a(t) \approx \frac{3\sqrt{3}}{2} t . \]
$a(t) \sim t$ is remarkably close to observation:

- age of univ. $13.9 \times 10^9 \text{y}$ from present Hubble parameter

artificial within GR,

natural in M.M., provided gravity emerges below cosm. scales

- can reasonably reproduce SN1a (without acceleration)

Big Bang:
shortly after the BB $\eta \gtrsim \eta_0$:

$$a(t) \propto c(t)^{\frac{1}{4}} \propto t^{1/7}$$

conformal factor & 4-volume form $|\theta^{\mu\nu}|$ responsible for singular expansion!
other features:

- $\exists$ Euclidean pre-BB era
- 2 sheets with opposite intrinsic “chirality” (i.e. $\mathbf{A}SD$)

$\exists$ higher-spin fluctuation modes

$\rightarrow$ higher-spin gauge theory

small $n$ possible (even $n = 0$)
other cosmological solutions

- “momentum embedding” (same $M^{3,1}_n$, different metric) $k = -1$
  
  M. Sperling & H.S. 1806.05907

- expanding closed universe $k = 1$

- recollapsing universe $k = 1$
  
  HS arXiv:1709.10480
momentum embedding:

\[ T^\mu := \frac{1}{R} \mathcal{M}^{\mu 4} \]

\[ \Box_T T^\mu = -3 \frac{1}{R^2} T^\mu . \]

... solution of IKKT model with mass

- \([T^\mu, X^\nu] = i f(t) \eta^{\mu \nu}, \text{momentum generator}\]
  
  (cf. Hanada, Kawai, Kimura hep-th/0508211]

- similar expansion of functions \( f(X) + f_\mu(X) T^\mu + \ldots \), higher-spin modes on \( \mathcal{M}^{3,1} \)

- similar eff. \( SO(3,1) \)-invariant FRW metric, \( k = -1 \)

- similar late-time behavior

- BB, initial \( a(t) \sim t^{1/5} \), no signature change

... work in progress M. Sperling & HS
∃ further FRW solutions with $k = +1$,
in presence of $SO(4, 1)$-breaking mass $-m^2 Y^i Y^i + m_0^2 Y^0 Y^0$

- expanding closed universe
  from projection of fuzzy $H^4_n$
- recollapsing closed universe
  from projection of fuzzy $S^4_N$

HS, arXiv:1709.10480
fluctuations & higher spin gauge theory on $H_n^4$

$$S[Y] = Tr(-[Y^a, Y^b][Y_a, Y_b] + m^2 Y^a Y_a) = S[U^{-1} Y U]$$

background solution: $S_N^4$, $H_n^4$

add fluctuations $Y^a = X^a + A^a$

expand action to second oder in $A^a$

$$S[Y] = S[X] + \frac{2}{g^2} \text{Tr} A_a \left( \Box + \frac{1}{2} \mu^2 \right) \delta^a_b + 2[[X^a, X^b], .] - [X^a, [X^b, .]] \right) A_b$$

$$\Box = [X^a, [X_a, .]]$$

- fluctuations $A_a$ describe gauge theory (NCFT) on $\mathcal{M}$
  
  ("open strings" ending on $\mathcal{M}$)

- for $S_N^4$, $H_n^4$: $A_a$ ... $\mathfrak{h}_5$-valued gauge field, incl. spin 2
4 indep. tangential fluctuation modes

\[ \mathcal{A}_a \in \text{End}(\mathcal{H}) \otimes (5) \]

\[ \mathcal{A}_a^{(1)} = \partial_a \phi(s), \]
\[ \mathcal{A}_a^{(2)} = \theta^{ab} \partial_b \phi(s) = \{ x^a, \phi(s) \} \]
\[ \mathcal{A}_a^{(3)} = \phi_a^{(s)} \]
\[ \mathcal{A}_a^{(4)} = \theta^{ab} \phi_b^{(s)}. \]

where \( \phi^{(s)} \in \text{End}(\mathcal{H}) \) ... spin \( s \) mode, \( \phi_a^{(s)} \propto \{ x^a, \phi^{(s)} \}_{s-1} \)

eigenmodes of \( \mathcal{D}^2 \):

\[ \mathcal{B}_a^{(1)} = \mathcal{A}_a^{(1)} - \frac{\alpha_s}{R^2 r^2} (\Box - 2r^2) \mathcal{A}_b^{(4)}, \]
\[ \mathcal{B}_a^{(2)} = \mathcal{A}_a^{(2)} + \alpha_s (\Box - 2r^2) \mathcal{A}_a^{(3)}, \]
\[ \mathcal{B}_a^{(3)} = \mathcal{A}_a^{(3)} \]
\[ \mathcal{B}_a^{(4)} = \mathcal{A}_a^{(4)} \]

can diagonalize \( \mathcal{D}^2 \)
all tangential modes are stable!

+ radial modes (unstable)

M. Sperling & H.S. 1806.05907
metric and vielbein

consider scalar field $\phi = \phi(X)$ (= transversal fluctuation)

kinetic term

$$-\text{Tr}[X^a, \phi][X_a, \phi] \sim \int e^a \phi e_a \phi = \int \gamma^\mu_\nu \partial_\mu \phi \partial_\nu \phi$$

vielbein

$$e^a := \{X^a, .\} = e^{a\mu} \partial_\mu$$

$$e^{a\mu} = \theta^{a\mu}$$

metric

$$\gamma^\mu_\nu = \eta_{\alpha\beta} e^{\alpha\mu} e^{\beta\nu} = \frac{1}{4} \Delta^4 g^{\mu\nu}$$

Poisson structure $\rightarrow$ frame bundle!
perturbed vielbein: \[ Y^a = X^a + A^a \]

\[ e^a := \{ Y^a, \cdot \} \sim e^{a\mu}[A] \partial_\mu \quad \text{... vielbein} \]

\[ \delta_A \gamma^{ab} =: H^{ab}[A] = \theta^{ca} \{ A_c, x^b \} + (a \leftrightarrow b) \]

linearize & average over fiber →

\[ G^{ab} = \gamma^{ab} + h^{ab}, \quad h^{ab} \sim [H^{ab}]_0 \]

spin 2 graviton:

\[ h_{ab}[\mathcal{B}^{(4)}] = 2\alpha_1 (\Box - 2r^2) \phi_{ab}, \quad \nabla^a h_{ab} = 0 \]

all other modes drop out: \[ h_{ab}[\mathcal{B}^{(i)}] = 0 \]
quadratic action for spin 2 graviton \( h_{ab}[B] = 2\alpha_1(\Box - 2r^2)\phi_{ab} \):

\[
S_2[h_{ab}] \propto \int B_a D^2 B^a \propto \int h_{ab}[B] h_{ab}[B]
\]

\( h_{ab} \) doesn’t propagate in classical model
due to field redefinitions via \((\Box - 2r^2)\)
coupling to matter:

\[
S[\text{matter}] \sim \int_M d^4 x \ h^{ab} T_{ab}
\]

\( \rightarrow \) auxiliary field \( h_{ab} \sim T_{ab} \)!

however:

1. quantum effects $\rightarrow$ induced gravity action $\sim \int h_{\mu\nu} \Box h^{\mu\nu}$
   $\rightarrow$ (lin.) Einstein equations (+ possibly c.c. and/or mass)

2. consider different action (however: UV/IR mixing)

3. for cosmological space-times:
   ... to be worked out

GR not renormalizable $\Rightarrow$ need different starting point

$\rightarrow$ emergent gravity ?

present model might be healthy candidate
towards higher-spin gravity on $\mathcal{M}^{3,1}$

momentum embedding $Y^a = T^a$ best suited

- space of modes = tangential modes on $H^4$, similar structure
- clean separation of higher spin modes
- manifest $SO(3,1)$, local Lorentz-invar. not guaranteed
  (could be bi-metric...)
- conjecture: no ghosts
- compute mass spectrum (to exclude tachyons, instabilities)

work in progress  M. Sperling, HS
summary

- **matrix models**: promising framework for quantum theory of space-time & matter
- $\exists$ nice cosmological FRW space-time solutions
  - reg. BB, finite density of microstates
  - IKKT allows to address origin of time!
- all ingredients for gravity, good UV behavior (SUSY)
  - $\rightarrow$ regularized higher spin theory, cf. Vasiliev
- may not lead to gravity at classical level; emergent gravity?
  - more work required for cosm. space-times

stay tuned!
gauge transformations:

\[ Y^a \rightarrow UY^a U^{-1} = U(X^a + A^a)U^{-1} \] leads to \((U = e^{i\Lambda})\)

\[ \delta A^a = i[\Lambda, X^a] + i[\Lambda, A^a] \]

expand

\[ \Lambda = \Lambda_0 + \frac{1}{2} \Lambda_{ab} M^{ab} + \ldots \]

\[ \ldots U(1) \times SO(5) \times \ldots \text{- valued gauge trasfos} \]

diffeos from \(\delta_v := i[v^\rho P^\rho , .]\)

\[ \delta h_{\mu\nu} = (\partial_\mu v_\nu + \partial_\nu v_\mu) - v^\rho \partial_\rho h_{\mu\nu} + (\Lambda \cdot h)_{\mu\nu} \]

\[ \delta A_{\mu\rho\sigma} = \frac{1}{2} \partial_\mu \Lambda_{\sigma\rho}(x) - v^\rho \partial_\rho A_{\mu\rho\sigma} + (\Lambda \cdot A)_{\mu\rho\sigma} \]

etc.
further solutions: expanding closed universe

\[ S[Y] = \frac{1}{g^2} \text{Tr} \left( [Y^a, Y^b][Y^a', Y^b']\eta_{aa'}\eta_{bb'} - m^2 Y^i Y^i + m_0^2 Y^0 Y^0 \right). \]

\[ \exists \text{ solution:} \]

\[ Y^i = X^i, \quad \text{for} \quad i = 1, \ldots, 4, \quad Y^0 = \kappa X^5 \]

FRW cosmology with spatial $S^3$, $k = 1$

cosm. scale factor: late time $a(t) \sim t^{1/3}$, BB $a(t) \sim t^{1/7}$
further solutions: recollapsing closed universe

\[
S[Y] = \frac{1}{g^2} \text{Tr} \left( [Y^a, Y^b][Y^{a'}, Y^{b'}] \eta_{aa'} \eta_{bb'} - m^2 Y^i Y^i + m_0^2 Y^0 Y^0 \right)
\]

\exists \text{ solution}

\begin{align*}
Y^i &= X^i, \quad \text{for } i = 1, \ldots, 4, \\
Y^0 &= \kappa X^5
\end{align*}

for \( X^a \) ... fuzzy \( S^4_N \)

FRW cosmology with spatial \( S^3 \), \( k = 1 \)

\[ a(t) \sim t^{1/7} \]