Matrix models and the quantum structure of space, time and matter

Harold Steinacker

Department of Physics, University of Vienna

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outline:

- status of “fundamental theory”
- quantum space-time & geometry
- matrix models
- 4D quantum spaces & cosmological space-times
- towards particle physics, outlook
present understanding of fundamental matter & interactions:

- standard model of elementary particle physics
  = quantum field theory
  governs fundamental constituents of matter & interactions except for gravity!

- general relativity (GR)
  = classical geometrical theory of gravity

however: class. Einstein equations inconsistent with QM:

\[ \mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \]

GR resists quantization, not renormalizable
Motivation

Quantum geometry

IKKT model, NC branes

Cosmological space-times

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  classical

  quantum

GR resists quantization, not renormalizable
underlying issue: superposition principle of quantum mechanics

superposition of massive objects

⇒ need quantum theory of geometry & gravity

requires new framework for quantum space-time
Observational issue
present cosmological concordance model: $\Lambda$CDM model

95% of the Universe is not understood!!
either
- General Relativity is wrong, or
- particle physics is incomplete, or
- both (most likely)
→ expect major change in fundamental physics!
approaches to quantum & gravity

- direct quantization of (pure) gravity:
  - loop quantum gravity, canonical QG
  - causal dynamical triangulations
  - asymptotic safety
  - ...

(all have problems!)

- quantization of very special models which include gravity
  - string theory
  - Matrix Models (this talk!)

(landscape? def?)

- (holography)

- “quantum” (noncommutative) geometry described by Matrix Models
  physics emerges from fluctuations
### fuzzyness of space-time

Q.M. & G.R. $\Rightarrow$ break-down of classical space-time

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$\Rightarrow (\Delta x)^2 \geq \frac{G\hbar}{c^3} = L_{Pl}^2, \quad L_{Pl} = 10^{-33} cm$

Classical geometry breaks down, need pre-geometric framework

“fuzzy”, “foam-like” structure of space-time

J. Wheeler 1955

“Unanwendbarkeit der Geometrie im Kleinen”

Schrödinger 1934

Why worry? Because virtual quantum effects in QFT probe UV scale, very significant!!

(cf. UV divergences in 4D QFT)

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virtual quantum effects in QFT probe UV scale, very significant!!
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what is the quantum structure of geometry?

... lots of possibilities, generic problem:

- modifications of geometry in UV → uncontrollable, unexpected effects in IR (cf. UV divergences in 4D QFT)
- most approaches will fail, need protection mechanism

more precise statement: Doplicher Fredenhagen Roberts 1995

\[ \sum \Delta x^\mu \Delta x^\nu \geq L^2_{Pl} \]

... space-time uncertainty relations, follows from

\[ [X^\mu, X^\nu] = i\theta^{\mu\nu} \]

(cf. Q.M. !)

... noncommutative (quantum) space-time
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... noncommutative (quantum) space-time
Motivation Quantum geometry IKKT model, NC branes Cosmological space-times

quantized space-time & geometry

old idea: Schrödinger, Heisenberg, Snyder, ...

borrow idea from QM:

QM = quantization of phase space $[X, P] = i\hbar$

quantum space = quantization $[X^\mu, X^\nu] = i\theta^{\mu\nu}$ of space-time

inherent uncertainty of space(time)

use Q.M. techniques (coherent states, ...) for spacetime & geometry
basic message of GR:

cannot separate space-time ↔ matter

need unified model governing space-time, matter & interaction

task:

- find correct model which governs dynamical quantum space-times
  
  (many approaches conceivable, most will fail)

- find correct space-time solutions

- identify mechanisms for emergent physics
  
  (gauge theory & gravity)

how to choose model & framework?
two crucial insights:

**insight 1 (around 2000):**

Yang-Mills gauge theory arises from fluctuations of quantum spaces in Matrix Models

\[
\begin{align*}
[X^\mu, X^\nu] &= i\theta^{\mu\nu}, \\
[X^\mu + A^\mu, X^\nu + A^\nu] &= i\theta^{\mu\nu} + i\partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]
\end{align*}
\]

simpler than in classical space-time!! (use \([X^\mu, A] \sim i\theta^{\mu\nu} \partial_\nu A\))

\[
S = \text{Tr}[X^\mu, X^\nu][X^\mu, X^\nu] \sim \int F^{\mu\nu} F_{\mu\nu} + c
\]

what about gravity?

**insight 2:**

Matrix Models \(S = \text{Tr}[X^\mu, X^\nu][X^\mu, X^\nu] \rightarrow \) dynamical quantum spaces
useful guideline: Schomerus, Chu-Ho, Seiberg-Witten, ... \( \sim \) 1999

NC spaces & gauge theory arise on D-branes in string theory
... “NonCommutative (quantum) field theory”

problem: UV/IR mixing Minwalla, van Raamsdonk, Seiberg 1999

- virtual quantum effects in QFT probe shortest scales
- string-like behavior \( |x\rangle \langle y| \) at high energies, non-local
- UV divergences in QFT \( \rightsquigarrow \) new IR divergences

\( \rightarrow \) naive approaches will fail

avoided in one special model: IKKT matrix model

2-fold origin/interpretation:
- maximally SUSY Yang-Mills theory
- nonperturbative string theory
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Matrix Models as candidates for a fundamental theory

IKKT or IIB model

\[ S[X, \psi] = -\text{Tr} \left( [X^a, X^b][X^a', X^b']\eta_{aa'}\eta_{bb'} + \bar{\psi}\gamma_a[X^a, \psi] \right) \]

\[ X^a = X^a\dagger \in \text{Mat}(N, \mathbb{C}), \quad a = 0, ..., 9, \quad N \text{ large} \]

gauge symmetry \[ X^a \rightarrow UX^aU^{-1}, \quad SO(9, 1), \quad \text{SUSY} \]

proposed as non-perturbative definition of IIB string theory

- maximally SUSY Yang-Mills in 3+1 dim.
- quantization via matrix “path integral”

\[ Z = \int dXd\psi \ e^{iS[X]} \]

- cf. BFSS model 1996: matrix quantum mechanics (class. time)
leads to "matrix geometry" ($\approx$ NC geometry):

- $S_E \sim \text{Tr}[X^a, X^b]^2 \Rightarrow$ config's with small $[X^a, X^b] \neq 0$ dominate

  i.e. "almost-commutative" configurations

- $\exists$ quasi-coherent states $|x\rangle$, minimize $\sum_a \langle x|\Delta X^2_a|x\rangle$

- $(X^a) \approx \text{diag.}$, spectrum $\equiv: \mathcal{M} \subset \mathbb{R}^{10}$

- $\langle x|X^a|x'\rangle \approx \delta(x - x')x^a, \quad x \in \mathcal{M}$

- "condensation" of matrices $\rightarrow$ geometry:

**NC branes embedded in target space $\mathbb{R}^{10}$**

$X^a \sim x^a: \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$

matrices = observables $X^a = \text{quantized functions } x^a$ (cf. Q.M)
examples of matrix geometries:

- three $2 \times 2$ matrices

$$X^3 = \begin{pmatrix} x(1) \\ x(2) \end{pmatrix} = x(1)|1\rangle\langle 1| + x(2)|2\rangle\langle 2|,$$

$$X^1 + iX^2 = \begin{pmatrix} x(12) \\ x(21) \end{pmatrix} = x(12)|2\rangle\langle 1|,$$

$$X^1 - iX^2 = \begin{pmatrix} x(12) \\ x(21) \end{pmatrix} = x(21)|1\rangle\langle 2|,$$

describe two points at $x(1), \quad x(2) \in \mathbb{R}^D$

- off-diagonal matrices $\approx$ strings connecting branes

spectrum of $X^a \leftrightarrow$ location in $\mathbb{R}^D$
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• ↔ •

(“point branes”)

off-diagonal matrices $\approx$ strings connecting branes

spectrum of $X^a \leftrightarrow$ location in $\mathbb{R}^D$

• $\rightarrow$ minimal fuzzy sphere $S^2 \hookrightarrow \mathbb{R}^3$

$X^a = \sigma^a, \quad X_1^2 + X_2^2 + X_3^2 = \frac{3}{4}$
examples of matrix geometries:

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spectrum of $X^a \leftrightarrow$ location in $\mathbb{R}^D$

- minimal fuzzy sphere $S^2 \leftrightarrow \mathbb{R}^3$

$X^a = \sigma^a,$ $X_1^2 + X_2^2 + X_3^2 = \frac{3}{4}$
generic class of matrix geometries:

**fuzzy spaces = quantized Poisson manifolds (cf. phase space, QM)**

\[
X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^D
\]

quantization map:

\[
\mathcal{Q} : \mathcal{C}(\mathcal{M}) \rightarrow \text{End}(\mathcal{H}) \cong \text{Mat}(N, \mathbb{C}) \quad \mathcal{H} = \mathbb{C}^N
\]

such that

\[
\begin{align*}
\mathcal{Q}(f) \mathcal{Q}(g) &= \mathcal{Q}(fg) + O(\theta), \quad \theta \sim \frac{1}{N} \\
[\mathcal{Q}(f), \mathcal{Q}(g)] &= \mathcal{Q}(i\{f, g\}) + O(\theta^2)
\end{align*}
\]

\[
f = f(X) = f^\dagger \in \text{Mat}(N, \mathbb{C}) \quad \text{... quantized algebra of functions on } \mathcal{M}
\]

(cf. QM!)

in particular:

\[
X^a = \mathcal{Q}(x^a)
\]

typically

\[
\text{Tr} \mathcal{Q}(f) \sim \int_\mathcal{M} f(x)
\]
Example: the fuzzy sphere $S^2_N$

**classical $S^2$:**

$x^a : S^2 \hookrightarrow \mathbb{R}^3$

$$(x^1)^2 + (x^2)^2 + (x^3)^2 = 1$$

**fuzzy sphere $S^2_N$:**

3 matrices $X^a := \frac{1}{\sqrt{C_N}} \pi_N(J^a)$ ... spin $\frac{N-1}{2}$ generators of $su(2)$

$$
(X^1)^2 + (X^2)^2 + (X^3)^2 = 1,

[X^a, X^b] = \frac{i}{\sqrt{C_N}} \varepsilon^{abc} X^c \sim \frac{i}{N}, \quad C_N = \frac{1}{4}(N^2 - 1)
$$

algebra $\mathcal{A} = \text{Mat}(N, \mathbb{C})$ ... quantized functions on $S^2_N$

$S^2_N$ ... quantization of $S^2$ with Poisson bracket $\{x^a, x^b\} = \frac{2}{N} \varepsilon^{abc} x^c$
covariance: \( SO(3) \) rotations

\[
\mathfrak{so}(3) \times \mathcal{A} \rightarrow \mathcal{A}
\]

\[
(J^a, \phi) \mapsto [\pi_N(J^a), \phi]
\]

\( \rightarrow \) fuzzy spherical harmonics, \( \text{UV cutoff} \)

\[
\mathcal{A} = \text{Mat}(N, \mathbb{C}) \cong (N) \otimes (\bar{N}) = (1) \oplus (3) \oplus \ldots \oplus (2N-1)
\]

\[
= \{ \hat{Y}_0 \} \oplus \{ \hat{Y}_1 \} \oplus \ldots \oplus \{ \hat{Y}_{N-1} \}
\]

\( S_N^2 \) is fully covariant!

metric encoded in NC Laplace operator

\[
\Box : \mathcal{A} \rightarrow \mathcal{A},
\]

\[
\Box \phi = [X^a, [X^b, \phi]]\delta_{ab} = \frac{1}{C_N} J^a J^a \phi
\]

compute

\[
\Box \hat{Y}_l^m = \frac{1}{C_N} l(l+1) \hat{Y}_m^l
\]

spectrum identical with classical case

\[
\Delta g\phi = \frac{1}{\sqrt{|g|}} \partial_\mu (\sqrt{|g|} g^{\mu \nu} \partial_\nu \phi)
\]

\( \Rightarrow \) effective metric of \( \Box = \) round metric on \( S^2 \)
can measure such matrix geometries $\{X^a\}$:


measure energy $E(x)$ of string connecting $\mathcal{M}$ with point at $x \in \mathbb{R}^D$
location of $\mathcal{M} \subset \mathbb{R}^D \leftrightarrow$ minima of $E(x)$

Mathematica package “Bprobe”

DOI 10.5281/zenodo.45045
Schneiderbauer - HS

examples:

squared fuzzy $\mathbb{C}P^2_N \subset \mathbb{R}^6$
fuzzy torus $T^2_N$

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Motivation: Quantum geometry, IKKT model, NC branes, Cosmological space-times

4D covariant quantum spaces

- **Issue**: NC spaces: \([X^\mu, X^\nu] =: i\theta^{\mu\nu} \neq 0\) breaks Lorentz invariance.

- **However**: \(\exists\) fully \(SO(5)\) covariant fuzzy four-sphere \(S^4_N\)

  Grosse-Klimcik-Presnajder 1996; Castelino-Lee-Taylor; Ramgoolam; Kimura; Hasebe; Medina-O’Connor; Karabail-Nair; Zhang-Hu 2001 (QHE!) ...

  Complication/bonus: “internal structure”

  - Fluctuations includes spin 2 modes \(\rightarrow\) higher spin theory
  - “Gravity” naturally emerges


Euclidean case unphysical
∃ analogous

covariant cosmological quantum space-time solutions of IKKT models


- exactly homogeneous & isotropic
- finite density of microstates
- mechanism for Big Bang
starting point: Euclidean fuzzy hyperboloid $H^4_n$

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4, 2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}).$$

choose “short” discrete unitary irreps $\mathcal{H}_n$ ("minireps", doubletons)

special properties:

- irreducible under $\mathfrak{so}(4, 1)$, multiplicity-free
- positive discrete spectrum

$$\text{spec}(\mathcal{M}^{05}) = \{E_0, E_0 + 1, \ldots\}, \quad E_0 = 1 + \frac{n}{2}$$

- eigenspaces of $\mathcal{M}^{05}$ finite-dim.
fuzzy hyperboloid $H^4_n$:

def. 5 hermitian matrices

$$X^a := rM^a{}^5, \quad a = 0, \ldots, 4$$

satisfy

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^21, \quad R^2 = r^2(n^2 - 4)$$

$$[X^a, X^b] = ir^2M^{ab} =: i\Theta^{ab}$$

one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4,1)$

note: induced metric is Euclidean
oscillator construction: 4 bosonic oscillators \([\psi_\alpha, \bar{\psi}^\beta] = \delta^\beta_\alpha\)

then

\[ X^a = r \bar{\psi} \gamma^a \psi \]

on Fock space \(\mathcal{H}_n\)

\(\mathcal{H}^4_n\) is really quantized \(\mathbb{C}P^{1,2} = S^2\) bundle over \(H^4\), selfdual \(\theta^{\mu\nu}\)

(analogous to \(S^4_N\))

\(\text{spec}(X^0 = \mathcal{M}^{05})\) discrete, finite degeneracy

\(\Rightarrow\) finite density of microstates!
open FRW universe from projected $H^4_n$

$Y^\mu := X^\mu$, for $\mu = 0, 1, 2, 3$ (drop $X^4$ !)

= projection of $H^4_n$ in $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \mathbb{C}P^{1,2} \rightarrow H^4 \xrightarrow{\Pi} \mathbb{R}^{1,3}.$$ 

is solution of IKKT with $m^2 = -3r^2$, since

$$[Y^\mu, [Y^\mu, Y^\nu]] = ir^2[Y^\mu, M^{\mu\nu}] \quad \text{(no sum)}$$

$$= r^2 \begin{cases} 
Y^\nu, & \nu \neq \mu \neq 0 \\
-Y^\nu, & \nu \neq \mu = 0 \\
0, & \nu = \mu
\end{cases}$$

hence

$$\Box Y^\mu = [Y^\nu, [Y_\nu, Y^\mu]] = 3r^2 Y^\mu \quad (= \text{eom of IKKT})$$
Motivation
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properties:

- $SO(3, 1)$ manifest $\Rightarrow$ foliation into $SO(3, 1)$-invariant space-like 3-hyperboloids $H^3_i$ (homogeneous & isotropic!)
- double-covered FRW space-time with hyperbolic ($k = -1$) spatial geometries

\[ ds^2 = dt^2 - a(t)^2 d\Sigma^2, \]

\[ d\Sigma^2 \ldots SO(3, 1)\text{-invariant metric on space-like } H^3 \]
effective metric:

\[ \Box_Y = [Y_\mu, [Y^\mu, .]] \sim \frac{1}{\sqrt{|G|}} \partial_\mu (\sqrt{|G|} G^{\mu\nu} \partial_\nu) : \]

\[ G^{\mu\nu} = \alpha g^{\mu'\nu'} [\theta_{\mu'\mu} \theta_{\nu'\nu}] S^2, \quad \alpha = \sqrt{|\theta_{\mu\nu}|} \]

\[ \cong \alpha \text{ diag}(c_0(\eta), c(\eta), c(\eta), c(\eta)) \]

where \([.]._S^2 \ldots \text{ averaging over the internal } S^2\]

\[ c(\eta) = 1 - \frac{1}{3} \cosh^2(\eta) \]

\[ c_0(\eta) = \cosh^2(\eta) - 1 \geq 0 \]

signature change at \(c(\eta) = 0\)

\[ \cosh^2(\eta_0) = 3 \quad \ldots \text{Big Bang!} \]

Euclidean for \(\eta < \eta_0\), Minkowski \((+----)\) for \(\eta > \eta_0\)
**FRW metric and scale factor**

\[ ds_G^2 = dt^2 - a^2(t)d\Sigma^2 \]

with

\[ \frac{dy_0}{dt} = \frac{c_0(y_0)^{1/2}}{|c(y_0)|^{3/4}}, \quad a(t) = |c(y_0)|^{1/2} y_0^2. \]

**Big Bang:**

shortly after the BB:

\[ a(t) \propto c(t)^{1/4} \propto t^{1/7} \]

![Graph showing the scale factor a(t) vs. time t]
late times:

**linear coasting cosmology**

\[ a(t) \sim \frac{3\sqrt{3}}{2} t. \]
$a(t) \sim t$ is remarkably close to observation:

- age of univ. $13.9 \times 10^9$ y from present Hubble parameter similar to Milne Univ;

![Graph showing scale factor $a(t)$ vs. time $t/t_0$.](image)

- artificial within GR, natural in M.M.
- gravity should emerge below cosm. scales (?!)
- can reproduce SN1a (without acceleration)
- different physics for early universe (CMB etc.)

other features:

- ∃ Euclidean pre-BB era
- 2 sheets with opposite intrinsic “chirality”

- fluctuations on internal $S^2 \rightarrow$ higher-spin fluctuation modes

$$A^\mu = \theta^{\mu \nu} h_{\nu \rho}(x) P^\rho$$

expect (higher-spin-extended) gravity (spin 2 = gravity!)

towards particle physics from the IKKT model

IKKT model has 9+1 “dimensions“

idea (cf. string theory):

consider solutions $\mathcal{M}^{3,1} \times \mathcal{K}_N$ with 6 fuzzy extra dimensions

requires embedding of standard model fields in adjoint of $SU(N)$:

indeed:

A. Chatzistavrakidis, H.S., G. Zoupanos arXiv:1107.0265

$$\psi = \begin{pmatrix} 0_2 & 0 & 0 & I_L & Q_L \\ 0 & 0 & e_R & Q_R \\ 0 & 0 & \nu_R & Q_R \\ 0 & 0 & 0_3 & 0 \end{pmatrix},$$

where

$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$,

$I_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$,

$Q_R = \begin{pmatrix} d_R \\ u_R \end{pmatrix}$

similar to brane constructions of the S.M. in string theory
3 generations

from projected quantized coadjoint $SU(3)$ orbits in extra dim.

$Y_a = \pi_\mu(T_a), \quad T_a \ldots su(3)$ root generators

$C[\mu] \mapsto \mathbb{R}^8 \quad \pi \mapsto \mathbb{R}^6$

$(y^a)_{a=1,\ldots,8} \mapsto (y^a)_{a=1,2,4,5,6,7}$

= solutions of IKKT model with cubic term,

4- or 6-dimensional self-intersecting variety in $\mathbb{R}^6$

chiral fermions from strings linking self-intersecting sheets

$\psi_{\alpha,\Lambda} = |\mu'\rangle\langle \mu|$, \quad $|\mu\rangle$ ... coherent states

triple self-intersection $\rightarrow$ 3 generations!

no obstacle in principle to get (extended) standard model from IKKT, no need for string compactifications

the simplest possible model might actually work !?
understanding of fundamental physics is **not** complete

→ opportunities!

**matrix models**: promising framework for quantum theory of space-time & geometry

- analogous math as in Q.M.
- extremely simple, good UV behavior (IKKT model)
- all ingredients for gravity & particle physics
- cosmological space-times & BB from fuzzy $H_n^4$
  finite d.o.f. per volume
- gravity expected to emerge

... exciting potential, seize the opportunity!