Intersecting branes, strings & chirality in softly deformed $\mathcal{N} = 4$ SYM

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DESY Theory Workshop 2018

the amazing richness of large-N gauge theory

- $SU(N)$ $\mathcal{N} = 4$ SYM, deformed by cubic potential

  $\Rightarrow$ rich set of vacua = fuzzy branes in extra dim's $\mathbb{R}^4 \times \mathcal{K}_N$

  - intersecting branes, strings connecting branes, 3 generations, chirality
  - classical (Higgs) mechanism

- long-term goal/motivation:

  geometry & chiral physics from matrix models (IKKT)

  (without string landscape)

outline

- $SU(N)$ $\mathcal{N}=4$ SYM with cubic ("flux") deformation
- vacua with $SU(3)$ structure & self-intersecting branes
- zero modes, string states & chiral fermions
- new exact solutions: branes connected by strings
- stacks of branes $\rightsquigarrow$ Patti-Salam-like models

H.S., M. Sperling arXiv:1803.07323;
H.S., J. Zahn arXiv:1409.1440,
H.S. arXiv:1504.05703, arXiv:1411.3139

deformed $\mathcal{N} = 4$ SYM & self-intersecting branes

starting point: $SU(N)$ $\mathcal{N} = 4$ SYM

$$\boxed{\begin{aligned} S &= \int d^4x \frac{1}{4g^2} tr\Big(- F^{\mu\nu}F_{\mu\nu} - 2D^\mu\Phi^a D_\mu\Phi_a + [\Phi^a,\Phi^b][\Phi_a,\Phi_b]\Big) \\ &\quad + tr\Big(\bar\Psi\gamma^\mu iD_\mu\Psi + \bar\Psi\Gamma^a[\Phi_a,\Psi]\Big) \end{aligned}}$$

$$D_\mu = \partial_\mu - i[A_\mu, .]$$

- 6 scalar fields Φ^a, global $SO(6)_R = SU(4)_R$
- $N = 1$ SYM in 10 D dim. red. to 4D
- $(\gamma^\mu, \Gamma^a) = \Gamma^A$ ... 10D Clifford generators, $\Psi \to$ 4 Weyl fermions

... beautiful but too ”round“ for physics

dimensionless scalar fields

$$\left.\begin{array}{ll} mY_1^\pm & = (\phi_4 \pm i\phi_5) \\ mY_2^\pm & = (\phi_6 \mp i\phi_7) \\ mY_3^\pm & = (\phi_1 \mp i\phi_2) \end{array}\right\} \quad Y_\alpha^\pm, \quad \alpha = 1,2,3$$

consider Y_α^+ as (3) under $SU(3)_R \subset SU(4)_R$ (non-regular!)

add $SU(3)_R$-invariant soft susy breaking terms to potential

$$\mathcal{V}[\Phi] = \frac{m^4}{g^2}\left(V_4[Y] + V_{\text{soft}}[Y]\right)$$

$$\boxed{\begin{aligned} V_4[\Phi] &= -\text{tr}[Y_i^+, Y_j^+][Y_i^-, Y_j^-] + \tfrac{1}{2}\text{tr}(\textstyle\sum_i [Y_i^+, Y_i^-])(\sum_j [Y_j^+, Y_j^-]) \,, \\ V_{\text{soft}}[\Phi] &= \tfrac{4}{3}\text{tr}\left(-\varepsilon_{ijk} Y_i^+ Y_j^+ Y_k^+ + \text{h.c.} + 3M_i^2 Y_i^- Y_i^+\right) \end{aligned}}$$

- known: for $M_1 = 0 = M_2, M_3 = \sqrt{2}$: preserves $\mathcal{N} = 1$ SYM
  $\rightarrow$ fuzzy sphere solutions $Y_i^\pm \sim J_i$ (cf. Polchinski-Strasssler)
- new: for $M_i < \frac{2}{\sqrt{3}}$: SUSY broken,
  $\rightarrow$ squashed coadjoint $SU(3)$ orbits H.S., J. Zahn arXiv:1409.1440

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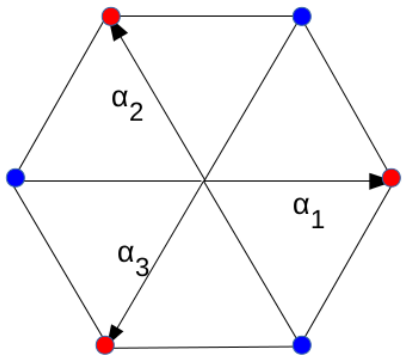
eom:

$$
\begin{aligned}
0 &= \left(\Box_4 + m^2(\Box_Y + 4M_i^2)\right) Y_i^+ + 4m^2 \varepsilon_{ijk} Y_j^- Y_k^- \\
\Box_Y &\equiv [Y_i^+, [Y_i^-, .]] + [Y_i^-, [Y_i^+, .]]
\end{aligned}
$$

squashed $SU(3)$ brane solutions

$$Y_i^\pm \equiv Y_{\pm\alpha_i} = r_i \, \pi(T_{\pm\alpha_i})$$

$T_{\pm\alpha_i}$ ... root generators of $\mathfrak{su}(3)$, any rep. π



(use $\mathfrak{su}(3)$ Lie algebra, weight lattice)

key:

view $Y_{\pm\alpha_i}$ as $(6) \subset (8)$ = (adjoint) of $SU(3)_Y \subset SU(N)$, dropping Cartan's

symmetries of background $Y_i^\pm \propto T_i^\pm$

- $SU(N)$ gauge symmetry (completely or partially) broken (Higgs) $\to$ massive KK modes!
- T_a are $SU(3)_Y \subset SU(N)$ intertwiners

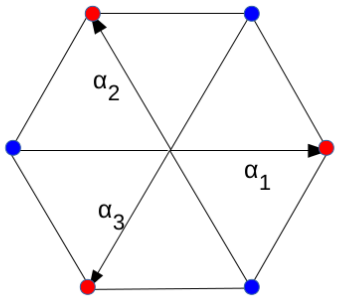
  $\to$ scalar fields $\phi_i^\pm$ transform as $\begin{cases} (3), (\bar{3}) & \text{under } SU(3)_R \\ (8) - 2 & \text{under } SU(3)_Y \end{cases}$
- breaks $SU(3)_R \nrightarrow U(1)_{K_1} \times U(1)_{K_2}$ up to gauge trafo

  residual symmetry generated by

$$K_i := \underbrace{2\tau_i}_{SU(3)_R} - \underbrace{[H_{\alpha_i}, .]}_{\text{gauge}}$$

$\Rightarrow$ $\mathfrak{su}(3)_{K_i}$ weight lattice respected

$\to$ organization of modes



internal geometry: squashed fuzzy coadjoint orbits

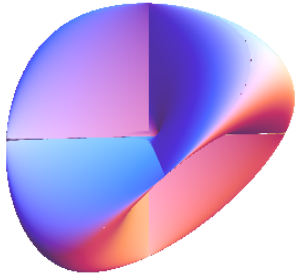
$Y_a = \pi_\mu(T_a)$ ... quantized coadjoint orbit projected along Cartans:

$$y^a : \quad \mathcal{C}[\mu] = \{g^{-1}\mu g;\ g \in SU(3)\} \quad \hookrightarrow \mathbb{R}^8 \cong \mathfrak{su}(3) \xrightarrow{\Pi} \mathbb{R}^6$$

... 4- or 6-dim. self-intersecting variety in $\mathbb{R}^6$ H.S., J. Zahn arxiv:1409.1440

- $\mu = (n, 0)$ or $\mu = (0, n)$:

  $\mathcal{C}[\mu]$ ... 4-dim. squashed $\mathbb{C}P^2$:



- $\mu = (n, m)$ generic:

  $\mathcal{C}[\mu]$... 6-dim. orbit $\xrightarrow{\Pi}$ 6 coinciding sheets

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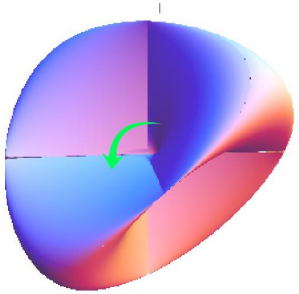
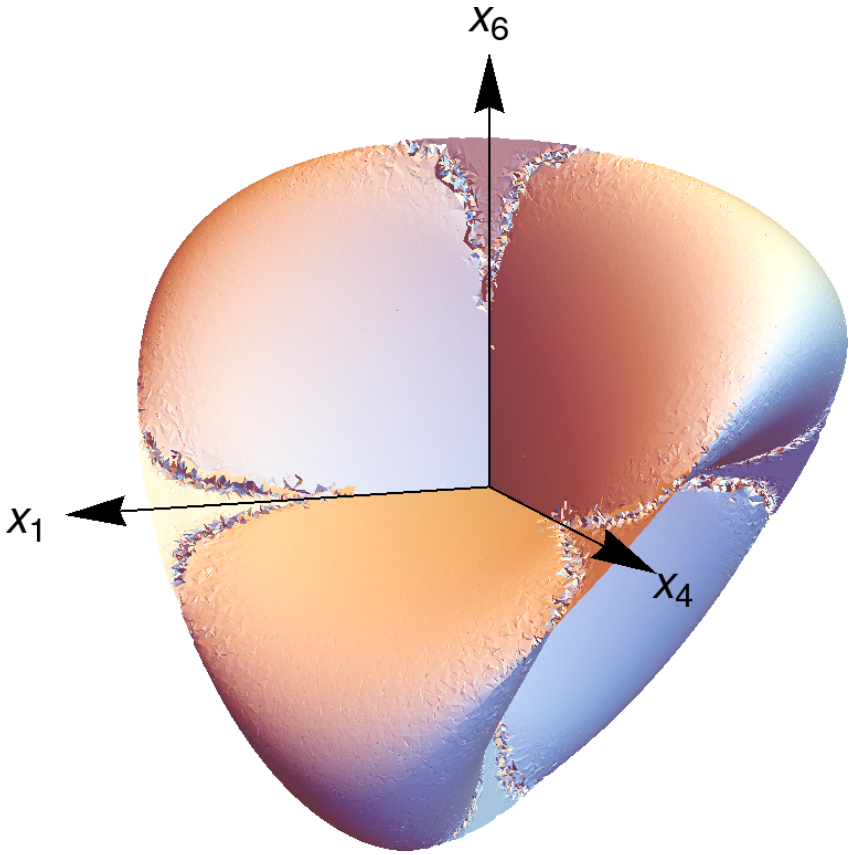


- $\mu = (n, m)$ generic:

  $\mathcal{C}[\mu]$... 6-dim. orbit $\xrightarrow{\Pi}$ 6 coinciding sheets

self-intersections $\Rightarrow$ zero-modes: strings linking sheets at origin

computer measurement of $Y^a = \pi_\mu(T^a)$:



squashed $\mathcal{C}_N[\mu]$ for $\mu = (20, 0)$.

L. Schneiderbauer, H.S. arXiv:1601.08007

$\mathcal{C}[\mu]$ is symplectic (Kirillov-Kostant) $\rightarrow$ correspondence

classical $\mathcal{C}[\mu]$	$\leftrightarrow$	quantized (fuzzy) space $\mathcal{C}_N[\mu]$
$\mu = (n_1, n_2)$ ... dom. int. weight	$\leftrightarrow$	$\mathcal{H}_\mu$ ... highest weight irrep
points $y \in \mathcal{C}[\mu]$	$\leftrightarrow$	coherent states $\vert y\rangle \in \mathcal{H}_\mu$
algebra of functions	$\leftrightarrow$	matrix algebra
$Fun(\mathcal{C}[\mu]) \cong \oplus_\Lambda m_\Lambda \mathcal{H}_\Lambda$	$\leftrightarrow$	$End(\mathcal{H}_\mu) \cong \mathcal{H}_\mu \otimes \mathcal{H}_\mu^*$
$\phi(y)$	$\overset{\mathcal{Q}}{\longleftrightarrow}$	$\int dy \phi(y) \vert y\rangle\langle y\vert$
Poisson brackets	$\leftrightarrow$	commutation relations
$\{y^a, y^b\} = f^{ab}_c \; y^c$		$[Y^a, Y^b] = i \, f^{ab}_c \; Y^c$
metric Laplacian $-\{y_a, \{y^a, .\}\}$	$\leftrightarrow$	matrix Laplacian $[Y_a, [Y^a, .]]$

fluctuation & extra dimensions

all fields (scalar, gauge fields, fermions) take values in

$\mathfrak{u}(N) = Mat(\mathcal{H}_\mu) \quad \cong$ functions on squashed $\mathcal{C}_N[\mu]$

... KK modes !

expand into $SU(3)_Y$ harmonics:

gauge fields: $A_\mu(x) = \sum A_\mu^{(\Lambda)}(x) \underbrace{\hat{Y}_\Lambda}_{SU(3)_Y \text{modes}} \equiv A_\mu(x,y)$

scalar fields: $\Phi^a = mY^a + \phi^a(x)$,

$$\phi_a(x) = \sum \phi_a^{(\Lambda)}(x)\hat{Y}_\Lambda \quad \equiv \phi_a(x,y)$$

fermions ... (similar)

dynamical generation of fuzzy extra dimensions,
recover massive KK tower

cf. Aschieri, Grammatikopoulos, HS and Zoupanos,hep-th/0606021

massive gauge bosons as KK modes

Higgs effect $\rightarrow$ gauge modes $A_\mu(x) = \sum A_\mu^{(\Lambda)}(x)\,\hat{Y}_\Lambda$ acquire mass

$$\int \mathrm{tr}(D_\mu \Phi_a)^\dagger D_\mu \Phi_a = \int \mathrm{tr}(D_\mu \phi_a^\dagger D_\mu \phi_a + \sum m_\Lambda^2 A^\dagger_{\mu,(\Lambda)} A^\mu_{(\Lambda)}) + S_{int}$$

using $D_\mu \Phi_a = (\partial_\mu + i[A_\mu, .])(mY_a + \phi_a)$

KK mass:

$$\begin{aligned} \Box_Y \hat{Y}_\Lambda \equiv [Y^a, [Y_a, \hat{Y}_\Lambda]] &= m_\Lambda^2 \hat{Y}_\Lambda \\ m_\Lambda^2 &= O(\tfrac{\Lambda^2}{g^2} m^2) > 0 \end{aligned}$$

$\Rightarrow$ tower of massive KK modes

geometric interpretation of Higgs effect

$A_\mu(x) = \sum A_\mu^{(\Lambda)}(x)\,\hat{Y}_\Lambda \equiv A_\mu(x,y)$ on $\mathbb{R}^4 \times \mathcal{C}[\mu]$

massive gauge modes = KK modes on $\mathcal{C}[\mu]$,

$\rightarrow$ effect. gauge theory on $\mathbb{R}^4 \times \mathcal{C}[\mu]$

similar for scalar fields & fermions

scalar modes & stringy zero modes on $\mathcal{C}[\mu]$

scalar fluctuations: $Y_\alpha \to Y_\alpha + \phi_\alpha$

quadratic potential for ϕ_α:

$$\begin{aligned} V_2[\phi] &= \mathrm{tr}\phi^\alpha \big(\Box_Y + 2\not{D}_{\mathrm{diag}} - 2\not{D}_{\mathrm{mix}}\big)\phi_\alpha, \\ (\not{D}_{\mathrm{mix}}\phi)_i^+ &= -\varepsilon_{ikj}[Y_k^-, \phi_j^-] \\ (\not{D}_{\mathrm{diag}}\phi)_\alpha &= [H_\alpha, \phi_\alpha] \qquad \text{(no sum)} \end{aligned}$$

respects $(U(1)\times U(1))_{K_i}$, $\quad \tau\not{D}_{\mathrm{mix}} = -\not{D}_{\mathrm{mix}}\tau$, $\quad \tau\phi_{\pm\alpha} = \pm\phi_{\pm\alpha}$

can show:

- no negative modes (!)
- $\exists$ zero modes: regular & exceptional H.S., J. Zahn arxiv:1409.1440
- stabilized e.g. by small M_i^2

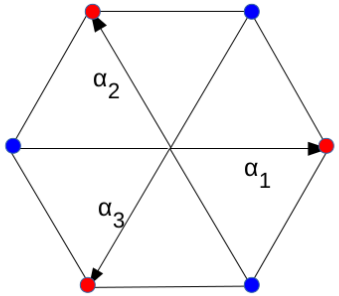
- regular zero modes: $\phi_i^{\pm}$, characterized by

decoupling condition:

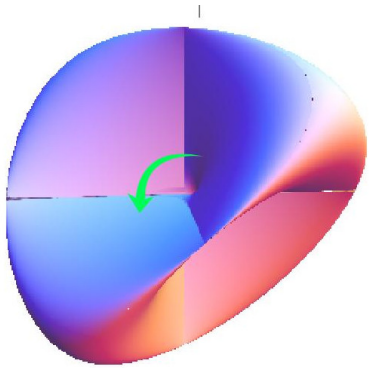
$$[Y_j^+, \phi_i^+] = 0 \quad \text{for } j \neq i, \quad \text{and} \quad [Y_i^-, \phi_i^+] = 0 \quad \text{(no sum).}$$

form ring, 3 generations H.S., M. Sperling arXiv:1803.07323

... 6 extremal charges of $U(1)_{K_i}$
for each $\mathfrak{su}(3)_Y$ irrep in $Mat(\mathcal{H}) \otimes (8)$



in particular:
strings $|\mu'\rangle\langle\mu|$ between intersecting sheets of $\mathcal{C}[\mu]$ a
$|\mu\rangle$ ... coherent states



"Higgs sector"

- exceptional zero modes:

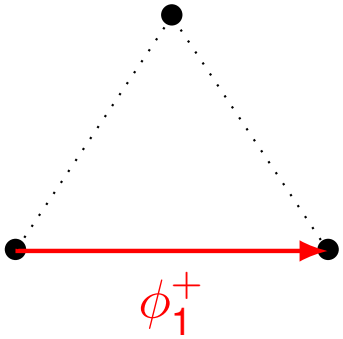
in particular: 6 Goldstone bosons (from $SU(3)/_{U(1)\times U(1)}$)

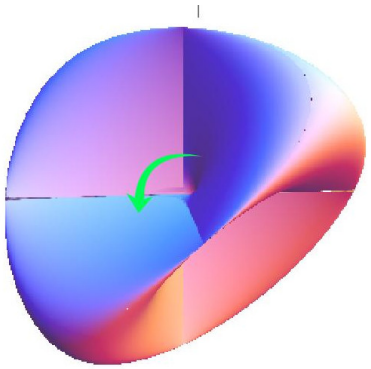
examples:

- squashed $\mathbb{C}P^2$ brane $\mathcal{C}[(N,0)]$ with regular zero mode:

$$\begin{aligned} Y_1^+ &= T_1^+ + \varepsilon_\alpha (T_1^-)^n, \qquad n = 1, ..., N \\ Y_2^+ &= T_2^+, \qquad Y_3^+ = T_3^+ \end{aligned}$$

maximal zero mode $(T_1^-)^N$ = string $|2\rangle\langle 1| \in End((N,0))$





- similarly: strings connecting stacks of branes

what about stability? full interactions?

FACTS for background brane + regular zero mode:

$$Y_\alpha = T_\alpha + \phi_\alpha$$

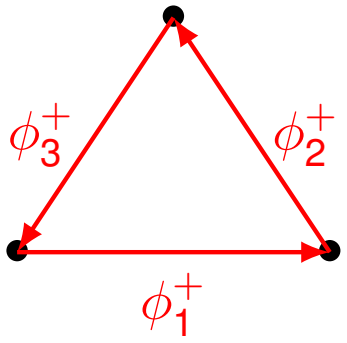
decoupling conditions $\Rightarrow$

- $V(Y+\phi) = V(Y) + V(\phi)$

  typically $V(Y), V(\phi) < 0$, "bound states"

- $eom(Y+\phi) = eom(Y) + eom(\phi)$

  $\Rightarrow$ $\exists$ exact solutions, "Higgs condensate" ϕ given by triple of maximal zero modes



because:

ϕ_α act on corners, form $(1,0)$ of $\mathfrak{su}(3)$

$\exists$ lots of solutions connecting stacks of branes

H.S., M. Sperling arXiv:1803.07323

stabilization of $Y_\alpha = T_\alpha + \phi_\alpha$ by masses M_i

- typically $\exists$ instabilities for $M_i = 0$
- can add masses $0 < M_i < \frac{1}{2}$, solutions persist
- numerical observation: stabilization achieved for range of M_i
- exact statement: rewrite potential as

$$V[X] = \frac{1}{2}\mathrm{tr}(F_l F_l^\dagger) + \frac{1}{2}\mathrm{tr}(DD^\dagger) + 4\sum_i (M_i^2 - \tfrac{2}{9})\mathrm{tr}(X_i^- X_i^+)$$

where

$$F_l = \sum_{i,j} \varepsilon_{lij}\left([X_i^+, X_j^+] - \tfrac{2}{3}\sum_k \varepsilon_{ijk} X_k^-\right), \qquad D = \sum_i [X_i^+, X_i^-]$$

$\Rightarrow$ $Y_\alpha = T_\alpha + \phi_\alpha$ exact global minima for

$$M_i = \frac{\sqrt{2}}{3} =: M^*$$

rich class of exact solutions H.S., M. Sperling arXiv:1803.07323
no instabilities, compact moduli space

fermions

Dirac operator on squashed $\mathcal{C}_N[\mu]$:

$$\not{D}_{(int)}\Psi \quad = \sum_{i=1}^{3} \left(\gamma_i[Y_i^+,.] + \gamma_i^\dagger[Y_1^-,.]\right)\Psi$$

zero modes:

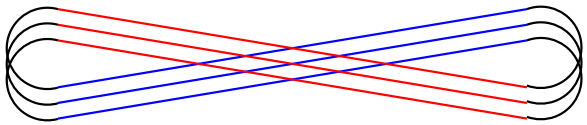
$$\not{D}_{(int)}\Psi_\alpha = 0$$

- in one-to-one correspondence to regular zero modes ϕ_α
  in particular: (fermionic) strings $\psi_\alpha \sim |x\rangle\langle 0|$ connecting $\mathcal{C}$

  ~~SUSY~~ no counterpart for exceptional zero modes
- have distinct chirality $\tau = \pm 1$ determined by flux on branes
- Yukawas from (stringy) zero mode condensates

chirality & zero modes on 6D branes $\mathcal{C}[(n,m)]$

3+3 locally space-filling branes with non-deg. flux



def. "chirality" generator

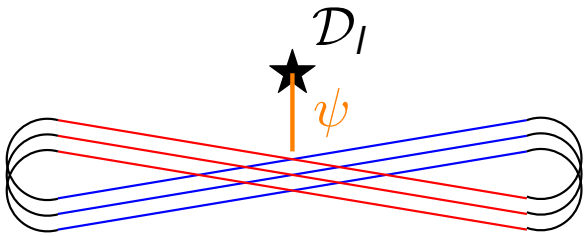
$$\begin{aligned}\chi &:= i\varepsilon^{(6)}_{abcdef}[T^a,T^b][T^c,T^d][T^e,T^f] \qquad \in \mathfrak{u}(N)\\ &\sim \varepsilon^{(6)}_{abcdef}\{y^a,y^b\}\{y^c,y^d\}\{y^e,y^f\} = \mathrm{Pf}\theta^{ab}\\ &\sim \mathbf{1}_L - \mathbf{1}_R\end{aligned}$$

corresponding "chiral" gauge field:

$$A_\mu = A_\mu(x)\chi\ .$$

- measures chirality of the L and R sheets,
- (among) lightest gauge modes

add point brane $\mathcal{D}_I$ $\Rightarrow$ 3L + 3R chiral fermionic zero modes Ψ:



$[\chi, \Psi] \sim \gamma_5 \Psi = \pm \Psi$

$\Rightarrow$ chirality of zero modes Ψ determined by charges of χ

$\rightarrow$ chiral gauge theory:

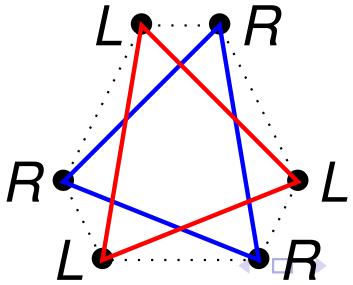
fermions with different chirality couple differently to (massive) gauge fields A_μ (cf. standard model in broken phase)

total index = 0

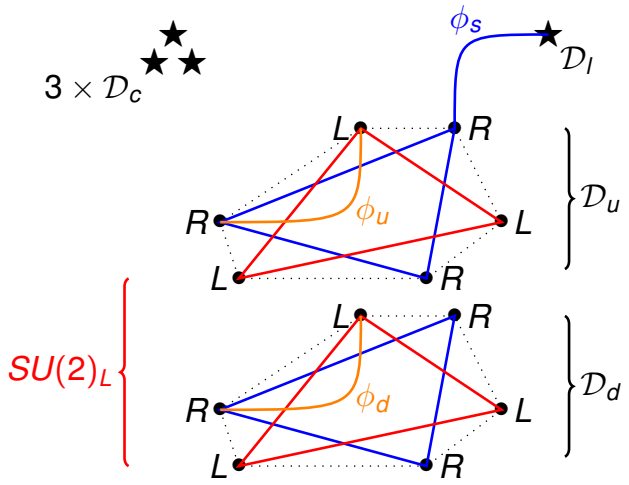
3L and 3R sheets stabilized & other modes removed by

chiral Higgs condensate:

=regular zero modes,
exact solution for $\mathcal{C}[(1,1)]$
conjectured for $\mathcal{C}[(N,1)]$



A Pati-Salam-like brane configuration



assume "Pati-Salam" Higgs ϕ_S, breaks gauge symmetry to

$$SU(3)_c \times U(1)_Q \times U(1)_{B'} \ .$$

$$\begin{aligned} Q &= \tfrac{1}{2}\big(\mathbf{1}_u - \mathbf{1}_d + \mathcal{L} - B\big) \ , \\ B &= \tfrac{1}{3}\mathbf{1}_c, \qquad \mathcal{L} = \mathbf{1}_l \\ Y &= \mathbf{1}_{Ru} - \mathbf{1}_{Rd} + \mathcal{L} - B \ . \end{aligned}$$

Higgs doublets ϕ_u, ϕ_d from zero modes

SM-like fermions, correct charges from zero modes linking branes

$$\Psi = \begin{pmatrix} *_2 & \tilde{H}_u & \tilde{H}_d & l_L & Q_L \\ & * & ? & \nu_R & u_R \\ & & * & e_R & d_R \\ & & & * & u' \\ & & & & *_3 \end{pmatrix},$$

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \qquad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix},$$

- 3 generations ($\mathbb{Z}_3$ Weyl rotations)
- all standard model fermions, superpartners, ν_R
- extra fermions $u', ?$, gauginos etc., no exotic charges
- rich Higgs sector (=zero modes), not fully understood
  2 Higgs doublets, 3 generations, Pati-Salam ϕ_S
- massless $U(1)_B$? (possibly resolved in matrix model)

summary

- rich class of vacuum solutions of deformed $SU(N)$ $\mathcal{N} = 4$ SYM
  $\rightarrow$ self-intersecting extra dim $\mathbb{R}^4 \times \mathcal{C}[\mu]$
- fluctuations $\rightarrow$ KK modes & string-like modes

large N gauge theory $\rightarrow$ small, rich low-energy sector
chiral low-energy physics possible, index 0

- can be fairly close to standard model (in broken phase)
- open issues:
  elaborate rich zero mode sector (Higgs) for $\mathcal{C}[(n, m)]$ branes
  quantum corrections (weak coupling)