The IKKT matrix model as a possible basis for (quantum) gravity & cosmology

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how does gravity fit into the quantum world?

Einstein equations:

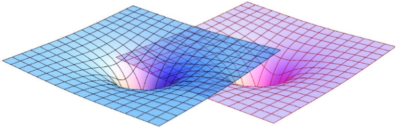
$$\underbrace{\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\mathcal{R} + \Lambda g_{\mu\nu}}_{\text{classical}} \stackrel{?}{=} 8\pi G_N \underbrace{T_{\mu\nu}}_{\text{quantum}}$$

not renormalizable ($G_N \sim L^2_{\rm pl}$ strong at short distances)

superposition of massive objects

$$\Downarrow$$

superposition of "gravitational potentials"

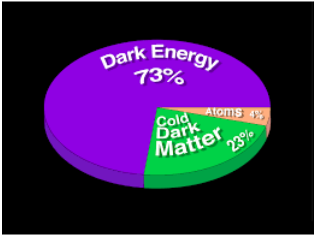


$\Rightarrow$ need quantum theory of geometry, different dof!

further motivation for modification of GR:

95 % of the universe is not understood !

ΛCDM model:



either

- general relativity is wrong, or
- particle physics is incomplete, or
- both (most likely)

$\rightarrow$ expect major change in fundamental physics sophisticated proposal: string theory

gives 9+1D (quantum) gravity ↯

$\rightarrow$ compactification $\rightarrow$ landscape ☹

non-perturbative approach defined by Matrix Models : IKKT, BFSS

- proposed as constructive def. of (corner of) string theory
- inherit magical properties of string theory (max. SUSY)

here: consider the IKKT = IIB model as fundamental starting point

can the IKKT model yield (near-)realistic $3+1$ dim. physics?

sophisticated proposal: string theory

gives 9+1D (quantum) gravity $\lightning$

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IKKT = IIB Matrix Model

Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S = Tr([Y^a, Y^b][Y_a, Y_b] + \bar{\Psi}\Gamma_a[Y^a, \Psi])$$

maximal SUSY, closely related to IIB string theory

Y^a, $a = 0, ..., 9$ $N \times N$ matrices, hermitian
Ψ ... matrix-valued MW spinor of $SO(1,9)$ hep-th/9612115

- gauge invariance $Y^a \to U^{-1} Y^a U$
- solutions / backgrounds = branes
  space-time = suitable 3+1d brane $\bar{Y}^a$ ("brane-world")
- well-suited for quantization:

$$Z = \int dY d\Psi e^{iS[Y,\Psi]}$$

1-loop $\to$ induced 3+1d gravity

unique model without pathological UV/IR mixing aside on string theory:

Q: isn't this just string theory in disguise?

A: maybe, but it provides a novel mechanism for gravity!

approach:

weakly coupled gauge theory on 3+1 dim. NC branes

$\to$ novel gauge theory for geometry/gravity (no holography!)

(NC $\mathcal{N} = 4$ SYM ... unique consistent 4D noncommutative QFT!)

physical modes on brane, nothing escapes into bulk (weak coupling!)

(much of 2000's literature on MM is about bulk physics)

avoids landscape: no compactification of target space!

relation with IIB string theory best seen via interactions of D-branes (1-loop) IKKT hep-th/9612115

(holographic dual, unphysical!)

can (and should!) put IKKT model on a computer!

extensive work by group around

Nishimura, Tsuchiya, Anagnostopoulos, etal. cf. 2307.01681

oscillating integral $\rightarrow$ requires sophisticated methods (Langevin, Lefshetz thimble, ...)

some evidence for SSB of $SO(9,1)$ & lower dimensional space-time brane, not yet conclusive novel mechanism for 3+1 gravity on branes $\mathcal{M}^{3,1} \times \mathcal{K}_N \subset \mathbb{R}^{9,1}$

- 1-loop $\to$ induced E-H action on brane
  for eff. metric (= open string metric)
- UV finite, reasonable cosmology without fine-tuning
- $\mathcal{K}_N$ finite, gives structure to low-energy gauge theory
- no compactification of target space, no landscape problem

"fuzzy space(time)" = quantized symplectic brane $\mathcal{M}^{3,1}$

(1-loop sugra = weak, short-range r^{-8} interaction on brane) novel mechanism for 3+1 gravity on branes $\mathcal{M}^{3,1} \times \mathcal{K}_N \subset \mathbb{R}^{9,1}$

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"fuzzy space(time)" = quantized symplectic brane $\mathcal{M}^{3,1}$

(1-loop sugra = weak, short-range r^{-8} interaction on brane) outline:

- geometric interpretation of Yang-Mills matrix models
- geometrical structures: frame, metric, torsion
- **quantization**: 1-loop effective action
  $\rightarrow$ Einstein-Hilbert action (+ extras)
- covariant FLRW quantum space-time $\mathcal{M}_n^{3,1}$ & higher spin

introductory review: arXiv:1911.03162
quantization & E-H action: arXiv:2303.08012, 2110.03936

book "Quantum Geometry, Matrix Models, and Gravity" (soon)

quantum spaces from matrix models

$$S = Tr([Y^a, Y^b][Y_a, Y_b] \; + ...)$$

expect: dominant configs = “almost-commuting” matrix configurations

$$[Y^a, Y^b] \approx 0$$

= quantized symplectic spaces

Y^a generates algebra of “functions” on $\mathcal{M}$

$$[Y^a, Y^b] \sim i\{y^a, y^b\} \qquad \text{...Poisson brackets}$$

semi-classical correspondence (cf. QM!)

$$\begin{aligned} \mathrm{End}(\mathcal{H}) \quad &\sim \quad \mathcal{C}(\mathcal{M}) \\ \Phi \quad &\sim \quad \phi(y) \\ [\Phi, \Psi] \quad &\sim \quad i\{\phi, \psi\} \\ \mathrm{Tr}\Phi \quad &\sim \quad \int_{\mathcal{M}} \Omega\phi, \qquad \Omega \text{ ... symp. volume} \end{aligned}$$

old idea: Schrödinger, Heisenberg, Snyder, ...

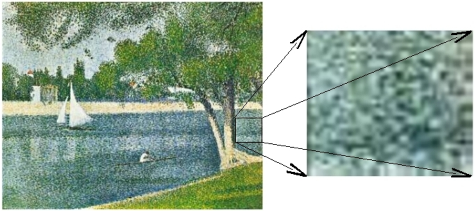
borrow idea from QM:

QM = quantization of phase space $[Q, P] = i\hbar$

$$Y^a = \begin{pmatrix} P \\ Q \end{pmatrix}$$ ... QM phase space = “quantum plane” $\mathbb{R}^2_\theta$

$\mathrm{End}(\mathcal{H})$ ... algebra of observables $\cong$ functions on phase space

quantum spacetime = quantization $[Y^a, Y^b] = i\theta^{ab}(Y)$ of spacetime



inherent uncertainty of space(time)

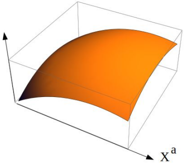
use Q.M. techniques (coherent states, ...) for spacetime & geometry in matrix model:

assume dominant / (meta-) stable

"vacuum" matrix configuration (solution) $Y^a \in \mathrm{End}(\mathcal{H})$

interpreted as embedded symplectic manifold $(\mathcal{M}, \omega)$ ("brane")

$$Y^a \sim y^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^D$$



physical meaning of Yang-Mills matrix models?

insight 1 (around 2000):

Yang-Mills gauge theory arises from fluctuations of quantum spaces in Matrix Models

$$
\begin{aligned}
[Y^a, Y^b] &= i\theta^{ab}, \qquad\qquad Y^a \to Y^a + \mathcal{A}^a \\
[Y^a + \mathcal{A}^a, Y^b + \mathcal{A}^b] &= i\theta^{ab} + i\theta^{aa'}\theta^{bb'}\underbrace{(\partial_a A_b - \partial_b A_a + i[A_a, A_b])}_{F_{ab}}
\end{aligned}
$$

simpler than on classical space-time !! (use $[Y^a, A] \sim i\theta^{ab}\partial_b A$)

$$
S = Tr[Y^a, Y^b][Y_a, Y_b] \sim \int F^{ab}F_{ab} + c
$$

... NC gauge theory; pathological upon quantization (UV/IR mixing, non-renormalizability) (Susskind/Toumbas, Jack, Jones etal) except

$\mathcal{N} = 4$ NC SYM $\cong$ IKKT model
contains geometrical theory encoded in $U(1)$ sector

what about gravity?

insight 2:

Matrix Models $S = Tr[Y^a, Y^b][Y_a, Y_b]$ $\rightarrow$ dynamical quantum spaces

Alekseev-Recknagel-Schomerus, IKKT, HS, ... 1999 ff

$\rightarrow$ "emergent gravity" (cf. Rivelles, HS, Yang, ...)

complementary / consistent with string theory

The effective metric in matrix models

consider transversal fluctuations = scalar fields $\phi \in \mathrm{End}(\mathcal{H})$

$$\begin{aligned} S[\phi] &= -Tr\eta_{ab}[Y^a,\phi][Y^b,\phi] \\ &\sim \int \rho_M\, \eta_{ab} E^{a\mu}\partial_\mu\phi E^{b\nu}\partial_\nu\phi \sim \int \sqrt{|G|}\, G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \end{aligned}$$

semi-classical frame & metric:

$$E^{a\mu} = \{Y^a, x^\mu\} \ \sim -i[Y^a, x^\mu]$$

divergence constraint $\nabla_\nu(\rho^{-2}E_a{}^\nu) = 0$ (Jacobi identity)

$$\begin{aligned} G^{\mu\nu} &= \rho^{-2}\eta_{ab}E^{a\mu}E^{b\nu} = \rho^{-2}\gamma^{\mu\nu} \\ \rho^2 &= \rho_M\sqrt{|\gamma^{\mu\nu}|} \qquad \text{....dilaton} \end{aligned}$$

governs all fluctuations in M.M, universal $\Rightarrow$ gravity !

no local Lorentz transformation of the frame!

Weitzenböck connection:

$$\nabla^{(W)} E_{\dot{a}} = 0 \qquad \text{(Weitzenböck)} \qquad \Rightarrow \qquad \nabla^{(W)} G^{\mu\nu} = 0$$

flat but torsion:

$$T_{\dot{a}\dot{b}} \equiv T[E_{\dot{a}}, E_{\dot{b}}] = \nabla_{\dot{a}} E_{\dot{b}} - \nabla_{\dot{b}} E_{\dot{a}} - [E_{\dot{a}}, E_{\dot{b}}]$$

can show (Jacobi):

$$\boxed{T_{\dot{a}\dot{b}}{}^{\mu} = \{\hat{\Theta}_{\dot{a}\dot{b}}, x^{\mu}\},} \qquad \hat{\Theta}_{\dot{a}\dot{b}} := -\{Y_{\dot{a}}, Y_{\dot{b}}\}$$

$$T_{\dot{a}} = dE_{\dot{a}}, \qquad E_{\dot{a}} = E_{\mu\dot{a}} dx^{\mu} \qquad \text{...coframe}$$

torsion tensor encodes field strength of the NC gauge theory

(HS arXiv:2002.02742 , cf. Langmann Szabo hep-th/0105094)

geometric form of the matrix eom $\{X^a, \{X_a, X^b\}\} = m^2 X^b$

Weitzenböck connection:

$$\boxed{\nabla^{(W)}_\nu T^\nu{}_{\rho\mu} + T_\nu{}^\sigma{}_\mu T_{\sigma\rho}{}^\nu = -m^2 \gamma_{\rho\mu}}$$

HS arXiv:2002.02742 , cf. Hanada-Kawai-Kimura hep-th/0508211

Levi-Civita connection:

$$\boxed{\nabla^{(G)\nu}\left(\rho^2 T_{\nu\mu}{}^{\dot{a}}\right) + \frac{1}{2} T^{(AS)\nu\sigma}{}_\mu T_{\nu\sigma}{}^{\dot{a}} = -m^2 E^{\dot{a}}{}_\mu}$$

and

$$\star T^{(AS)} = \tilde{T}_\mu dx^\mu, \qquad \tilde{T}_\mu = \rho^{-2}\partial_\mu \tilde{\rho}$$

..." gravitational axion"

Fredenhagen, HS arXiv: 2101.07297

- E-H action in terms of torsion: identity

$$\int d^4x\sqrt{|G|}\mathcal{R} = -\int d^4x\sqrt{|G|}\Big(\frac{7}{8}T^{\mu}{}_{\sigma\rho}\,T_{\mu\sigma'}{}^{\rho}G^{\sigma\sigma'} + \frac{3}{4}\tilde{T}_{\nu}\tilde{T}_{\mu}G^{\mu\nu}\Big)$$

(cf. teleparallel gravity) S. Fredenhagen, H.S. arxiv:2101.07297

- on-shell Ricci tensor

$$\begin{aligned}\mathcal{R}_{\nu\mu} &= \tfrac{1}{4}T^{(AS)\sigma}{}_{\rho\mu}\,T^{(AS)}{}_{\sigma}{}^{\rho}{}_{\nu} - T_{\mu\sigma}{}^{\rho}T_{\nu}{}^{\sigma}{}_{\rho} + 2\rho^{-2}\partial_\nu\rho\partial_\mu\rho \\ &\quad + \tfrac{1}{4}G_{\nu\mu}\big(T^{\sigma}{}_{\nu\delta}\,T_{\sigma}{}^{\nu}{}_{\rho}\,G^{\delta\rho} - \tfrac{1}{3}T^{(AS)\sigma}{}_{\rho\mu}\,T^{(AS)}{}_{\sigma}{}^{\rho}{}_{\nu}\,G^{\mu\nu}\big)\end{aligned}$$

quadratic in T and $\partial\rho$ $\Rightarrow$ linearized on-shell metric fluctuations on flat background are Ricci-flat

pre-gravity from classical matrix model:

dynamical geometry, lin. Ricci-flat, differs from GR at non-lin level

- bare action: $S \sim \int \frac{1}{g^2} \Theta_{\dot a \dot b} \Theta^{\dot a \dot b}$ ... 2 derivatives less than E-H

$$\int d^4x \sqrt{|G|}\mathcal{R} = \int d^4x \sqrt{|G|}\Big(-\frac{3}{4}\tilde T_\nu \tilde T_\mu G^{\mu\nu} - \frac{7}{8} T^\mu{}_{\sigma\rho} T_{\mu\sigma'}{}^\rho G^{\sigma\sigma'}\Big)$$

since

$$T^{\dot a \dot b \mu} = \{\Theta^{\dot a \dot b}, x^\mu\} \sim \partial \Theta^{\dot a \dot b} \qquad \big(\Theta^{\dot a \dot b} = \{Y^{\dot a}, Y^{\dot b}\} \big)$$

$\Rightarrow$ different from GR, expected to dominate on large scales

quantization is well-behaved!

- on covariant quantum spaces (later):
  - all gravitational dof, no ghosts, lin. Schwarzschild etc.

    Sperling, HS 1901.03522 , HS 1905.07255 ff
  - "reasonable" cosmology without any fine-tuning

    BBounce, $a(t) \sim \frac{3}{2} t$ at late times
  - Feynman propagator

    Karczmarek, HS 2207.00399; Battista, HS 2207.01295

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1-loop effective action and induced gravity

SUSY $\rightarrow$ mild quantum effects:

Idea:

Einstein-Hilbert action (+ extra) in the 1-loop effective action on $\mathcal{M}^{3,1}$ (cf. Sakharov '67)

$$\Gamma_{1-\mathrm{loop}} \ni \int\limits_{\mathcal{M}} T_{\nu\lambda}{}^{\mu} T_{\nu\lambda}{}^{\mu} + \ldots \sim \int\limits_{\mathcal{M}} d^4x \sqrt{G}\, m_{\mathcal{K}}^2 \mathcal{R}[G] + \ldots$$

requires presence of fuzzy extra dimensions $\mathcal{K}$

finite, no UV divergence / cutoff !!

nonperturbative quantization of MM:

$$Z = \int dY d\Psi e^{iS[Y,\Psi]}, \qquad S = S_{\rm IKKT} + i\varepsilon Y^a Y^b \delta_{ab}$$

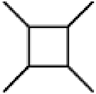
cf. numerical work (Nishimura, Tsuchiya, Anagnostopoulos etal.)

1-loop effective action

$$e^{i\Gamma_{\text{1-loop}}[Y]} = \int\limits_{\text{1 loop}} d\mathcal{A} d\Psi e^{iS[Y+\mathcal{A},\Psi]}$$

$$\begin{aligned}\Gamma_{\rm 1loop}[Y] &= \frac{1}{2}\mathrm{Tr}\Big(\log(\Box - M_{ab}[\Theta^{ab},.]) - \frac{1}{2}\log(\Box - M^{(\psi)}_{ab}[\Theta^{ab},.]) - 2\log(\Box)\Big)\\ &= \frac{1}{2}\mathrm{Tr}\Big(\sum_{n=4}^{\infty}\frac{1}{n}\Big((\Box^{-1}M_{ab}[\Theta^{ab},.])^n - \frac{1}{2}(\Box^{-1}M^{(\psi)}_{ab}[\Theta^{ab},.])^n\Big)\Big)\end{aligned}$$

UV-finite on 4D backgrounds due to max. SUSY !!

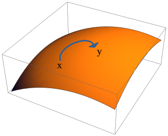


evaluate trace use string mode formalism

$$\mathrm{Tr}_{\mathrm{End}(\mathcal{H})}\mathcal{O} = \frac{1}{(2\pi)^m}\int\limits_{\mathcal{M}\times\mathcal{M}} dxdy \left(\begin{smallmatrix}x\\y\end{smallmatrix}\middle|\mathcal{O}\middle|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

string modes:

$$\boxed{\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) := |x\rangle\langle y|} \qquad \in \mathrm{End}(\mathcal{H})$$



$|x\rangle$ ... coherent state on $\mathcal{M}$

... “string” from x to y, extreme UV but non-local on any NC space

H.S. arXiv:1606.00646, cf. Iso Kawai Kitazawa hep-th/0001027

H.S., J. Tekel arXiv:2203.02376

$\approx$ diagonalize kinetic operators:

$$[Y^a, \left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)] \approx (x^a - y^a)\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

$$\Box\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right) \approx (|x-y|^2 + 2\Delta^2)\left|\begin{smallmatrix}x\\y\end{smallmatrix}\right)$$

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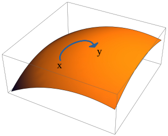

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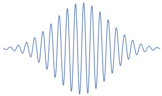
evaluation of 1-loop trace of IKKT model using string modes:

max. SUSY, UV-finite ⇒ short string modes ≅ plane wave packets dominate:

$$\Psi^{(L)}_{k;y} := \int d^4z\, e^{-|y-z|^2/L^2} \Big| {z+\tfrac{k}{2} \atop z-\tfrac{k}{2}} \Big) \cong e^{ikx} e^{-|x-y|^2/L^2}$$



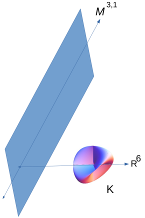
locally diagonalize kinetic operators in IR:

$$\begin{aligned} \Box \Psi^{(L)}_{k;y} &\approx \gamma^{\mu\nu}(x) k_\mu k_\nu \Psi^{(L)}_{k;y} \\ [\theta^{ab}, \Psi^{(L)}_{k;y}] &\approx -\{\theta^{ab}, x^\mu\} k_\mu \Psi^{(L)}_{k;y} \end{aligned}$$

trace formula for UV-finite traces on NC spaces:

$$\mathrm{Tr}\mathcal{O} = \frac{1}{(2\pi)^m} \int\limits_{\mathcal{M}\times\mathcal{M}} \Omega_x \Omega_y \left({x \atop y} \Big| \mathcal{O} \Big| {x \atop y} \right) \approx \frac{1}{(2\pi)^m} \int\limits_{\mathcal{M}} \sqrt{G} dx \int \frac{1}{\sqrt{G}} dk \langle \Psi^{(L)}_{k,x}, \mathcal{O} \Psi^{(L)}_{k,x} \rangle$$

use this to evaluate 1-loop eff. action



a priori: 4-derivative action ☹

however: brane $\mathcal{M} \times \mathcal{K} \subset \mathbb{R}^{9,1}$ with fuzzy extra dim.

from 6 transversal directions $\langle \phi^i \rangle \neq 0$

mixed term $(\delta\Theta^{\alpha\beta}\delta\Theta^{\alpha\beta})(\delta\Theta^{ij}\delta\Theta^{ij})$ leads to induced E-H action ☺

$$\begin{aligned}\{\theta^{\alpha\beta},\{\theta^{\alpha\beta},\psi_{k;y}\}\} &\approx -\{\theta^{\alpha\beta},x^\mu\}\{\theta^{\alpha\beta},x^\nu\}k_\mu k_\nu \psi_{k;y}\\ &= -T^{\alpha\beta\mu}k_\mu T^{\alpha\beta\nu}k_\nu\psi_{k;y}\end{aligned}$$

(torsion $T^{\alpha\beta\mu} = \{\theta^{\alpha\beta},x^\mu\}$)

$$\boxed{\begin{aligned}\Gamma_{\rm 1loop} &\sim -\int_{\mathcal{M}} d^4x\sqrt{G}\,c_{\mathcal{K}}^2 m_{\mathcal{K}}^2\, T^\rho{}_{\sigma\mu}T_{\rho'}{}^\sigma{}_\mu G^{\mu\mu'}\\ &\sim \int d^4x\sqrt{G}\,c_{\mathcal{K}}^2 m_{\mathcal{K}}^2\left(8\mathcal{R}[G] + 6\tilde{T}_\nu\tilde{T}_\mu G^{\mu\nu}\right)\end{aligned}}$$

where $m_{\mathcal{K}}^2$ ... KK scale on $\mathcal{K}$ HS 2110.03936 bottom line:

- $\Gamma_{\rm 1loop}$ includes Einstein-Hilbert action, eff. Newton constant

$$G_N \sim \frac{\rho^2}{c_{\mathcal{K}}^2 m_{\mathcal{K}}^2}$$

set by Kaluza-Klein mass scale on $\mathcal{K}$

- large vacuum energy

$$\Gamma_{\rm 1loop}^{\mathcal{K}} \sim -\int\limits_{\mathcal{M}} \Omega\, \rho^{-2} m_{\mathcal{K}}^4 \sum_{\Lambda s} \frac{V_{4,\Lambda}}{\mu_\Lambda^4} + \ldots$$

not c.c., leads to stabilization of $m_{\mathcal{K}}$ at one loop !

HS 2303.08012

- $\mathcal{K}$ also leads to interesting low-energy gauge theory (e.g. self-intersecting brane & chiral Ψ, 1803.07323)

$$S \sim \int \Theta^{ab}\Theta^{ab} + S_{E-H}$$

two scaling regimes:

- cosmic scale: bare M.M. $S \sim \int \Theta^{ab}\Theta^{ab}$ dominates, stabilizes FLRW space-time irrespective of matter

  M. Sperling, HS 1901.03522

- "intermediate" scale: 1-loop term dominates, expect to recover $\approx$ GR + extra modes

4D covariant quantum spaces & $\mathfrak{hs}$

issues on "basic" NC branes:

- Poisson structure $\theta^{\mu\nu}$ breaks Lorentz / rotation invariance
- enough dof for metric, frame ?

quantized twistor space as brane:

$$\mathbb{C}P^{1,2}_N \overset{loc}{\cong} S^2 \times \mathcal{M}^{3,1} \qquad \subset \mathbb{R}^{9,1}$$

... sympl. equivariant S^2- bundle over space(time) $\mathcal{M}^{3,1}$

- $\langle\theta^{\mu\nu}\rangle_{\mathcal{M}} = 0$!
- price to pay: higher-spin theory, all dof for metric on $\mathcal{M}^{3,1}$
- vol.-preserving diffeos on $\mathcal{M} \subset$ higher-dim symplectomorphisms

HS : 1606.00769 , M. Sperling, HS 1806.05 ff, HS , T. Tran 2203.05436

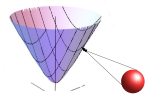
MM description: 2-step procedure

- MM background

$$Y^a := \frac{1}{R}\mathcal{M}^{a5}, \;\; a = 0, ..., 4$$

for $\mathcal{H}_n$ ... doubleton unitary irrep of $\mathfrak{so}(4,2)$

$\cong \mathbb{C}P^{1,2}_n \;\; = \;$ quantized S^2_n -bundle over H^4_n



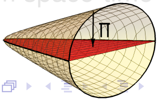
harmonics:

$$\mathrm{End}(\mathcal{H}_n) \cong \; \mathcal{C}(\mathbb{C}P^{1,2}) \; \cong \; \bigoplus_{s=0}^{n} \mathcal{C}^s$$

would-be KK modes $\to$ spin s modes on H^4 in $\mathfrak{hs} = \oplus \begin{array}{|c|c|c|}\hline \; & \; & \; \\ \hline \; & \; & \; \\ \hline \end{array}$

matrix model $\to$ higher spin gauge theory, truncated at n

- further projection $H^4 \to \mathcal{M}^{3,1}$ ... FLRW quantum space-time
manifest homogeneous & isotrop, Big Bounce



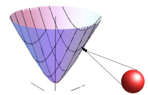
MM description: 2-step procedure

- MM background

$$Y^a := \frac{1}{R}\mathcal{M}^{a5}, \;\; a = 0, ..., 4$$

for $\mathcal{H}_n$ ... doubleton unitary irrep of $\mathfrak{so}(4,2)$

$\cong \mathbb{C}P^{1,2}_n \;\; = \;$ quantized S^2_n -bundle over H^4_n



harmonics:

$$\mathrm{End}(\mathcal{H}_n) \cong \; \mathcal{C}(\mathbb{C}P^{1,2}) \; \cong \bigoplus_{s=0}^{n} \mathcal{C}^s$$

would-be KK modes $\to$ spin s modes on H^4 in $\mathfrak{hs} = \oplus\, \boxed{}\boxed{}\boxed{}$

matrix model $\to$ higher spin gauge theory, truncated at n

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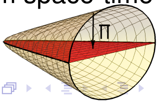
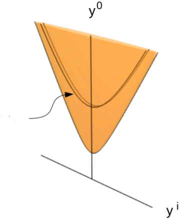


cosmological FLRW space-time solution:

very simple: $Y^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}$



- manifest $SO(3,1)$ $\Rightarrow$ foliation into space-like 3-hyperboloids H^3_τ
- $\Box = [\bar{Y}^\mu, [\bar{Y}_\mu, .]] \sim \Box_G$ encodes FLRW metric

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

  double-covered space-time ($k = -1$)

  $a(t) \propto t\ ,\qquad t \to \infty$ ... 'reasonable', coasting universe

- Big Bounce $a(t) \sim (t - t_0)^{1/5}$

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summary & outlook

gravity arises as quantum effect on 3+1-dim. quantum space-time in the IKKT matrix model

- MM = "pre-gravity", suitable for quantization
- quantization $\rightarrow$ induced Einstein-Hilbert action, no c.c. problem (?)
- cross-over GR $\leftrightarrow$ cosm. regime
- covariant quantum spaces = twisted S^2 bundles over $\mathcal{M}^{3,1}$
  $\rightarrow$ higher spin gauge theory
  rotation invariance manifest
- new physics (axion, dilaton, $\mathfrak{hs}$ ...)

IKKT = distinguished model for emergent near-realistic (?) physics
string theory without compactification

modified Einstein equation at one loop:

$$\begin{aligned}\mathcal{R}_{\mu\lambda} - \tfrac{1}{2} G_{\mu\lambda}\mathcal{R} \quad &= 8\pi G_N \Big[T^{(m)}_{\mu\lambda} - \rho^{-4} G_{\mu\lambda}\big(2\rho^2 \mathcal{F}_{\mathcal{K}}^2 m_{\mathcal{K}}^4 - C_1 m_{\mathcal{K}}^4 + \frac{C_2}{R^4} + 3\frac{C_3}{R^8 m_{\mathcal{K}}^4}\big) \\ &\quad + 4\big(C_{\mu\lambda} - \tfrac{1}{2} G_{\mu\lambda} C\big)\Big] \\ &\quad + 2\big(\partial_\mu\sigma\partial_\lambda\sigma - \partial_\mu\partial_\lambda\sigma + G_{\mu\lambda}(\Box_G\sigma - \tfrac{3}{2}\partial\sigma\cdot\partial\sigma)\big)\end{aligned}$$

$C_{\mu\nu}$ ... ”anharmonicity“

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Fuzzy extra dimensions $\mathcal{K}$

consider backgrounds with product structure

$$\mathcal{M}^{3,1} \times \mathcal{K} \qquad\qquad (\subset \mathbb{R}^{9,1}!)$$

$\mathcal{K}$ ... quantized compact symplectic space, e.g. $S^2_N, \ldots$

realized by

$$\begin{aligned} Y^{\dot a} &\sim y^{\dot a}: \qquad \mathcal{M} \hookrightarrow \mathbb{R}^{3,1}, \qquad \dot a = 0, \ldots, 3 \\ Y^{i} &\sim y^{i}: \qquad \mathcal{K} \hookrightarrow \mathbb{R}^{6}, \qquad i = 4, \ldots, 9 \end{aligned}$$

matrix d'Alembertian decomposes as

$$\Box = [Y^{\dot a}, [Y_{\dot a}, .]] + [Y^i, [Y_i, .]] = \Box_{\mathcal{M}} + \Box_{\mathcal{K}} \ .$$

internal $\Box_{\mathcal{K}}$ has a positive spectrum

$$\Box_{\mathcal{K}} \lambda_\Lambda = m_\Lambda^2 \lambda_\Lambda$$

hence

$$\Box \phi_\Lambda = (\Box_{\mathcal{M}} + m_\Lambda^2)\phi_\Lambda \ , \qquad\qquad [\Theta^{ij}, [\Theta^{ij}, \lambda_\Lambda]] = m_{\mathcal{K}}^4 C_\Lambda^2 \lambda_\Lambda$$

(= Higgs mechanism!)

gauge transformations as diffeos

... arise from $Y^a \to U^{-1} Y^a U$ on NC branes $\mathcal{M}$

scalar fields:

$$\delta_\Lambda \phi = \{\Lambda, \phi\} = \xi^\mu \partial_\mu \phi = \mathcal{L}_\xi \phi, \qquad \xi^\mu = \{\Lambda, x^\mu\}$$

vector fields (frame!):

$$\begin{aligned} \delta_\Lambda Y_{\dot\alpha} &= \{\Lambda, Y_{\dot\alpha}\} \\ \delta_\Lambda E_{\dot\alpha} &= \{\Lambda, \{Y_{\dot\alpha}, .\}\} - \{Y_{\dot\alpha}, \{\Lambda, .\}\} = \mathcal{L}_\xi E_{\dot\alpha} \qquad \text{(Jacobi)} \end{aligned}$$

hence

$$\boxed{\delta_\Lambda E_{\dot\alpha}{}^\mu = \mathcal{L}_\xi E_{\dot\alpha}{}^\mu, \qquad \delta_\Lambda G^{\mu\nu} = \mathcal{L}_\xi G^{\mu\nu}}$$

diffeos from NC gauge trafos!

$\{\Lambda, .\}$ ... Hamiltonian VF

on covariant quantum space $\mathcal{M}^{3,1}$: all dof for vol-preserving diffeos more precisely:

semi-classical correspondence:

approximate isometry in IR regime

$$\begin{array}{ccc} \mathrm{End}(\mathcal{H}) & & \mathcal{C}(\mathcal{M}) \\ \cup & & \cup \\ \mathrm{Loc}(\mathcal{H}) & \cong & \mathcal{C}_{\mathrm{IR}}(\mathcal{M}) \end{array}$$

"almost-commutative" = sufficiently large semi-classical IR regime