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Quantized D-branes on group manifolds

Harold Steinacker

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hep-th/0203110, J. Pawelczyk, H. Steinacker

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Outline:

- Review of D-branes in WZW models
- The quantum algebraic description
- Comparison with the CFT results
- Examples: Fuzzy S_q^2, $\mathbb{C}P_q^N$

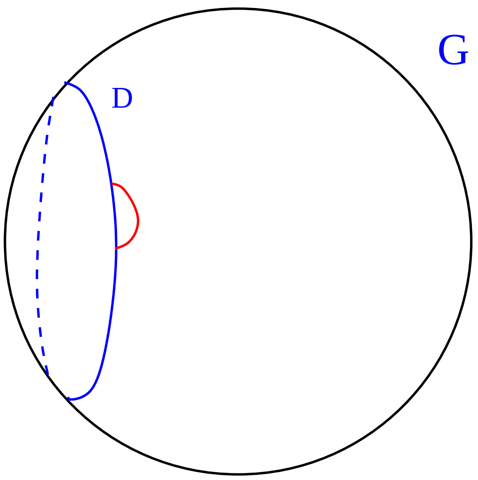
D-branes on groups: CFT description

(Alekseev, Schomerus; Bachas, Douglas, Schweigert; ...)

consider D-branes on (matrix) group

$$G = SU(N),\ SO(N),\ USp(N)$$

with boundaries (open strings):



described by WZW model on G at level k

$$S_\Sigma = \frac{k}{4\pi} \int_\Sigma tr(\partial_+ g \partial_- g^{-1}) + \int_\Sigma B + \int_{\partial\Sigma} A$$
$$H = \frac{k}{12\pi} tr(g^{-1} dg)^3 = dB$$

nontrivial B field

$\hookrightarrow$ expect non-commutative D-branes

(B)CFT results:

for simplest = untwisted D-branes:

boundary conditions preserve one copy

$$\widehat{\mathfrak{g}}_V \subset \widehat{\mathfrak{g}}_L \times \widehat{\mathfrak{g}}_R$$

of the bulk symmetries (ALA).

($\widehat{\mathfrak{g}}_V$ = "unbroken" symmetry algebra of (B)CFT)

$\rightarrow$ Cardy boundary states:

one for each primary field λ in the theory:

$$\boxed{\lambda \in P_k^+ = \{\lambda \in P^+;\ \lambda \cdot \theta \leq k\}; \quad \theta = \text{highest root}}$$

= integrable reps of $\widehat{\mathfrak{g}}_V$.

geometrical picture:

e.g. by scattering of closed strings

(Felder, Fuchs, Fröhlich, Schweigert; ...)

untwisted D-branes = conjugacy classes in G

$$\mathcal{C}(t) = \{gtg^{-1}; \quad g \in G\}$$

Result:

$$\boxed{D_\lambda = \mathcal{C}(t_\lambda),} \quad t_\lambda = q^{2(H_\lambda + H_\rho)}$$

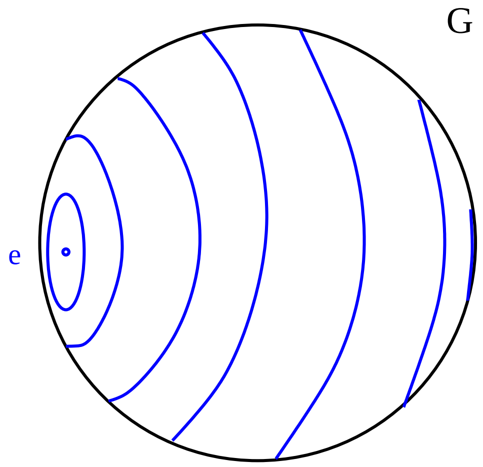
for $\lambda \in P_k^+$

... finite set of stable D-branes

geometrical picture:

$$\boxed{\mathcal{C}(t) = \{gtg^{-1}; \quad g \in G\}} \quad \cong G/K_t,$$

$$K_t = \{g \in G : \ [g,t] = 0\} \text{ ... stabilizer}$$



Symmetries: for untwisted branes:

geometric: $G_V \subset G_L \times G_R$

algebraic: $\widehat{\mathfrak{g}}_V \subset \widehat{\mathfrak{g}}_L \times \widehat{\mathfrak{g}}_R$ (ALA at level k)

Fact: $\exists$ close relation $\widehat{\mathfrak{g}} \leftrightarrow U_q(\mathfrak{g})$

(= Drinfeld-Jimbo quantized univ. enveloping algebra)

for

$$\boxed{q = e^{\frac{i\pi}{k+g^\vee}}}$$

representation theory is the same

Further results:

Position of D_λ in G:

conveniently measured by

$$\boxed{s_n = Tr(\mathcal{C}(t_\lambda)^n) = Tr(t_\lambda^n) = \sum_{\nu \in V_N} q^{2n(\rho+\lambda)\cdot\nu}}$$

$n = 1, 2, ..., rank(G)$.

alternatively: characteristic polynomial

$$P_\lambda(t) = \prod_{\nu \in V_N} (t - q^{2(\lambda+\rho)\cdot\nu}).$$

Energy of D_λ:

$$\boxed{E_\lambda = \dim_q(V_\lambda) = \prod_{\alpha>0} \frac{\sin\left(\pi \frac{\alpha\cdot(\lambda+\rho)}{k+N}\right)}{\sin\left(\pi \frac{\alpha\cdot\rho}{k+N}\right)}}$$

V_λ ... highest-weight representation of $\mathfrak{g}$

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on other hand: D-branes can be interpreted as

bound states of $D0$-branes

leading contribution in $1/k$ (then $SU(2)$ becomes flat):

$\exists$ description in terms of matrix models *(Myers, ...)*

$\hookrightarrow$ fuzzy spheres

Goal: effective description of these D-branes for all k.

Functions on a D-brane

determined by

OPE = algebra of boundary vertex op's in WZW model

Space of harmonics on classical $\mathcal{C}(t)$ (under adjoint):

recall $\mathcal{C}(t) \cong G/K_t$:

$$\mathrm{Fun}(\mathcal{C}(t)) \cong \bigoplus_{\mu \in P^+} N_t^\mu \, V_\mu$$

$N_t^\mu = mult_{\mu^+}^{(K_t)}$ = dim. of K_t-invariant subspace of V_{μ^+}

for $SU(2)$:

$$\mathrm{Fun}(S^2) \cong \bigoplus_{L \in \mathbb{N}} \{Y_m^L\}$$

Space of harmonics on D-brane D_λ (boundary primaries):

$$\boxed{\mathrm{Fun}(D_\lambda) \cong \bigoplus_{\mu \in P^+}^{\max_k(\mu)} \hat{N}_\lambda^\mu \, V_\mu .}$$

where $\max_k(\mu)$ ... some cutoff in μ

$$\hat{N}_\lambda^\mu = \hat{N}_{\lambda\lambda^+}^\mu \quad \text{... fusion rules of } \hat{\mathfrak{g}} \text{ or } U_q(\mathfrak{g})$$

can show: $\hat{N}_\lambda^\mu \approx N_{t_\lambda}^\mu$

Algebra of harmonics on D-brane D_λ (OPE)

for $SU(2)$:

$$Y_i^I(x)\, Y_j^J(x') \quad \sim \sum_{K,k} (x-x')^{h_I+h_J-h_K}.$$

$$\cdot \underbrace{\begin{bmatrix} I & J & K \\ i & j & k \end{bmatrix}}_{Clebsch} \underbrace{\left\{ \begin{matrix} I & J & K \\ \lambda & \lambda & \lambda \end{matrix} \right\}_q}_{6j\ (q)} Y_k^K(x')$$

x, x' ... boundary of worldsheet

Y_i^I ... boundary primaries $\cong$ harmonics on the brane

$h_I = \frac{I(I+1)}{k+2}$ ... conformal weights

for general G: analogous; involves

fusion matrices = generalized $6j$ - symbols of $U_q(\mathfrak{g})$

$$\boxed{q = e^{\frac{i\pi}{k+g^\vee}}}$$

simplify:

- omit world-sheet dependence x, x'

  (exact for $k \to \infty$)

  $\to$ quasi-associative algebra

  (Alekseev, Recknagel,Schomerus)

- Drinfeld – twist $\Rightarrow$ associative

$\hookrightarrow$ effective algebra

$$\boxed{Y_i^I \; Y_j^J = \sum_{K,k} \left[\begin{array}{ccc} I & J & K \\ i & j & k \end{array} \right]_q \left\{ \begin{array}{ccc} I & J & K \\ \lambda & \lambda & \lambda \end{array} \right\}_q Y_k^K}$$

$= S^2_{q,\lambda}$ ... q-deformed fuzzy sphere

associative!

cutoffs: $K \leq \min(I + J, k - I - J, \lambda, k - \lambda)$

generalizes to other groups G.

involves fusion matrices of $\hat{\mathfrak{g}} \cong 6j$ - symbols of $U_q(\mathfrak{g})$.

can be interpreted as non-commutative algebra of functions on the brane.

nice for ONE brane, BUT:

want a global picture of branes on the group G

explicit NC algebra of functions on branes, group G

(cp. $[x_i, x_j] = i\theta_{ij}$ for flat branes)

guidelines:

- OPE
- $\exists$ analog of $G_V \subset G_L \times G_R$ covariance
- $U_q(\mathfrak{g}_V) \sim \hat{\mathfrak{g}}_V$ on branes

approach: "educated guess" & compare with CFT

Quantized algebra of functions on G

Let $M = M^i_j$ ... NC coordinate functions on G

Reflection equation: *Kulish; Majid*

$$\boxed{\begin{aligned} R_{21} M_1 R_{12} M_2 &= M_2 R_{21} M_1 R_{12} \\ \det_q(M) &= 1 \end{aligned}}$$

where $R_{12} = R^{ij}_{kl}$ ... "R - matrix" of $U_q(\mathfrak{g})$.

(& other constraints for $SO(N), USP(N)$)

generates

$\boxed{\mathcal{G} \sim Fun_q(G)}$ ... quantized functions on G

explicitly:

$$R^k{}_a{}^i{}_b \; M^b_c \; R^c{}_j{}^a{}_d \; M^d_l = M^k_a \; R^a{}_b{}^i{}_c \; M^c_d \; R^d{}_j{}^b{}_l \, .$$

for $q \to 1$ $(i.e.\ k \to \infty)$:

$$[M^i_j, M^k_l] = 0$$

(also: $\exists \, *$ - structure)

Symmetries

want: analog of action of $G_V \subset G_L \times G_R$ on G

this classical group of motions does not act on $\mathcal{G}$.

However: $\exists$ analogous (co)actions of (FRT)-quantum group of left and right motions

$$G_L^q \times G_R^q$$

which (co)acts on $\mathcal{G}$.

$G_L^q \times G_R^q$ = quasitriangular Hopf algebra generated by matrices $l = l_j^i, \;\; r = r_j^i$ satisfying

$$\begin{aligned} l_2 r_1 R_{12} &= R_{12}\, r_1 l_2,\; r_2 r_1 R_{12} = R_{12}\, r_1 r_2 \\ \Delta(r) &= r \otimes r,\; \Delta(l) = l \otimes l \\ rS(r) &= 1 = lS(l), \quad \ldots \end{aligned}$$

... FRT quantum groups

(quantized functions on $G_L \times G_R$)

(co)action on $\mathcal{G}$ ($\mathcal{G}$ ... module algebra):

$$\begin{aligned} M &\to l^{-1} M r, \\ M_j^i &\to (l^{-1})_k^i M_l^k r_j^l \end{aligned}$$

is CONSISTENT.

vector "subgroup":

$v = v^i_j$ generates (FRT)-quantum "sub"group

$$G^q_V \subset G^q_L \times G^q_R$$

which acts by

$$M \to v^{-1} M v$$

More precisely (dual):

$\mathcal{G}$ is module-algebra under the quantum group

$$U_q(\mathfrak{g}_L) \bowtie_R U_q(\mathfrak{g}_R) \quad \supset \quad U_q(\mathfrak{g}_V)$$

(quasitriangular Hopf algebras)

$\underline{U_q(\mathfrak{g}_L) \bowtie_R U_q(\mathfrak{g}_R)}$:

... require $U_q(\mathfrak{g}_V)$ - sub(Hopf)algebra.

"standard" $U_q(\mathfrak{g}_L \times \mathfrak{g}_R) = U_q(\mathfrak{g}_L) \otimes U_q(\mathfrak{g}_R)$.

$\exists$ natural embedding of the "vector" (sub)algebra

$$U_q(\mathfrak{g}_V) \to U_q(\mathfrak{g}_L) \otimes U_q(\mathfrak{g}_R) :$$
$$d : u \mapsto (u_1 \otimes u_2) = \Delta(u) = \text{coproduct of } U_q(\mathfrak{g})$$

is not compatible with standard coproduct of $U_q(\mathfrak{g}_L \times \mathfrak{g}_R)$: $\quad \Delta \circ d \neq (d \otimes d) \circ \Delta$

Solution to this problem: <u>twisting</u>:

Consider the modified coproduct

$$\begin{aligned} \Delta_{\mathcal{R}} : U_q^L \otimes U_q^R &\to (U_q^L \otimes U_q^R) \otimes (U_q^L \otimes U_q^R), \\ u^L \otimes u^R &\mapsto \mathcal{R}_{23}^{-1}(u_1^L \otimes u_1^R) \otimes (u_2^L \otimes u_2^R)\mathcal{R}_{23} \end{aligned}$$

satisfies $\Delta_{\mathcal{R}} \circ d = (d \otimes d) \circ \Delta$

universal R- matrix:

$$\mathcal{R}_{II} = \mathcal{R}_{41}^{-1}\mathcal{R}_{13}\mathcal{R}_{24}\mathcal{R}_{23}$$

... defines a *quantum group*, i.e. a quasitriangular Hopf algebra

subspaces (branes):

classically:

$$\iota : \quad D \hookrightarrow G$$

dualizing:

$$\iota^* : \quad \mathcal{G} \longrightarrow \mathcal{D} \;\cong\; \mathcal{G}/_{Ker\ \iota^*}$$

How to find $Ker\ \iota^*$ = relations?

<u>fact</u>:

$$\boxed{Tr_q(M^n) \equiv Tr(M^n q^{-2\rho}) \in \mathcal{G}}$$

are

- central in $\mathcal{G}$ $\Rightarrow$ take values $\in \mathbb{C}$ on irreps of $\mathcal{G}$
- invariant under $U_q(\mathfrak{g}_V)$: $\quad M \rightarrow v^{-1} M v$
- constant on (untwisted) branes ($q = 1$):

  measure position of $\mathcal{C}(t) \subset G$

therefore:

$$\boxed{D := \mathcal{G}/_{(Tr_q(M^n) - c_n)}}$$

c_n can be calculated on IRREPS of $\mathcal{G}$

$\hookrightarrow$ $\boxed{D\text{-branes are given by irreps of } \mathcal{G}}$

irreps of $\mathcal{G}$:

Fact: $\exists$ algebra map

$$\begin{aligned} \mathcal{G} &\to U_q(\mathfrak{g}), \\ M^i_j &\to L^+ S L^- \end{aligned}$$

where $L^+ = (id \otimes \pi)(\mathcal{R}), \quad SL^- = (\pi \otimes id)(\mathcal{R})$

$\mathcal{R} \in U_q(\mathfrak{g}) \otimes U_q(\mathfrak{g})$ ... universal R-"matrix"

$\hookrightarrow$ need irreps of $U_q(\mathfrak{g})$ for $q = e^{\frac{i\pi}{k+g^\vee}}$

<u>simplest irreps</u>: highest weight reps

$$\boxed{V_\lambda, \quad \lambda \in P^+_k}$$

<u>why $\lambda \in P^+_k$?</u>

- then V_λ are unitary reps of $U_q(\mathfrak{g})$ ($\Rightarrow$get $*$ - representation of $\mathcal{G}$)
- $\dim_q(V_\lambda) > 0$
- correspond precisely to integrable reps of ALA $\hat{\mathfrak{g}}$ at level k

there are other reps, but they are “ill-behaved” (?)

claim: $\boxed{\text{quantized } D\text{-branes on } G}$

are given by

$$\boxed{D_\lambda := \mathcal{G}/_{(Tr_q(M^n)-c_n(\lambda))} \cong \pi_\lambda(U_q(\mathfrak{g})) \cong Mat(V_\lambda)}$$

for $\lambda \in P_k^+$

justify it by comparing with CFT results

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1) Position = values of Casimirs

$$c_n(\lambda) = Tr_q(M^n) \text{ on } D_\lambda \qquad \text{for } n = 1, 2, ..., rank(G)$$

result:

$$\begin{aligned} c_1(\lambda) &= Tr(q^{2(H_\rho+H_\lambda)}) = \sum_{\{\nu\in V_N\}} q^{2((\lambda+\rho)\cdot\nu)}, \\ c_n(\lambda) &= \sum_{\{\nu\in V_N;\ \lambda+\nu\in P_k^+\}} q^{2n((\lambda+\rho)\cdot\nu-\lambda_N\cdot\rho)} \frac{\dim_q(V_{\lambda+\nu})}{\dim_q(V_\lambda)} \\ &\approx s_n = Tr(q^{2n(H_\rho+H_\lambda)}) \end{aligned}$$

exact agreement for $n = 1$, approximate for $n > 1$

$\exists$ also characteristic equation:

$$P_\lambda(M) = \prod_{\nu\in V_N} (\ M - q^{2(\lambda+\rho)\cdot\nu-2\lambda_N\cdot\rho}) = 0.$$

as classically (almost, up to shift $\lambda_N \cdot \rho$)

2) Energy of D_λ = quantum dimension of D_λ:

$$E_\lambda = \dim_q(V_\lambda) = Tr_q(1) = \prod_{\alpha>0} \frac{\sin(\pi \frac{\alpha\cdot(\lambda+\rho))}{k+g^\vee})}{\sin(\pi \frac{\alpha\cdot\rho}{k+g^\vee})}$$

3) Harmonics on D_λ under (co)action of G_V^q

assume $\lambda \leq \frac{k}{2}$ for simplicity

$$\boxed{D_\lambda \cong Mat(V_\lambda) = V_\lambda \otimes V_\lambda^* \cong \oplus_\mu N_{\lambda\lambda+}^\mu \, V_\mu,}$$

can show:

$$N_{\lambda\lambda+}^\mu = mult_{\mu^+}^{(K_\lambda)} = N_{t'_\lambda}^\mu$$

hence

$$\boxed{D_\lambda \cong \mathcal{F}(\mathcal{C}(t'_\lambda))}$$

up to some cutoff in μ, where $t'_\lambda = q^{2H_\lambda}$.

all cutoffs match exactly with string theory, due to

$$\boxed{q = e^{\frac{i\pi}{k+g^\vee}}}$$

$\exists$ degenerate branes with smaller dimensions than "regular" ones, if λ has nontrivial stabilizer group K_λ.

e.g. fuzzy $\mathbb{C}P_q^N$, ...

(for $\lambda > \frac{k}{2}$ use "truncated tensor product") most detailed information:

4) Algebra of functions: (e.g. for $SU(2)$)

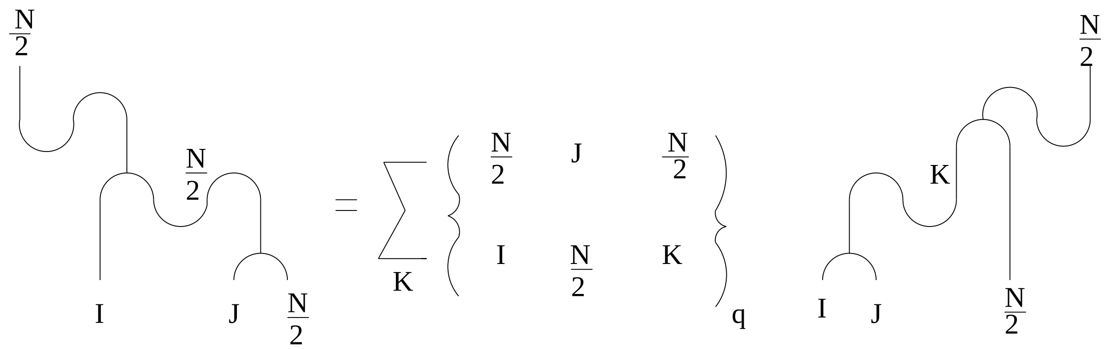
decompose D_λ into harmonics Y_i^I under G_V^q

They satisfy the algebra

$$Y_i^I \; Y_j^J = \sum_{K,k} \begin{bmatrix} I & J & K \\ i & j & k \end{bmatrix}_q \begin{Bmatrix} I & J & K \\ \lambda & \lambda & \lambda \end{Bmatrix}_q Y_k^K$$

matches with (twisted) OPE

Derivation:



$(\lambda = N/2)$

Example: $G = SU(2)$

$$M = x^i \sigma_i + x^0 \sigma_0 = \begin{pmatrix} x^4 - i x^0 & -i q^{-3/2} \sqrt{[2]} x^+ \\ i q^{-1/2} \sqrt{[2]} x^- & x^4 + i q^{-2} x^0 \end{pmatrix}$$

x^i ... NC generators

Reflection equation & constraint:

$$\boxed{\begin{gathered} [x^4, x^k] = 0, \quad \epsilon^k_{ij}\, x^i x^j = i(q - q^{-1}) x^4 x^k \\ \det_q(M) = (x^4)^2 + g_{ij} x^i x^j = 1. \end{gathered}}$$

Note: ϵ^k_{ij}, g_{ij} ... q-deformed

Casimir:

$$c_1 = tr_q(M) = [2]\, x^4$$

realization in $U_q(su(2))$:

$$M = \begin{pmatrix} q^H & q^{-\frac{1}{2}} (q - q^{-1}) q^{H/2} X^- \\ q^{-\frac{1}{2}} (q - q^{-1}) X^+ q^{H/2} & q^{-H} + q^{-1} (q - q^{-1})^2 X^+ X^- \end{pmatrix}$$

take irreps $V_\lambda = \mathbb{C}^{n+1}, \quad n = 0, 1, ..., k$:

one finds

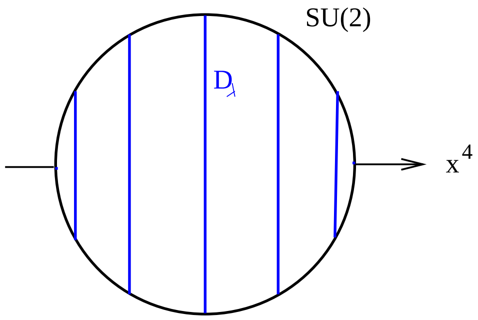
$$\boxed{x^4 = \cos(\tfrac{(n+1)\pi}{k+2})/\cos(\tfrac{\pi}{k+2})}$$

$n = 0, 1, ..., k$

$\Rightarrow \quad D_\lambda \;=$ q-deformed fuzzy sphere $S^2_{q,\lambda}$ with radius

$$r_n^2 = g_{ij}x^i x^j \approx \sin^2(\frac{n\pi}{k+2}) \quad \text{for large } k.$$

however: $r_0 = r_k = 0$! ($D0$ - branes)



everything is invariant under $n \to k - n$

<u>Energy</u>: assume one-brane configuration $S^2_{q,n}$

$\mathcal{G}$ acts on $\mathbb{C}^{n+1}$. observe

$dim_q(\mathbb{C}^{n+1}) = tr_q(\mathbf{1}) = [n+1]_q = \frac{1}{\sin(\frac{\pi}{k+2})}\ \sin(\frac{n+1}{k+2}\pi)$

agrees precisely with BCFT (and DBI) result.

Example: $G = SU(N)$

fuzzy $\mathbb{C}P^{N-1}_{q,n}$ $\cong$ degenerate brane

$\cong$ quantized conjugacy class through $q^{2(H_{n\Lambda_1})}$

$$M = x^\alpha \lambda_\alpha = \sum_a x^a \lambda_a + x^0 \lambda_0$$

$$c_1 = tr_q(M) = x^0 \quad \text{central, const. on branes}$$

acting on $V_{n\Lambda_1}$:

$$f_{ab}^c x^a x^b = \alpha\, x^0 x^c = \alpha c_1\, x^c, \quad d_{ab}^c x^a x^b = \beta_n x^c$$

for suitable α, β_n

Harmonics:

$$D_{n\Lambda_1} \cong \oplus_n V_{(n,0,\ldots,0,n)}$$

up to some cutoff

Gauge theory on $S^2_{q,N}$

Algebra of functions:

$$\boxed{\begin{aligned} (\varepsilon_q)^{ij}_k x_i x_j &= \Lambda_{q,N} x_k, \\ (g_q)^{ij} x_i x_j &= R^2. \end{aligned}} \quad \dots\ S^2_{q,N}$$

$$\Lambda_{q,N} = R\,\frac{[2]_{q^{N+1}}}{\sqrt{[N]_q[N+2]_q}}, \qquad [n]_q = \frac{q^n - q^{-n}}{q - q^{-1}}.$$

$(\varepsilon_q)^{ij}_k$ ... q–Clebsch–Gordan coeff.

$(g_q)^{ij}$ ... q–invariant tensor

$S^2_{q,N}$ is same <u>algebra</u> $Mat(N+1,\mathbb{C})$ as S^2_N,

<u>but:</u> $\boxed{S^2_{q,N} \text{ is } U_q(su(2)) - \text{module algebra}}$

i.e. $\exists$ different additional structure (rotations, ...),

$$S^2_{q,N} = (1) \oplus (3) \oplus \dots \oplus (2N+1)$$

under q–adjoint action of $U_q(su(2))$

Invariant integral of functions $f \in S^2_{q,N}$:

$$\int u \triangleright_q f = \varepsilon(u) \int f, \qquad u \in U_q(su(2))$$

is given by

$$\int f(x) := \frac{1}{C_N} Tr_q f = \frac{1}{C_N} Tr(f q^{-H}),$$

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furthermore:

- Covariant differential $\star-$ calculus:
  - frame θ^a of one-forms, $a = 1, 2, 3$!

    $$\Theta := x \cdot \theta \ (= \lambda_a \theta^a), \qquad \text{singlet}$$

    satisfies $\boxed{df := [\Theta, f] \in \Omega^1}$, $f \in S^2_{q,N}$
  - $\exists$ n-forms: $dim(\Omega^n) = (1, 3, 3, 1)$
  - Hodge-star $*_H(\theta^a) = -\varepsilon^a_{cb} \theta^b \theta^c$
- Stokes theorem, etc.

Gauge fields

(abelian)

consider 1-forms

$$B = B_a\theta^a = \theta^a B_a \quad \in \Omega^1$$

form $S^2_{q,N}$ – (bi)module.

Note: have 3 components $a = 1, 2, 3$!

$\Rightarrow \exists$ fluctuations in radial direction,

radial component of gauge field is scalar field.

"curvature":

$$\boxed{F := B^2 - *_H B} = (B_a B_b + B_c\, \varepsilon^c_{ba})\theta^a\theta^b$$

why?

1. good limit $q = 1$: if gauge transformations $B \to U^{-1}BU$, then $F \to U^{-1}FU$
2. consider

$$B = \Theta + A, \qquad B_a = \lambda_a + A_a \qquad (\lambda_a \approx [N]\, x_a)$$

(assume $\langle A\rangle = 0$, B fluctuates around $B_0 = \Theta$). Then

$$\begin{aligned} F &= dA + A^2 \\ &= (\lambda_a A_b + A_a\lambda_b + A_a A_b + A_c\, \varepsilon^c_{ba})\theta^a\theta^b \end{aligned}$$

actions:

just polynomials in B, no explicit derivative terms.

simplest possible forms:

$$\begin{aligned} S_2 &= \int B *_H B = \int B_a B_b g^{ba}, \\ S_3 &= \int B^3 = \int B_a B_b B_c \varepsilon^{cba} \end{aligned}$$

Linear combination

$$S_{CS} := \frac{1}{3} S_3 - \frac{1}{2} S_2 = C_N + \int AdA + \frac{2}{3} A^3$$

is the unique linear combination which contains no linear terms in A. "Chern–Simons"

4th order terms:

$$S_4 = \int B^2 *_H B^2, \quad \text{(and 2 others)}$$

Let

$$S_{YM} = \int F *_H F.$$

Note: for $q \to 1$, recover actions of *Alekseev, Recknagel, Schomerus* for string–induced field theory on D–branes in $SU(2)$.

separate A in scalar and "tangential" components:

$$A_a = \frac{x_a}{R}\phi + A^t_a, \quad x_a A^t_b \, g^{ab} = 0,$$

find for $N \to \infty$ e.g.

$$S_{YM} = -\int\limits_{S^2} \quad 2F^t_{ab}F^{t}{}_{ab} + \frac{2}{R^2}\varepsilon^{n}_{ab}x_n\phi F^{t\,ab}$$
$$+\frac{1}{R^2}(\phi^2 + [\lambda_a, \phi][\lambda^a, \phi])$$

<u>However</u>:

want "global" theory on group manifold,

not just one brane

(work in progress)

Summary

- (untwisted) D - branes on group manifolds are very well described by $U_q(\mathfrak{g})$ - covariant quantum algebras
- symmetry pattern $G_V \subset G_L \times G_R$ "quantized"
- precise agreement with CFT results for
  - positions
  - energy
  - harmonics (number and algebra)

  $\exists$ degenerate branes

  in particular, find (q)-fuzzy $\mathbb{C}P^N$, ...
- works for any k, not just leading $1/k$

  discrete, finite, explicit mathematical structure

Open problems:

- strings induce gauge theory on these q-deformed spaces (work in progress)
- other branes ? ...