Higher spin gauge theory on fuzzy space(-times) from the IKKT matrix model

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Action MP 1405
Quantum Structure of Spacetime

Motivation

goal: quantum theory of spacetime & gravity

guidelines:

- simple
- gauge theory (Minkowski signature!)
- finite dof (per volume), pre-geometric ($\rightarrow$ NCG ?)
- GR should arise as effective field theory (not starting point)

starting point:

Matrix Models as fundamental theories

- simple !
- describe dynamical NC / fuzzy spaces, gauge theory
- generic models of NCG: serious UV/IR mixing problem

  preferred model: maximal SUSY: IKKT model
- preserves power of string theory, may avoid “landscape”

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strategy: find good background space-time (solution)

$\rightarrow$ typically issues with Lorentz invariace

way out: “covariant” quantum space $\rightarrow$ higher spin !

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outline:

- matrix models & matrix geometry
- *4D covariant quantum spaces*: fuzzy S^4_N, H^4_n
- cosmological space-times: $\mathcal{M}^{3,1}$ & BB!
- fluctuations $\rightarrow$ higher spin gauge theory
- propagating modes, towards gravity on $\mathcal{M}^{3,1}$

HS, arXiv:1606.00769
M. Sperling, HS arXiv:1707.00885
HS, arXiv:1709.10480, arXiv:1710.11495
M. Sperling, HS arXiv:1806.05907
M. Sperling, HS arXiv:1901.xxxx

The IKKT model

IKKT or IIB model — Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[X,\Psi] = -Tr\Big([X^a,X^b][X^{a'},X^{b'}]\eta_{aa'}\eta_{bb'} \; + m^2 X^a X_a \; + \bar\Psi\gamma_a[X^a,\Psi]\Big)$$

$$X^a = X^{a\dagger} \in Mat(N,\mathbb{C})\,, \qquad a = 0,...,9, \qquad N \text{ large}$$

gauge symmetry $X^a \to UX^aU^{-1}$, $SO(9,1)$, SUSY

- quantized Schild action for IIB superstring
- reduction of $10D$ SYM to point, N large
- equations of motion:
  $\Box Y^a + m^2 Y^a = 0, \qquad \Box \equiv \eta_{ab}[Y^a,[Y^b,.]]$
- quantization: $\quad Z = \int dXd\Psi\, e^{iS[X]}$

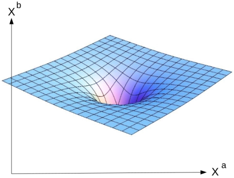
"matrix geometry" ($\approx$ NC geometry):

- $S_E \sim Tr[X^a, X^b]^2 \Rightarrow$ config's with small $[X^a, X^b] \neq 0$ dominate

  i.e. "almost-commutative" configurations

- $\exists$ quasi-coherent states $|x\rangle$, minimize $\sum_a \langle x|\Delta X_a^2|x\rangle$

  $X^a \approx$ diag., spectrum $=: \mathcal{M} \subset \mathbb{R}^{10}$



NC branes embedded in target space $\mathbb{R}^{10}$

$$X^a \sim x^a : \qquad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

typical examples: quantized Poisson manifolds

- Moyal-Weyl quantum plane $\mathbb{R}^4_\theta$:

$$[X^a, X^b] = i\theta^{ab}\,\mathbb{1} \qquad \text{(Heisenberg algebra)}$$

quantized symplectic space $(\mathbb{R}^4, \omega)$

admits translations $X^a \to X^a + c^a\mathbb{1}$, no rotation invariance

- fuzzy 2-sphere S^2_N

$$X_1^2 + X_2^2 + X_3^2 = R_N^2, \qquad [X_i, X_j] = i\epsilon_{ijk}X_k$$

fully covariant under $SO(3)$ (Hoppe; Madore)

derivations:

$$[X^\mu, \phi] =: i\theta^{\mu\nu}\partial_\nu\phi$$

fluctuations in M.M. $\to$ NC gauge theory

consider fluctuations around background X^a

$$Y^a = X^a + \mathcal{A}^a$$

$$\begin{aligned} [Y^\mu, \phi] &= i\theta^{\mu\nu} D_\nu \phi, \qquad D_\mu = \partial_\mu + i[A_\mu, .] \\ [Y^\mu, Y^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} \underbrace{(\partial_{\mu'} A_{\nu'} - \partial_{\nu'} A_{\mu'} + [A_{\mu'}, A_{\nu'}])}_{F_{\mu'\nu'}} \\ S &= Tr([Y^\mu, Y^\nu][Y_\mu, Y_\nu]) \sim \int F_{\mu\nu} F^{\mu\nu} + c. \end{aligned}$$

$\to$ YM gauge theory

$\leftrightarrow$ dynamical geometry ("emergent gravity")

review: H.S. arXiv:1003.4134

covariant quantum spaces from equivariant bundles

- issue: symplectic form ω breaks local (Lorentz/Euclid.) invar.
- way out: quantized equivariant bundles over $\mathcal{M}^4$

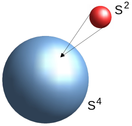
  $\rightarrow$ higher spin gauge theory
- example: fuzzy four-sphere S_N^4

  Grosse-Klimcik-Presnajder; Castelino-Lee-Taylor; Ramgoolam; Kimura; Karabail-Nair; Zhang-Hu 2001 (QHE!) ...
- noncompact H_n^4 $\rightarrow$ projection $\rightarrow$ cosmological space-time $\mathcal{M}_n^{3,1}$
- spin 2 modes on $\mathcal{M}_n^{3,1} \stackrel{\text{conjecture}}{\rightarrow}$ gravity

(work in progress)

$\mathbb{C}P^3$ as S^2 bundle over S^4

Hopf map:



$$\begin{array}{ccc} \mathbb{C}P^3 & \ni & \psi \\ \downarrow & & \downarrow \\ S^4 & \ni & x^a = \psi^\dagger \gamma^a \psi \end{array}$$

... $SO(5)$ intertwiner, equivariant S^2 bundle

$\mathbb{C}P^3$ is coadjoint orbit of $SU(4)$ or $SO(6) \supset SO(5)$

$$\mathbb{C}P^3 \cong \mathcal{O}[\Lambda] = \{g \cdot \Lambda \cdot g^{-1};\ g \in SO(6)\} \quad \subset \mathfrak{so}(6), \qquad \Lambda = (0, 0, \lambda)$$

embedding functions

$$\begin{array}{rlll} m^{ab}: & \mathcal{O}[\Lambda] & \hookrightarrow \mathbb{R}^{15} & \\ & \xi & \mapsto m^{ab} = tr(\xi\, \Sigma^{ab}) \\ & \xi & \mapsto x^a = tr(\xi\, \Sigma^{a6}) \end{array}$$

where $\quad \xi = \psi\psi^\dagger, \qquad \Sigma^{ab} \in \mathfrak{so}(6), \qquad \gamma^a = \Sigma^{a6}$

quantized (“fuzzy”) $\mathcal{O}_\Lambda$:

(Kirillov-Kostant-Souriau) symplectic form on $\mathcal{O}[\Lambda]$, Poisson structure

$$\{x^a, x^b\} = f^{abc} x^c$$

Λ ... dominant integral weight, $\mathcal{H}_\Lambda$ ... h.w. irrep

$End(\mathcal{H}_\Lambda)$ ... quantized algebra of functions on $\mathcal{O}_\Lambda$

generators

$$[X^a, X^b] = i f^{abc} X^c$$

covariant fuzzy four-sphere S^4_N

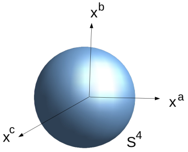
start with fuzzy $\mathbb{C}P^3$ given by $End(\mathcal{H}_\Lambda)$ for $\Lambda = (0,0,N)$

quantized embedding function

$$X^a = \mathcal{M}^{a6} \sim x^a: \quad \mathbb{C}P^3 \to S^4 \subset \mathbb{R}^5$$

... quantized Hopf map !

(Grosse-Klimcik-Presnajder 1996)



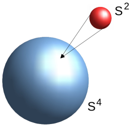
relations:

$$\begin{aligned} \textstyle\sum_a X_a^2 &= R_N^2 \mathbf{1}, \qquad R_N^2 \sim \tfrac{1}{4}N^2 \qquad \text{(sphere)} \\ \epsilon^{abcde} X_a X_b X_c X_d X_e &= (N+2) R^2 \qquad \text{(volume quantiz.)} \\ [\mathcal{M}_{ab}, X_c] &= i(\delta_{ac} X_b - \delta_{bc} X_a), \qquad \text{(covariance)} \end{aligned}$$

$\mathcal{M}_{ab}$ ... $so(5)$ generators acting on $\mathcal{H}_N$

underlying fact: $(0,0,N)$ of $\mathfrak{so}(6)$ is irreducible under $\mathfrak{so}(5)$

equivariant <u>bundle structure</u>



Poisson tensor $\theta^{\mu\nu}(x,\xi) \sim -i[X^\mu, X^\nu]$

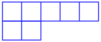
local $SO(4)_x$ rotates fiber $\xi \in S^2$

averaging over fiber $\rightarrow$ $[\theta^{\mu\nu}(x,\xi)]_0 = 0$, local $SO(4)$ preserved!

... 4D "covariant" quantum space

fields and harmonics on S^4_N

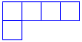
algebra of "functions":

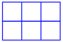
$$End(\mathcal{H}_N) \cong \bigoplus_{s=0}^{N} \mathcal{C}^s \qquad \mathcal{C}^s = \bigoplus_{n=0}^{N} (n, 2s) \ni$$ 

$(n, 0)$ modes = scalar functions on S^4:

$$\phi(X) = \phi_{a_1 \ldots a_n} X^{a_1} \ldots X^{a_n} \quad =$$ 

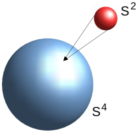
$(n, 2)$ modes = selfdual 2-forms on S^4

$$\phi_{bc}(X)\theta^{bc} = \phi_{a_1 \ldots a_n b;c} X^{a_1} \ldots X^{a_n} \theta^{bc} \quad =$$ 

$End(\mathcal{H}) \cong$ fields on S^4 taking values in $\mathfrak{hs} = \oplus$ 

higher spin modes = would-be KK modes on S^2

(local $SO(4)$ acts on S^2 fiber)



relations:

$$
\begin{aligned}
\mathcal{M}^{ab}\mathcal{M}^{ac} &= \frac{R^2}{\theta} P_T^{bc}, \\
\varepsilon_{abcde}\mathcal{M}^{ab}\mathcal{M}^{cd} &= 8\frac{R}{\theta} x^e ,
\end{aligned}
$$

($\sim$ cf. bundle version of Joseph ideal ...)

relation with spin s fields: one-to-one map

$$End(\mathcal{H}_N) \cong \bigoplus_{s=0}^{N} \mathcal{C}^s \quad \cong \{\text{symmetric tensor field on } S^4\}$$

$$\phi^{(s)} = \phi^{(s)}_{b_1 \ldots b_s; c_1 \ldots c_s}(x)\, \theta^{b_1 c_1} \ldots \theta^{b_s c_s} \quad \mapsto \phi^{(s)}_{c_1 \ldots c_s}(x) = \phi^{(s)}_{b_1 \ldots b_s; c_1 \ldots c_s} x^{b_1} \ldots x^{b_s}$$

$$\{x^{c_1}, \ldots, \{x^{c_s}, \phi^{(s)}_{c_1 \ldots c_s}(x)\} \ldots\} \leftarrow \phi^{(s)}_{c_1 \ldots c_s}(x)$$

... "symbol" of $\phi \in \mathcal{C}^s$

M. Sperling & HS, arXiv:1707.00885

$\mathcal{C}^s \cong$ symm., traceless, tang., div.-free rank s tensor field on S^4

$$\begin{aligned} \phi_{c_1 \ldots c_s}(x) x^{c_i} &= 0\,, \\ \phi_{c_1 \ldots c_s}(x) g^{c_1 c_2} &= 0\,, \\ \partial^{c_i} \phi_{c_1 \ldots c_s}(x) &= 0\,. \end{aligned}$$

similarly:

cosmological quantum space-times $\mathcal{M}_n^{3,1}$:

- exactly homogeneous & isotropic, Big Bounce
- on-shell modes obtained
- starting point: fuzzy hyperboloid H_n^4

Euclidean fuzzy hyperboloid H^4_n $(=EAdS^4_n)$

Hasebe arXiv:1207.1968

$\mathcal{M}^{ab}$ ... hermitian generators of $\mathfrak{so}(4,2)$,

$$[\mathcal{M}_{ab}, \mathcal{M}_{cd}] = i(\eta_{ac}\mathcal{M}_{bd} - \eta_{ad}\mathcal{M}_{bc} - \eta_{bc}\mathcal{M}_{ad} + \eta_{bd}\mathcal{M}_{ac}) \ .$$

$$\eta^{ab} = \mathrm{diag}(-1,1,1,1,1,-1)$$

choose "short" discrete unitary irreps $\mathcal{H}_n$ ("minireps", doubletons)

special properties:

- irreps under $\mathfrak{so}(4,1)$, multiplicities one, minimal oscillator rep.
- positive discrete spectrum

$$\mathrm{spec}(\mathcal{M}^{05}) = \{E_0, E_0+1, ...\}, \qquad E_0 = 1 + \frac{n}{2}$$

lowest eigenspace is $n+1$-dim. irrep of $SU(2)_L$: fuzzy S^2_n

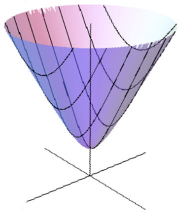
fuzzy hyperboloid H_n^4

def.

$$\begin{aligned} X^a &:= r\mathcal{M}^{a5}, \qquad a = 0, ..., 4 \\ [X^a, X^b] &= ir^2\mathcal{M}^{ab} =: i\Theta^{ab} \end{aligned}$$

5 hermitian generators $X^a = (X^a)^\dagger$ satisfy

$$\eta_{ab}X^aX^b = X^iX^i - X^0X^0 = -R^2\mathbf{1}, \qquad R^2 = r^2(n^2-4)$$



one-sided hyperboloid in $\mathbb{R}^{1,4}$, covariant under $SO(4,1)$

note: induced metric = Euclidean AdS^4

oscillator construction: 4 bosonic oscillators $[\psi_\alpha, \bar\psi^\beta] = \delta^\beta_\alpha$

$\mathcal{H}_n$ = suitable irrep in Fock space

Then

$$\mathcal{M}_{ab} = \bar\psi \Sigma_{ab} \psi, \qquad \gamma_0 = diag(1, 1, -1, -1)$$
$$X^a = r \bar\psi \gamma^a \psi$$

$H^4_n \;=\;$ quantized $\mathbb{C}P^{1,2} = S^2$ bundle over H^4, selfdual $\theta^{\mu\nu}$

analogous to S^4_N

fuzzy "functions" on H^4_n:

$$(End(\mathcal{H}_n) \to) \qquad HS(\mathcal{H}_n) \cong \bigoplus_{s=0}^{n} \mathcal{C}^s = \int\limits_{\mathbb{C}P^{1,2}} d\mu\, f(m)\, |m\rangle \langle m|$$

$\approx$ fields on H^4 taking values in $\mathfrak{hs} = \oplus_s \begin{array}{|c|c|c|}\hline & & \\ \hline & & \\ \hline\end{array} \ni \mathcal{M}^{a_1 b_1} \ldots \mathcal{M}^{a_s b_s}$

spin s sectors $\mathcal{C}^s$ selected by spin Casimir

$$\mathcal{S}^2 = \sum_{a<b\leq 4} [\mathcal{M}^{ab}, [\mathcal{M}_{ab}, \cdot]] + r^{-2}[X_a, [X^a, \cdot]] \,,$$

can show:

$$\mathcal{S}^2|_{\mathcal{C}^s} = 2s(s+1), \qquad s = 0, 1, ..., n$$

M. Sperling & H.S. 1806.05907

i.e. higher spin gauge theory, truncated at n

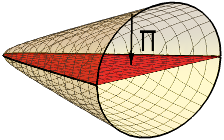
open FRW universe from H^4_n

HS arXiv:1710.11495

$\mathcal{M}^{3,1}_n$ = H^4_n projected to $\mathbb{R}^{1,3}$ via

$$Y^\mu \sim y^\mu : \ \mathbb{C}P^{1,2} \to H^4 \subset \mathbb{R}^{1,4} \xrightarrow{\Pi} \mathbb{R}^{1,3} .$$

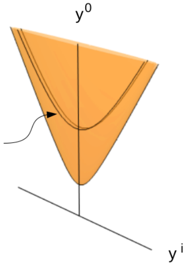
induced metric has Minkowski signature!



algebraically: $\mathcal{M}^{3,1}_n$ generated by

$$\boxed{Y^\mu := X^\mu, \quad \text{for} \ \ \mu = 0,1,2,3 \qquad (\text{drop } X^4\ )}$$

geometric properties:



- $SO(3,1)$ manifest $\Rightarrow$ foliation into $SO(3,1)$-invariant space-like 3-hyperboloids H^3_τ
- double-covered FRW space-time with hyperbolic ($k = -1$) spatial geometries

$$ds^2 = d\tau^2 - a(\tau)^2 d\Sigma^2,$$

$d\Sigma^2$ ... $SO(3,1)$-invariant metric on space-like H^3

"functions" on $\mathcal{M}^{3,1}_n$:

generated by $X^\mu = r\mathcal{M}^{\mu 5} \sim x^\mu$ and $T^\mu = \frac{1}{R}\mathcal{M}^{\mu 4} \sim t^\mu$

$$\begin{aligned} \{t^\mu, x^\nu\} &= \tilde{a}(\tau)\eta^{\mu\nu} \\ x_\mu t^\mu &= 0 \\ t_\mu t^\mu &= r^2(\tau) \end{aligned}$$

hence: t^μ ... space-like S^2, derivations

self-duality on H^4 implies

$$\theta^{\mu\nu} = c(x^\mu t^\nu - x^\nu t^\mu) + b\epsilon^{\mu\nu\alpha\beta} x_\alpha t_\beta$$

$\to$ expand modes as

$$\boxed{\phi = \phi(x) + \phi_\mu(x) t^\mu + \phi_{\mu\nu} t^\mu t^\nu + \ldots}$$

"space-like gauge" ($\to$ no ghosts!)

$\mathcal{M}^{3,1}$ realization in IKKT model:

background solution

$$T^\mu := \frac{1}{R}\mathcal{M}^{\mu 4}$$

satisfies

$$\Box T^\mu = 3R^{-2}T^\mu, \qquad \Box = [T^\mu,[T_\mu,.]]$$

- $[\Box, \mathcal{S}^2] = 0$
  $\Rightarrow$ higher-spin expansion $\phi = \phi(X) + \phi_\mu(X)T^\mu + ... \in HS(\mathcal{H})$ on $\mathcal{M}^{3,1}$
- $\Box$ encodes effective FRW metric, asymptotically coasting

  Big Bounce, initial $a(\tau) \sim \tau^{1/5}$ singularity

... work in progress M. Sperling & HS

fluctuations & higher spin gauge theory

$$S[Y] = Tr(-[Y^a, Y^b][Y_a, Y_b] + m^2 Y^a Y_a) = S[U^{-1} Y U]$$

background solution: $S^4_N,\ H^4_n,\ \mathcal{M}^{3,1}_n$

add fluctuations $Y^a = \overline{Y}^a + \mathcal{A}^a,$

gauge trafos $\mathcal{A}^a \to [\Lambda, \mathcal{A}^a] + [\Lambda, \overline{Y}^a], \qquad \Lambda \in End(\mathcal{H})$

expand action to second oder in $\mathcal{A}^a$

$$S[Y] = S[\overline{Y}] + \frac{2}{g^2}\mathrm{Tr}\mathcal{A}_a\Big(\underbrace{(\Box + \frac{1}{2}m^2)\delta^a_b + 2[[\overline{Y}^a, \overline{Y}^b], .]}_{\mathcal{D}^2} - \underbrace{[\overline{Y}^a, [\overline{Y}^b, .]]}_{g.f.}\Big)\mathcal{A}_b$$

- fluctuations $\mathcal{A}_a$ describe gauge theory (NCFT) on $\mathcal{M}$ ("open strings" ending on $\mathcal{M}$)
- for $S^4_N,\ H^4_n$: $\mathcal{A}_a$ ... $\mathfrak{hs}$-valued gauge field, incl. spin 2

on S_N^4 and H_n^4: background $\overline{Y}^a = X^a$

- $\mathcal{A}_a \in End(\mathcal{H}) \otimes (5)$
  $\rightarrow$ 4 indep. tangential fluctuation modes for each spin $s \leq n$

$$
\begin{aligned}
\mathcal{A}_a^{(1)} &= \eth_a \phi^{(s)} \ \in \mathcal{C}^s, \qquad \eth^a = x_b\{\theta^{ba}, .\} \\
\mathcal{A}_a^{(2)} &= \theta^{ab}\eth_b \phi^{(s)} \ = \{x^a, \phi^{(s)}\} \\
\mathcal{A}_a^{(3)} &= \phi_a^{(s)}, \qquad \phi^{(s)} = \{x^a, \phi_a^{(s)}\} \\
\mathcal{A}_a^{(4)} &= \theta^{ab}\phi_b^{(s)}
\end{aligned}
$$

+ radial mode $\mathcal{A}_a^{(r)} = x_a \phi^{(s)}$

- diagonalize $\rightarrow$ eigenmodes of $\mathcal{D}^2$

(details: M. Sperling & H.S. 1707.00885, 1806.05907)

all tangential modes are stable !

- radial modes are unstable on H_n^4
  $\rightarrow$ project to cosmological space-time $\mathcal{M}^{3,1}$

on $\mathcal{M}_n^{3,1}$: background $\overline{Y}^\mu = T^\mu$ M. Sperling and HS, work in progress

symmetry reduced to space-like $SO(3,1)$
underlying $SO(4,1)$ & $SO(4,2)$ extremely useful

- 2 physical modes in
  $\mathcal{H}_{\rm phys} = \{\mathcal{D}^2\mathcal{A} = 0, \ \ \mathcal{A} \ \text{gauge fixed}\}/\{\text{pure gauge}\}$

$$\begin{aligned}\mathcal{A}_\mu^{(2+)}[\phi^{(s)}] &= \{x_\mu, \phi^{(s)}\}_+ \\ \mathcal{A}_\mu^{(2-)}[\phi^{(s)}] &= \{x_\mu, \phi^{(s)}\}_- \ .\end{aligned}$$

  pure gauge mode

$$\mathcal{A}_\mu^{(g)}[\phi^{(s)}] = \{t_\mu, \phi^{(s)}\}$$

  time-like mode $\mathcal{A}_\mu^{(4)}[\phi^{(s)}] = x_\mu \phi^{(s)}$ (not in $\mathcal{H}_{\rm phys}$)
  conjecture: no ghosts (cf. YM !)

- all physical on-shell modes found,
  2 physical propagating modes for each spin $s \leq n$

further aspects of $\mathcal{M}^{3,1}$:

- symmetry $SO(3,1)$ only space-like
  $\Rightarrow$ "Lorentz-violating" structures: $x_\mu\{t^\mu,.\} \sim \frac{\partial}{\partial\tau}$ ... time-like VF
  (cosmic background!)
- same speed (of light) for all physical modes

vielbein, metric & dynamical geometry

consider scalar field $\phi = \phi(X)$ (e.g. transversal fluctuation)

kinetic term

$$-Tr[Y^a, \phi][Y_a, \phi] \sim \int e^a \phi e_a \phi = \int \gamma^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

vielbein

$$\begin{aligned} e^a &:= \{Y^a, .\} = e^{a\mu} \partial_\mu \\ e^{a\mu} &= \theta^{a\mu} \end{aligned}$$

metric

$$\gamma^{\mu\nu} = \eta_{\alpha\beta} e^{\alpha\mu} e^{\beta\nu}$$

dynamical frame bundle, metric

perturbed vielbein: $Y^a = \overline{Y}^a + \mathcal{A}^a$

$$e^a := \{\overline{y}^a, .\} = e^{a\mu}[\mathcal{A}]\partial_\mu \qquad \text{... vielbein}$$

$$\delta_{\mathcal{A}}\gamma^{ab} =: \{\overline{y}^c, x^a\}\{\mathcal{A}_c, x^b\} + (a \leftrightarrow b)$$

linearize & average over fiber $\rightarrow$ $h^{ab} = [\delta_{\mathcal{A}}\gamma^{ab}]_0$

coupling to matter:

$$S[\text{matter}] \sim \int_{\mathcal{M}} d^4x\, h^{ab} T_{ab}$$

on H^4: quadratic action:

$$S_{YM} \propto \int h_{ab} h_{ab}$$

$h_{ab} \sim T_{ab}$, doesn't propagate on H^4 in classical model

HS, arXiv:1606.00769, M. Sperling, HS arXiv:1707.00885 perturbed vielbein: $Y^a = \overline{Y}^a + \mathcal{A}^a$

$$
\begin{aligned}
e^a &:= \{\overline{y}^a, .\} = e^{a\mu}[\mathcal{A}]\partial_\mu \qquad \text{... vielbein} \\
\delta_{\mathcal{A}}\gamma^{ab} &=: \{\overline{y}^c, x^a\}\{\mathcal{A}_c, x^b\} + (a \leftrightarrow b)
\end{aligned}
$$

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towards higher-spin gravity on $\mathcal{M}^{3,1}$

perturbed metric: $\delta_{\mathcal{A}}\gamma^{\mu\nu} \propto \{\mathcal{A}^\mu, x^\nu\} + (\mu \leftrightarrow \nu)$

- contains all required dof for gravity !
- YM action $\to$ 2 $\sim$ Ricci-flat grav. waves
  +3 propagating modes (cf. massive gravity)
- linearized Einstein-Hilbert action obtained (gauge invariance), expected to be induced by quantum effects (cf. Sakharov) (conformal mode non-standard?)
- YM-action replaces $\int d^4x\,\sqrt{g}$, stabilizes $\mathcal{M}^{3,1}$
  no cosm. const. problem ?!
- expect
  - $\approx$ linearized GR
  - significant differences at cosmic scales

M. Sperling and HS, work in progress

summary

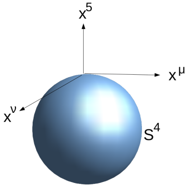
- matrix models:
  natural framework for quantum theory of space-time & matter
- 4D covariant quantum spaces $\rightarrow$ higher spin theories
- $\exists$ nice cosmological FRW space-time solutions
  reg. BB, finite density of microstates
- Yang-Mills structure $\rightarrow$ emergent gravity rather than GR
- good UV behavior (SUSY), well suited for quantization

... intriguing, needs more work

local description: pick north pole $p \in S^4$

$\to$ tangential & radial generators

$$X^a = \begin{pmatrix} X^\mu \\ X^5 \end{pmatrix}, \qquad \mu = 1, ..., 4 \text{...tangential coords at } p$$



separate $SO(5)$ into $SO(4)$ & translations

$$\mathcal{M}^{ab} = \begin{pmatrix} \mathcal{M}^{\mu\nu} & \mathcal{P}^\mu \\ -\mathcal{P}^\mu & 0 \end{pmatrix}$$

Poisson algebra $\{P_\mu, X^\nu\} \approx \delta^\nu_\mu$ locally local form of spin 2 harmonics:

$$\phi^{(2)} = h_{\mu\nu}(x) P^\mu P^\nu + \omega_{\mu:\alpha\beta}(x) P^\mu \mathcal{M}^{\alpha\beta} + \Omega_{\alpha\beta;\mu\nu}(x) \mathcal{M}^{\alpha\beta}\mathcal{M}^{\mu\nu}$$

recall $End(\mathcal{H}) = \oplus \mathcal{C}^s$, $\mathcal{C}^s \cong$ rank s tensor fields $\phi_{a_1 \dots a_s}(x) \cong (n, 2s)$

unique irrep $(n, 2s) \Rightarrow$ constraints!

$$\begin{aligned} \omega_{\mu;\alpha\beta} &\propto \partial_\alpha h_{\mu\beta} - \partial_\beta h_{\mu\alpha} \\ \Omega_{\alpha\beta;\mu\nu} &\propto \mathcal{R}_{\alpha\beta\mu\nu}[h] \end{aligned}$$

... linearized spin connection and curvature determined by $h_{\mu\nu}$

local spn 1 gauge trafos:

$$\Lambda = \Lambda^{(1)}_{ab}(x)\theta^{ab} = v_\mu(x)P^\mu + \omega_{\mu\nu}(x)\mathcal{M}^{\mu\nu}.$$

$\omega_{\mu\nu}$ ... field strength of vector field v_μ

$$\delta_\Lambda x^\mu = \{\Lambda, x^\mu\} = -\underbrace{v^\mu}_{\text{diffeo}} + (\underbrace{\partial_\nu v_\rho}_{\text{pure gauge graviton}} P^\rho + \partial_\nu\omega_{\rho\sigma}\mathcal{M}^{\rho\sigma})\theta^{\mu\nu} \quad \in \ \mathcal{C}^0 + \mathcal{C}^2$$