Abstract. The aim of these notes is to show the non-vanishing of local, non-archimedean Shalika newvectors at the identity matrix $g = \text{id}$. This is also meant as some sort of addendum to our paper [Gro-Rag14], where this result was used in Lem. 3.8.1 without comment. It has to be pointed out that the proof presented here is due to very generous explanations of Nadir Matringe, leaving basically only editorial work to us. We are grateful for Nadir’s permission to present these arguments in these notes.

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1. The setup

1.1. Fields and characters. Throughout these notes, $F$ will denote a local, non-archimedean field with ring of integers $\mathcal{O}$, valuation $v$ and normalized absolute value $|\cdot| = |\cdot|_v$. We fix a non-trivial, unramified additive continuous character $\psi : F \to \mathbb{C}$ and let $\eta$ be a continuous character $\eta : F^* \to \mathbb{C}^*$. The trivial character is denoted $1$. By $M_k$ we shall denote the algebra of $k \times k$-matrices with entries in $F$ and by $N_k$ the subset of upper triangular elements in $M_k$. If $X$ is any subset of $M_k$, we will denote $1_X$ the characteristic function of $X$.

1.2. Local groups. For any $k \geq 1$, we abbreviate $G_k := \text{GL}_k(F)$, the $F$-points of the split general linear group over $F$. The group $G_k$ has some well-known subgroups:

$$P_k := \left\{ p = \begin{pmatrix} g_{k-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} id_{k-1} & x \\ 0 & 1 \end{pmatrix} \mid g_{k-1} \in G_k, x \in F^{k-1} \right\},$$

the so-called mirabolic subgroup, and $U_k$, the subgroup of upper triangular unipotent matrices. If $k = 2n$ is even, then there are also the following subgroups:

$$L_{2n} := \left\{ \ell = \begin{pmatrix} g_n \\ 0 \end{pmatrix} \begin{pmatrix} 0 & h_n \\ 0 & g_n \end{pmatrix} \mid g_n, h_n \in G_n \right\},$$

the Levi subgroup of the “Siegel parabolic”, and

$$S_{2n} := \left\{ s = \begin{pmatrix} g_n & 0 \\ 0 & g_n \end{pmatrix} \begin{pmatrix} id_n & X \\ 0 & id_n \end{pmatrix} \mid g_n \in G_n, X \in M_n \right\},$$

called the Shalika subgroup.
All measures in this note are normalized to give volume 1 to the respective maximal compact subgroups, consisting of $O$-points.

1.3. **Non-archimedean Shalika models.** The characters $\eta$ and $\psi$ can be extended to a character of $S_{2n}$,

$$s = \begin{pmatrix} g_n & 0 \\ 0 & g_n \end{pmatrix} \begin{pmatrix} 1 & X \\ 0 & 1 \end{pmatrix} \mapsto (\eta \otimes \psi)(s) := \eta(\det(g_n))\psi(\text{Tr}(X)).$$

We will also write $\eta(s) = \eta(\det(g_n))$ and $\psi(s) = \psi(\text{Tr}(X))$.

**Definition 1.** Let $\pi$ be an irreducible admissible representation of $G_{2n}$. We say that $\pi$ has a $(\eta, \psi)$-Shalika model $S_{\psi}^\eta(\pi)$, if there is a $G$-submodule

$$S_{\psi}^\eta(\pi) \subseteq \text{Ind}_{S_{2n}}^{G_{2n}}[\eta \otimes \psi],$$

which is isomorphic to $\pi$.

Shalika models, if they exist, are unique: This has been established by Jacquet–Rallis [Jac-Ral96] for $\eta = 1$ and in general by Chen–Sun in [Che-Sun19], Thm. A.

In this note we will first concentrate on the case when $\eta = 1$. Doing so, we will abbreviate

$$S_{\psi}(\pi) := S_{\psi}^1(\pi).$$

2. **Whittaker models vs. Shalika models**

2.1. **Model comparison.** Let $\pi$ be an irreducible admissible representation of $G_{2n} = \text{GL}_{2n}(F)$, which admits a $(1, \psi)$-Shalika model $S_{\psi}(\pi)$. As for the final purpose of these notes we shall only be interested in local components of cuspidal automorphic representations, it is no harm, if we additionally assume that $\pi$ is unitary and that it has a (unique) Whittaker model $W_{\psi}(\pi) \subseteq \text{Ind}_{U_{2n}}^{G_{2n}}[\psi]$, where, as usual, $\psi$ is extended to $U_{2n}$ by the rule

$$\begin{pmatrix} 1 & u_{1,2} & \cdots \\ & 1 & u_{2,3} \\ & & \ddots \\ & & & 1 \\ & & & & u_{2n-1,2n} \end{pmatrix} \mapsto \psi(u_{1,2} + u_{2,3} + \cdots + u_{2n-1,2n}).$$

By uniqueness of Whittaker– and Shalika–models, $\text{Hom}_{G_{2n}}(W_{\psi}(\pi), S_{\psi}(\pi))$ is of dimension 1. We will now exhibit an explicit, non-trivial intertwining operator

$$\Theta : W_{\psi}(\pi) \rightarrow S_{\psi}(\pi).$$

We start off with

**Proposition 2.** We have

$$\text{Hom}_{S_{2n} \cap P_{2n}}(\pi, \psi) = \text{Hom}_{S_{2n}}(\pi, \psi).$$

**Proof.** By Frobenius reciprocity and uniqueness of local Shalika models, we obviously have

$$\mathbb{C} \cong \text{Hom}_{S_{2n}}(\pi, \psi) \subseteq \text{Hom}_{S_{2n} \cap P_{2n}}(\pi, \psi).$$

The result then follows from a combination of results of Matringe: By [Mat14], Prop. 4.3, the latter space $\text{Hom}_{S_{2n} \cap P_{2n}}(\pi, \psi)$ embeds into $\text{Hom}_{L_{2n} \cap P_{2n}}(\pi, 1)$, whereas it follows from the proof of Cor.

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1Local components of cuspidal automorphic representations are unitary times a twist by a potentially non-unitary character. The assumption that our representation admits a $(1, \psi)$-Shalika model hence implies that it is itself unitary.
4.18 in [Mat15], that the again latter space, \( \text{Hom}_{L_{2n} \cap P_{2n}}(\pi, 1) \), has dimension at most 1. This implies the assertion.

Because of Prop. 2, if \( \lambda \) is a non-zero element of \( \text{Hom}_{S_{2n} \cap P_{2n}}(W_{\psi}(\pi), \psi) \), then

\[
\Theta : W \mapsto (g \mapsto \lambda(g \cdot W)),
\]

is a non-zero element of \( \text{Hom}_{G_{2n}}(W_{\psi}(\pi), S_\psi(\pi)) \). In what follows we will determine such a \( \lambda \).

To this end, consider the Weyl group representative-matrix \( w_{2n} \), corresponding to the permutation of \( \{1, \ldots, 2n\} \):

\[
\begin{pmatrix}
1 & 2 & \ldots & n & n+1 & n+2 & \ldots & 2n \\
1 & 3 & \ldots & 2n-1 & 2 & 4 & \ldots & 2n
\end{pmatrix}
\]

and put

\[
\lambda(W) := \int_{U_n \cap P_n} \int_{N_n \cap M_n} W \left( w_{2n} \begin{pmatrix} id_n & X & 0 \\ id_n & 0 & 0 \\ id_n & 0 & 0 \end{pmatrix} \right) \psi^{-1}(Tr(X)) dX dp.
\]

If well-defined, i.e., absolutely convergent, this integral is in \( \text{Hom}_{S_{2n} \cap P_{2n}}(W_{\psi}(\pi), \psi) \). Moreover, \( \lambda \) will be non-zero as \( W_{\psi}(\pi)|_{P_{2n}} \) contains \( \text{Ind}_{U_{2n}}(\psi) \), cf. [Ber-Zel77]. It hence remains to prove the following

**Proposition 3.** For any \( W \in W_{\psi}(\pi) \) the integral defining \( \lambda(W) \) is absolutely convergent.

**Proof.** Following Matringe, let us introduce some notation: Let \( A_n \) denote the standard maximal torus of \( G_n \) and recall that the map \( a \in A_n \mapsto \mu_n(a) \in A_n \),

\[
\mu_n(a) := \text{diag}(a_1 \cdots a_n, a_2 \cdots a_n, \ldots, a_n a_n, a_n),
\]

is a group isomorphism. It will be sometimes convenient to use the notation

\[
\mu_n(a_1, \ldots, a_n) := \mu_n(a)
\]

for \( a \in A_n \). Put

\[
\mathfrak{S}_k(W, a) = \int_{N_k \cap M_k} W \left( w_{2n} \begin{pmatrix} id_k & 0 & X & 0 \\ id_{n-k} & 0 & 0 & 0 \\ id_k & 0 & 0 \end{pmatrix} \right) \psi^{-1}(Tr(X)) dX.
\]

This definition is inspired by [Jo19, Lemma 3.14]: In fact, as our first observation, by the Iwasawa decomposition, the convergence of \( \lambda(W) \) for any \( W \) is reduced to that of

\[
\int_{a \in A_{n-1}} \mathfrak{S}_n(W, a) \delta_{B_{n-1}}(a)^{-1} da,
\]

where \( \delta_{B_{n-1}} \) denotes as usual the modulus character of the standard Borel \( B_{n-1} \) of \( G_{n-1} \). Now, by the proof of [Jo19, Lemma 3.14] the convergence of the latter integral is reduced to that of

\[
\int_{A_{n-1}} \prod_{k=0}^{n-1} |a_k|^{-k(n-k)} W \left( w_{2n} \begin{pmatrix} \mu_{n-1}(a) \\ \mu_{n-1}(a) \end{pmatrix} \right) \delta_{B_{n-1}}(\mu_{n-1}(a))^{-1} da.
\]

Note that

\[
w_{2n} \text{diag}(\mu_{n-1}(a), 1, \mu_{n-1}(a), 1) w_{2n}^{-1} = \mu_{2n}(1, a_1, 1, a_2, \ldots, 1, a_{n-1}, 1, 1).
\]
Hence applying the asymptotic expansion of [Mat11, Proposition 2.9] or – as pointed out by Matringe – rather its corrected version in https://arxiv.org/abs/1004.1315v2 (see also [Jo19, Proposition 3.1]), it is sufficient to check that integrals of the form converge:

$$\int_{A_{n-1}} \prod_{k=1}^{n-1} |a_k|^{-k(n-k)} \prod_{k=1}^{n-1} |a_k|^{2k(n-k)} \prod_{k=1}^{n-1} \omega_k(a_k) \nu(a_k)^{m_k} \Phi(a_k) \delta_{B_{n-1}}(\mu_{n-1}(a))^{-1} da.$$ 

Here, $\omega_k$ is the central character of an irreducible subquotient of the derivative $\pi^{(2n-2k)}$ of $\pi$ (see [Ber-Zel77]), $m_k \in \mathbb{N}$ and $\Phi$ is a Schwartz function on $F$. However, this integral equals

$$\int_{A_{n-1}} \prod_{k=1}^{n-1} |a_k|^{k(n-k)} \prod_{k=1}^{n-1} \omega_k(a_k) \nu(a_k)^{m_k} \Phi(a_k) \delta_{B_{n-1}}(\mu_{n-1}(a))^{-1} da$$

$$= \int_{A_{n-1}} \prod_{k=1}^{n-1} \omega_k(a_k) \nu(a_k)^{m_k} \Phi(a_k) \operatorname{det}(\mu_{n-1}(a))|da$$

whence, we are reduced to the one-dimensional case of integrating characters against Schwartz functions. Now, recalling that $\pi$ is unitary, one has $|a_k|^k \omega_k(a_k) = |a_k|^r \omega_k(a_k)$ for some $r_k > 0$ and a $u_k$ unitary character, due to [Ber84, Section 7.3]. The convergence of the last integral now follows from classical results of Tate.

\[\square\]

3. NON-VANISHING OF $\lambda(W^\circ)$

3.1. Preparatory results. According to our definition of $\Theta$ it remains to show that $\lambda(W^\circ) \neq 0$ for one (and hence, by [Jac-PS-Sha81, Thm. 5.1], every) non-trivial Whittaker newvector $W^\circ \in W_\psi(\pi)$. We may hence work with a choice of a Whittaker newvector: Fix $W^\circ_\pi$ defined by $W^\circ_\pi(id) = 1$, cf. [Miy14, Cor. 4.4] for its existence.

We will now follow [Ana17] closely, where the analog computation is performed for local Flicker periods, but the computation of the present note is more involved. To start, we recall a form of [Mat13, Theorem 3.1]:

**Theorem 5.** There is an integer $r$, $0 \leq r \leq 2n-1$ and an unramified standard module $\pi_u$ of $G_r$ such that the spherical Whittaker function $W^\circ_{\pi_u} \in W_\psi(\pi_u)$, normalized by $W^\circ_{\pi_u}(id) = 1$ satisfies

$$W^\circ_{\pi} (\mu_{2n}(a_1, \ldots, a_{2n-1}, 1))$$

$$= W^\circ_{\pi_u}(\mu_r(a_1, \ldots, a_r)) \left| \operatorname{det}(\mu_r(a_1, \ldots, a_r)) \right|^{(2n-r)/2} \mathbf{1}_\mathbb{O}(a_r) \prod_{j=r+1}^{2n-1} \mathbf{1}_\mathbb{O}(a_j)$$

Now, set $w_{2n+1} = \text{diag}(w_{2n}, 1) \in G_{2n+1}$ and let

$$\mathcal{P}_m(\mathcal{O}) := \left\{ p = \begin{pmatrix} g_{m-1} & 0 \\ 0 & 1 \end{pmatrix} \left( \begin{array}{cc} id_{m-1} & x \\ 0 & 1 \end{array} \right) \big| g_{m-1} \in \text{GL}_m(\mathcal{O}) \right\}.$$ 

The integrals $\mathfrak{J}_k$ and their “odd” cousins

$$\mathfrak{J}_k(W, a) := \int_{N_k \setminus M_k} W \left( \begin{array}{cccc} id_k & 0 & X & 0 \\ id_{n-k} & 0 & 0 & 0 \\ id_k & 0 & 0 & 0 \\ id_{n-k} & 0 & 0 & 1 \end{array} \right) \left( \begin{array}{c} a \\ a \\ 1 \end{array} \right) \psi^{-1}(\text{Tr}(X)) dX$$

for $a \in A_n$ will naturally appear in our computation. We record the following useful relations satisfied by them as a lemma.
Lemma 6. Let $\pi$ be a standard module of $G_m$ and $W \in W_\psi(\pi)$ which is fixed by $P_m(O)$. If $m = 2n$ is even, then

$$\mathcal{J}_n(W, \mu_n(a)) = \prod_{k=1}^{n-1} |a_k|^{-k(n-k)} W \left( w_{2n} \begin{pmatrix} \mu_n(a) \\ \mu_n(a) \end{pmatrix} \right) = \prod_{k=1}^n |a_k|^{-k(n-k)} W \left( w_{2n} \begin{pmatrix} \mu_n(a) \\ \mu_n(a) \end{pmatrix} \right).$$

If $m = 2n + 1$ is odd, then

$$\mathcal{J}'_n(W, \mu_n(a)) = \prod_{k=1}^{n-1} |a_k|^{-k(n-k)} W \left( w_{2n+1} \begin{pmatrix} \mu_n(a) \\ \mu_n(a) \\ 1 \end{pmatrix} \right) = \prod_{k=1}^n |a_k|^{-k(n-k)} W \left( w_{2n+1} \begin{pmatrix} \mu_n(a) \\ \mu_n(a) \\ 1 \end{pmatrix} \right).$$

Proof. We provide the argument for $\mathcal{J}_n(W, \mu_n(z))$ (the proof of the second assertion being completely analogous): Right $P_n(O)$-invariance of $W$ implies that

$$\mathcal{J}_{k+1}(W, \mu_n(a)) = \prod_{i=1}^{k} |a_i|^{-1} \mathcal{J}_k(W, \mu_n(a)).$$

as it follows from the end of the proof of [Jo19, Lemma 3.14] (put $\phi = 1_O$ there). Note that the proof in question only deals with $\mu_n(a)$ with $a_n = 1$, but it remains valid for any $\mu_n(a)$. This already shows the claim.

3.2. The main result. We are now ready to compute $\lambda(W^\circ_\pi)$. To this end, note that

$$\lambda(W^\circ_\pi) = \int_{U_n \backslash P_n} \int_{N_n \backslash M_n} W^\circ_\pi \left( w_{2n} \begin{pmatrix} \text{id}_n & X \\ id_n & id_n \end{pmatrix} \begin{pmatrix} p \\ p \end{pmatrix} \right) \psi^{-1}(Tr(X)) \, dX \, dp$$

$$= \int_{U_n \backslash P_n} \int_{N_n \backslash M_n} W^\circ_\pi \left( w_{2n} \begin{pmatrix} \text{id}_n & X \\ id_n & id_n \end{pmatrix} \begin{pmatrix} \text{diag}(g,1) \\ \text{diag}(g,1) \end{pmatrix} \right) \psi^{-1}(Tr(X)) \, dX \, dg.$$

By the Iwasawa decomposition and because $W^\circ_\pi$ is right $P_{2n}(O)$-invariant, this simplifies to

$$\lambda(W^\circ_\pi) = \int_{A_{n-1}} \mathcal{J}_n(W^\circ_\pi, \text{diag}(a,1)) \delta_{B_{n-1}}^{-1}(a) \, da.$$

Applying first our Lem. 6, a simple coordinate change and finally Eq. (4), this integral becomes

$$\int_{A_{n-1}} W^\circ_\pi \left( w_{2n} \begin{pmatrix} \text{diag}(\mu_{n-1}(a),1) \\ \text{diag}(\mu_{n-1}(a),1) \end{pmatrix} \right) \prod_{k=1}^{n-1} |a_k|^{-k(n-k)} \delta_{B_{n-1}}^{-1}(\mu_{n-1}(a)) \, da$$

$$= \int_{A_{n-1}} W^\circ_\pi \left( w_{2n} \begin{pmatrix} \text{diag}(\mu_{n-1}(a),1) \\ \text{diag}(\mu_{n-1}(a),1) \end{pmatrix} \right) w_{2n}^{-1} \prod_{k=1}^{n-1} |a_k|^{-k(n-k)} \delta_{B_{n-1}}^{-1}(\mu_{n-1}(a)) \, da$$

$$= \int_{A_{n-1}} W^\circ_\pi(\mu_{2n}(1, a_1, \ldots, 1, a_{n-1}, 1, 1)) \prod_{k=1}^{n-1} |a_k|^{-k(n-k)} \delta_{B_{n-1}}^{-1}(\mu_{n-1}(a)) \, da.$$
We will now use Matringe’s Thm. 5, distinguishing two cases: First, suppose that \( r = 2r' \) is even. Then we get

\[
\lambda(W_π^∞) = \int_{A_{r'}} \left[ W_π^∞(μ_{r'}(1, a_1, \ldots, 1, a_{r'})) \right] 1_Ω(a_{r'}) \det(μ_{r'}(1, a_1, \ldots, 1, a_{r'})) |^{-n-r'} \]

\[
\times \prod_{k=1}^{r'} |a_k|^{-k(n-k)\delta_{B_{n-1}}^-} (\text{diag}(μ_{r'}(a), \text{id}_{n-1-r'})) da
\]

\[
= \int_{A_{r'}} \left[ W_π^∞(μ_{r'}(1, a_1, \ldots, 1, a_{r'})) \right] 1_Ω(a_{r'}) |^{-2(n-r')} \]

\[
\times \prod_{k=1}^{r'} |a_k|^{-k(n-k)\delta_{B_{n-1}}^-} (\text{diag}(μ_{r'}(a), \text{id}_{n-1-r'})) da
\]

\[
= \int_{A_{r'}} \prod_{k=1}^{r'} |a_k|^{-k(n-k)\delta_{B_{n-1}}^-} (\text{diag}(μ_{r'}(a), \text{id}_{n-1-r'})) da
\]

\[
= \int_{A_{r'}} \prod_{k=1}^{r'} |a_k|^k (\text{diag}(μ_{r'}(a), \text{id}_{n-1-r'})) 1_Ω(a_{r'}) \delta_{B_{r'}^-}^{-1}(μ_{r'}(a)) \det(μ_{r'}(a)) |^{-n-r'+1} da
\]

\[
= \int_{A_{r'}} \prod_{k=1}^{r'} |a_k|^k \left( w_{2r'} \left( \mu_{r'}(a) \right) \right) 1_Ω(a_{r'}) \delta_{B_{r'}^-}^{-1}(μ_{r'}(a)) \det(μ_{r'}(a)) |^{-n-r'+1} da
\]

\[
= \int_{A_{r'}} \prod_{k=1}^{r'} |a_k|^k \left( w_{2r'} \left( \mu_{r'}(a) \right) \right) 1_Ω(a_{r'}) \delta_{B_{r'}^-}^{-1}(μ_{r'}(a)) \det(μ_{r'}(a)) | da
\]

\[
= \int_{A_{r-1}} \prod_{k=1}^{r'-1} |a_k|^k (\text{diag}(μ_{r'-1}(a), 1)) | da \int_{F^×} ω_π(a_{r'}) 1_Ω(a_{r'}) | a_{r'} |'^r da_{r'}
\]

which, according to Lemma 6 again, becomes

\[
= \int_{A_{r-1}} 3_π(ω_π^∞, \text{diag}(μ_{r'-1}(a), 1)) \delta_{B_r^-}^{-1}(μ_{r'-1}(a)) | da \int_{F^×} ω_π(a_{r'}) 1_Ω(a_{r'}) | a_{r'} |'^r da_{r'}
\]

Now, set \( e_{r'} := (0, \ldots, 0, 1) \) \((a 1 \times r' \text{ unit-vector})\) and denote by \( J(s, W, Φ) \) the Jacquet-Shalika integral

\[
J(s, W, Φ) := \int_{U_{r'} \setminus G_{r'}} \int_{J_{N_{r'}} \setminus M_{r'}} W \left( w_{2r'} \left( \text{id}_{r'} X \right) \right) \psi^{-1}(\text{Tr}(X)) \Phi(e_{r'}, g) | \det(g) |^s dX dg.
\]

Once more by the Iwasawa decomposition we get that \( \lambda(W_π^∞) = J(1, W_π^∞, 1_{Ω_{r'}}) \), whence, applying the unramified computation of the exterior square \( L \)-function from [Jac-Sha90, Section 7.2], we finally obtain

\[
\lambda(W_π^∞) = L(1, π_u, Λ^2) \neq 0.
\]

Now assume \( r = 2r' + 1 \) is odd. As the variable \( b_{2r'+1} = b_{2r'+2} \) of \( b = μ_{2n}(a) \) in the integral must vary in \( O^× \), whereas \( W_π^∞ \) is \( \text{GL}_r(O) \)-invariant, we obtain:

\[
\lambda(W_π^∞) =
\]
we obtain thanks to Iwasawa decomposition
Following the steps of the even computation we arrive at
and let
As a consequence, any (non-zero) Shalika newvector in
Asthissmallnoteisalsomeantasanaddendumtoourpaper[Gro-Rag14], where other twisting
First, assume that


we summarize this computation in the following

**Theorem 7** (Matringe). Let \( \Theta: \mathcal{W}_\psi(\pi) \rightarrow \mathcal{S}_\psi(\pi) \) be the nonzero intertwining operator given by

\[
\Theta(W)(g) = \int_{U_n \backslash P_n} \int_{N_n \backslash M_n} W \left( \begin{array}{ccc}
\id & X & p \\
0 & \id & 0 \\
p & 0 & 1 
\end{array} \right) \psi^{-1}(Tr(X)) \, dX \, dp.
\]

and let \( S^\circ_\pi = \Theta(W^\circ_\pi) \), where \( W^\circ_\pi \) is the unique new vector in \( \mathcal{W}_\psi(\pi) \), satisfying \( W^\circ_\pi(\id) = 1 \). Then
\( S^\circ_\pi(\id) = L(1, \pi_u, \Lambda^2) \neq 0. \)

As a consequence, any (non-zero) Shalika newvector in \( \mathcal{S}_\psi(\pi) \) does not vanish at \( g = \id \).
4.3. Now, let $\eta = \bar{\eta} \cdot |w|$, with $w \in \mathbb{Z}$ and $\bar{\eta}$ a unitary unramified character. Let $\pi$ be an irreducible generic representation of $G_{2n}$, which has a $(\eta, \psi)$-Shalika model $S^{0}_{\psi}(\pi)$. Then, every newvector $S$ in $S^{0}_{\psi}(\pi)$ is of the form $S = |\det(\cdot)|^{w/2} \cdot \tilde{S}$, with $\tilde{S}$ a newvector in $S^{0}_{\psi}(\pi)$. See also [Gro-Rag14], Lem. 5.1.1. As a consequence, every non-zero newvector in $S^{0}_{\psi}(\pi)$ does not vanish at $g = id$.

We summarize the latter observations in the following

**Corollary 8.** Let $\eta = \bar{\eta} \cdot |w|$, with $w \in \mathbb{Z}$ and $\bar{\eta}$ a unitary unramified character. Let $\pi$ be an irreducible generic representation of $G_{2n}$, which has a $(\eta, \psi)$-Shalika model $S^{0}_{\psi}(\pi)$. Then, for every newvector $S$ in $S^{0}_{\psi}(\pi)$,

$$S(id) \neq 0.$$ 

**References**


