Debt Maturity Structure and Liquidity Shocks

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December 28, 2018

We analyze the role of a firm's debt maturity structure when refinancing its debt after a liquidity shock that reduces the firm's cash flow. Staggered debt diminishes the share of outstanding debt that a firm has to refinance at any given time, which should be most beneficial for highly levered firms. However, we show that for highly levered firms, a firm's ability to roll over its maturing debt hinges on its ability to prefinance its outstanding debt expiring in future periods. Prefinancing involves holding sufficient cash to repay the outstanding debt when it expires and eliminates the potential benefits of staggered debt. If agency problems prevent a firm from holding sufficient cash to implement this strategy, then staggered debt can reduce a firm's ability to withstand a negative cash flow shock.

Keywords: staggered debt; default boundary; liquidity shock.

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1. Introduction

We explore a firm's capacity to withstand unexpected, temporary, liquidity shocks with different debt structures. With concentrated debt, a firm rolls over its entire debt whenever this debt is due. With staggered debt, a firm rolls over only a part of its debt at any given time while the remaining debt matures at some future date. Staggered debt and different funding sources can reduce the expected cost of rolling over debt in the presence of shocks to the financial system that adversely affects firms' financiers, and thus results in a funding shock for the firm (Almeida et al., 2011; He and Xiong, 2012b). We focus on shocks to a firm's assets, which result in temporarily lower cash flows and increased funding needs. We show that a firm with staggered debt that has to refinance only part of its debt might actually find it more difficult to withstand such a liquidity shock than a firm that has to refinance its total debt.

The reason why a staggered debt structure can be detrimental is that the outstanding debt creates a negative externality on the pricing of new debt for highly levered firms. When a firm experiences a negative cash flow shock it can be optimal for a firm to use long-term funding to minimize the likelihood of roll over failures that result in costly financial distress (e.g., Brunnermeier and Yogo, 2009). If the firm refinances only the maturing part of the debt, it is still exposed to the rollover risk of of the part of its debt that matures in the near future. Failure to roll over this debt results in financial distress, the cost of which are borne by all debt holders. Thus, the risk of defaulting on outstanding debt spills over to newly issued debt and thus the conditions at which the firm can raise this debt. To overcome this externality, it might be necessary for the firm to prefinance its outstanding debt. Prefinancing involves issuing new long-term debt today and storing cash to repay the outstanding debt when it matures. The need to prefinance outstanding debt that matures in the future limits potential benefits from low levels of debt maturing today. Agency costs of holding cash can impede a firm's ability to prefinance future debt. In this case, a firm whose entire debt matures today might be able to roll over its debt, while a firm with the same total debt, but only part of it maturing today, might not be able to roll over its maturing debt.

We model a firm whose assets generate a fixed cash flow stream in normal times (high state). The firm may encounter a liquidity shock in which case the cash flow temporarily dries out (low state). After a liquidity shock, the time until the firm returns to the high state is uncertain. Firms finance themselves by issuing short-term and long-term debt in a liquid, competitive debt market. Without the possibility to run into a liquidity shock, the debt structure does not matter in our setting and the debt is risk free. However, the firm's ability to survive a liquidity shock depends on both the level and maturity structure of its outstanding debt as well as the maturity structure of the debt it uses to refinance the outstanding debt.

If the firm cannot roll over its debt, the firm goes bankrupt, which is costly. The maximum amount of debt the firm can raise when the low cash flow state hits, depends on the maximum amount of new debt the firm can repay if it returns to the high state before this newly issued debt matures and the probability with which it will return to the high state. The price of such risky debt claims reflects the risk and costs of bankruptcy.

The firm's debt maturity structure determines its funding needs and debt policies after a cash flow shock. A firm with staggered debt has always some debt maturing shortly after a cash flow shock. The amount of this debt is lower than that for a firm with concentrated debt that matures right after the liquidity shock. Thus, the firm with staggered debt has to issue less debt. However, a firm with staggered debt can only repay its newly issued debt if it does not default on its additional outstanding debt before the new debt matures. Because the firm's outstanding debt affects the probability of default, it imposes an externality on newly issued debt. Investors anticipate the rollover risk, which reduces the firm's ability to refinance its maturing debt.

Firms can eliminate the rollover risk by prefinancing their outstanding debt. To do so, the firm issues sufficient new debt today and stores the proceeds as cash to be able to repay the outstanding debt when it matures. As a result, the firm's outstanding debt becomes risk free and the firm's new debt structure functionally resembles a concentrated debt structure with long-term debt. Prefinancing helps to delay the next round of refinancing and gives the firm more time to recover from the low cash flow state. Alternatively, the firm can issue new debt that it can only repay if the cash flow recovers before its outstanding debt matures. In this case, it has less time to recover, but needs to raise less debt. Such gambling on a short-term recovery, exposes the firm to refinancing risk and dilutes the outstanding debt.

The firm will optimally trade off the benefits of prefinancing and gambling on a short-term recovery. Prefinancing increases the market value of new debt, but increases the amount of financing that the firm must raise. Hence, prefinancing is optimal when the increase in the firm's financing need (outstanding debt) is relatively low. With prefinancing, a firm's ability to roll over its debt does not depend on the relative amounts of maturing and outstanding debt and is the same with staggered and concentrated debt. If instead the amount of outstanding debt relative to maturing debt is sufficiently high, the firm stops prefinancing its outstanding debt and gambles on a short-term recovery. In this case, while the new debt that the firm must issue to repay maturing debt carries a higher risk premium, the lower amount of new debt the firm must raise with this strategy makes it easier for the firm to roll over its debt. Fully staggered debt results in intermediate ratios of outstanding to maturing debt. We derive conditions under which prefinancing is optimal when the firm's debt is fully staggered.

Prefinancing is not possible if the amount of cash that a firm can hold is limited due to agency problems. One example of such an agency problem is the risk of cash diversion by the firm's management (when the firm is in or close to financial distress). This constraint decreases the firm's ability to roll over its debt. As a result, firms with concentrated debt can better withstand negative cash flow shocks.

Our model generates several testable predictions. For example we predict that firms with staggered debt and weak corporate governance that results in high agency costs of holding cash will be more likely to default when they become financially distressed. We also predict that firm's with staggered debt that experience financial distress will increase their cash holdings and move to a less staggered debt structure.

Several papers analyze debt refinancing and maturity structure. One strand of literature focuses on firms' ability to refinance maturing debt by raising new debt. This literature has analyzed dynamic models of rolling over concentrated debt (Acharya et al., 2011; Brunnermeier and Yogo, 2009; Liang et al., 2014) and fully staggered debt (He and Xiong, 2012a; Schroth et al., 2014).[1] In both Acharya et al. (2011) and Brunnermeier and Yogo (2009), firms' ability to roll over their debt following drops in asset value increases when the firm can issue long-term debt to decrease its future roll-over risk. In He and Xiong (2012a), the impact of firms' debt maturity on their roll over ability depends on the volatility of a firm's cash flow process. These papers do not compare firms' ability to roll over concentrated versus staggered debt.

Our paper adopts a similar modeling approach as Acharya et al. (2011) and introduces the possibility of using partially and fully staggered debt. Thereby, we can compare the impacts of staggered versus concentrated debt on firms' ability to roll over their debt. We show that, in order to roll over staggered debt, firms must sometimes prefinance their outstanding debt in order to increase their debt maturity and reduce future roll-over risk. If prefinancing is not possible, staggered debt reduces firms' ability to withstand negative cash flow shocks.

Another strand of literature focuses on firms' optimal financing structures, allowing the firm to raise equity, in dynamic models with endogenous default. Early seminal papers

[1]The literature on bank runs Diamond and Dybvig (1983); Goldstein and Pauzner (2005); Rochet and Vives (2004) analyzes banks' ability to secure sufficient debt financing when all depositors simultaneously decide whether to continue financing the bank.

include Leland (1994) and Leland and Toft (1996) and relevant recent contributions are He and Milbradt (2016) and Chaderina (2018). In particular, He and Milbradt (2016) show that firms with fully staggered debt have an incentive to increase their debt maturities following drops in asset value in order to reduce future roll-over losses. Closely related to our paper, Chaderina (2018) analyzes a model of optimal financing in which firms can issue debt with different maturities, store cash, and issue costly equity. She also finds that firms can have incentive to prefinance outstanding debt to reduce the expected costs of future financing. Our paper characterizes the optimality of prefinancing for firms that must raise sufficient debt financing to roll over their debt at the default boundary. We also show that agency costs of holding cash severely reduce firms' ability to roll over staggered debt.

In two related papers, Choi et al. (2017a,b) also explore firms' choice between staggered and concentrated debt. In their models, staggered debt limits the amount of funds the firm needs at each repayment date. The firm thereby diminishes its roll-over risk because it reduces the amount of asset liquidation or forgone investment opportunities when a firm cannot roll over its debt.[2] Implementing staggered debt is costly in their model, as issuing two different types of bonds is more costly than one type of bond. We identify another cost of staggered debt and show that staggered debt can reduce firms ability to raise new financing. The reason is that outstanding debt increases future roll over risks and can make it more difficult to refinance the maturing part of the debt than if the entire debt would expire today.

Choi et al. (2017a) empirically document that firms affected by a negative shock to market wide liquidity subsequently stagger their debt more. In related work, Choi et al. (2017b) show that larger and more mature firms, with better investment opportunities, higher leverage, and lower profitability have more staggered debt. Norden et al. (2016) document that more staggered debt correlates with lower financing costs and firms being less financially constrained. None of these empirical papers specifically focuses on the impact of staggered debt on firms close to financial distress. In this regard, our paper develops a series of new empirical predictions relating the probability of default to the maturity structure of firms' net debt.

[2]In Darst and Refayet (2017) firms choose dispersed debt maturities to cater to investors with different beliefs about the future.

2. Model

2.1. Project and liquidity shock

A firm has a project that generates a cash flow r_t at the end of each period t over an infinite horizon. The level of the cash flow depends on whether the firm incurs a liquidity shock. In normal times, which we refer to as the high cash flow state H, the cash flow is $r^H > 0$. If the firm incurs a liquidity shock, it enters a low cash flow state L with zero cash flow $r^L = 0$. Possible causes of such a liquidity shock include demand shocks, payment problems of customers and borrowers, a breakdown of a firm's machines, a strike of employees, and regulatory constraints such as the temporary withdrawal of a license for the time of an investigation.

After a liquidity shock, the project's cash flow can recover with probability p_{LH} every period t. If the firm recovers from the liquidity shock, it enters the high cash flow state again. We assume that once the firm recovers, it never leaves the high state again. The corresponding state transition matrix after the firm incurred a liquidity shock is given by

$$\begin{pmatrix} p_{HH} & p_{LH} \\ p_{HL} & p_{LL} \end{pmatrix} = \begin{pmatrix} 1 & 1-p \\ 0 & p \end{pmatrix}$$

where p_{ij} denotes the transition probability from state i to state j.

2.2. Financing

A firm finances its project with debt. At the beginning of each period t, which we refer to as date t, the firm can issue any combination of short-term and long-term debt. Short-term debt issued at date t matures at date $t+1$, long-term debt matures at date $t+2$. The total payment obligations from debt issued at date t are d_t after one period and D_t after two periods. If the long-term debt pays coupons, these payments are included in d_t. The firm's financing choices are depicted in Figure 1.

The firm needs to attract new investors whenever it issues a new debt claim. The market is competitive with risk-neutral investors, who discount expected future cash flows with a discount factor $\delta \in (\frac{1}{2}, 1)$.

2.3. Cash holdings

At every date t, the firm can store cash C_t. We assume that holding cash is costly and model the cost by assuming that cash earns zero return. Cash holdings play an important

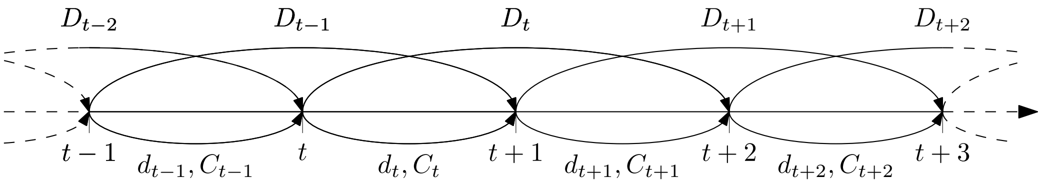

Figure 1: Time-line of the firm's financing decisions. At every date t, the firm must repay the maturing debt it has issued either one and two periods earlier, d_{t-1} and D_{t-2}. The firm can issue new short-term and long-term debt d_t and D_t, and decides how much cash C_t it holds.

role in a firm's financing decisions and we discuss the implications if the amount of cash that a firm can store is limited by agency problems.

The amount of cash that firms can store may be limited as high cash holdings can intensify the agency conflicts in a firm because it is easier to divert cash than a firm's productive assets. Cash diversion can take many forms including perquisite consumption, self-dealing, transfer pricing, or embezzlement. Holding cash is particularly problematic following a liquidity shock that decreases a firm's cash flows and increases the risk of financial distress. Investors anticipate the risk of misappropriation and the firm is unable to raise capital to hold cash levels that involve high agency costs.[3] We do not explicitly model the agency problem that limits the firm's ability to hold cash. Instead, we simply assume that following a liquidity shock, the investors will not finance cash holdings in excess of $\bar{C}$.

2.4. Timing and information

At the beginning of each date t, the firm simultaneously issues new debt, repays maturing debt, and stores cash. Possible state transitions occur after debt is issued. Hence, when the firm issues debt at date t, it knows only state s_{t-1}. If the firm is in the low state in period $t-1$ after a liquidity shock and refinances its debt at date t, it does not know when its cash flow will recover.. There is no asymmetric information between the firm and investors, and investors have the same information about the state as the firm.

2.5. Debt structure

At the beginning of any date t, the firm's debt structure consists of the maturing debt $R_t \equiv d_{t-1} + D_{t-2}$, the outstanding debt D_{t-1} and the cash stored at the previous date

[3]For recent empirical evidence on the effect of agency problems on corporate cash holdings see Nikolov and Whited (2014); Dittmar and Mahrt-Smith (2007).

C_{t-1}.

We call a firm's debt structure staggered when it has both maturing and outstanding debt, such that $R_t > 0$ and $D_{t-1} > 0$. It is fully staggered when $R_t = D_{t-1}$. A firm's debt structure is concentrated if its entire debt either matures at date t , such that $R_t > 0$ and $D_{t-1} = 0$, or at date $t + 1$, such that $R_t = 0$ and $D_{t-1} > 0$.

2.6. Cost of financial distress

The firm is bankrupt if it cannot repay its maturing debt at date t or if it is certain that it cannot repay its outstanding debt at date $t + 1$. When firms raise new debt, the consequences of financial distress are priced into the new debt. In order to simplify the exposition, we assume that the recovery value of assets is zero in bankruptcy. This assumption allows us to abstract from claim dilution among different classes of the firm's creditors in bankruptcy. We discuss recovery values in bankruptcy and debt seniority in Section 6.2.

As is common in the literature on costly financial distress, we assume that firms cannot renegotiate their outstanding debt to avoid the cost of financial distress due to creditor dispersion. We also assume that the firm can neither call nor buy back its debt. If instead firms were able to costlessly restructure their debt when they face financial distress, then financial distress would not be costly (e.g., Haugen and Senbet, 1987). For a discussion of why firms cannot efficiently renegotiate or buy back their public debt see, for example, Gertner and Scharfenstein (1991).

3. Refinancing Maturing Debt

3.1. Debt capacity

At any date t, the firm must repay its maturing debt R_t. In order to repay its debt, the firm can use its internal funds and issue new debt. The firm's internal funds at date t consist of the cash C_{t-1},which the firm stored at date $t - 1$, and the cash flow r_{t-1}, which the project generated at the end of period $t - 1$.

Let $V(d_t, D_t \mid D_{t-1}, C_t, s_{t-1})$ denote the total market value of new short-term and long-term debt that the firm issues at date t. The market value depends on the probability with which the firm can make the promised payments. This probability depends on the firm's outstanding payment obligation D_{t-1}, which is due at date $t + 1$, the amount of cash C_t, which the firm stores over the next period, and the current state s_{t-1} of its project. At any date t, the firm's decision variables are d_t, D_t, and C_t.

The firm can roll over its maturing debt when the following constraint is satisfied

$$R_t - C_{t-1} - r_{t-1} \leq \max_{d_t, D_t, C_t} [V(d_t, D_t \mid D_{t-1}, C_t, s_{t-1}) - C_t]. \tag{1}$$

The left hand side of the inequality represents the firm's external funding need at date t. The maximization term on the right hand side represents the firm's ability to raise external funds that it can use to repay its maturing debt. We call this maximization term the firm's net debt issuance capacity (or simply debt capacity for future reference)

$$B(D_{t-1}, s_{t-1}) \equiv \max_{d_t, D_t, C_t} V(d_t, D_t \mid D_{t-1}, C_t, s_{t-1}) - C_t. \tag{2}$$

The debt capacity at date t is the maximum amount of new debt the firm can borrow against its future expected cash flows net of the cash it stores at date t, C_t. The debt capacity is a function of outstanding (long-term) debt D_{t-1} and the current state of the project s_{t-1}. The external funding need at date t does not affect the debt capacity. The outstanding debt D_{t-1} limits the amount of new debt the firm can issue.

Storing cash C_t reduces the amount of funds a firm can use to repay maturing debt, but it is sometimes necessary to ensure that the firm can repay its outstanding debt D_{t-1}. Hence, a firm that reduces its cash holdings might also reduce the amount of external funds it can raise and this effect might dominate the direct effect of storing cash on the debt capacity.

The maximization in (1) and (2) is subject to the relevant non-negativity constraints for debt and cash holdings and the firm surviving with positive probability. If the amount of outstanding debt D_{t-1} exceeds the present value of the firm's expected future cash flow, the firm will only be able to survive if it has accumulated sufficient cash from internal funds and does not issue any new debt. In this case, the firm's debt capacity $B(D_{t-1}, L)$ is negative and describes the required amount of internal funds net of debt repayment that allows the firm to survive with positive probability. The rationale for this set up is that a firm must declare bankruptcy if it knows that it will fail with certainty. If $B(D_{t-1}, L)$ is positive, the firm can issue new debt to roll over maturing debt at date t.

3.2. Refinancing after reentering the high state

After reentering the high state, the firm's future cash flows are certain. Since cash flows accrue at the end of a period, their present value is given by

$$v^H = \frac{\delta r^H}{1 - \delta}.$$

To maximize its borrowing against the future cash flows, the firm must pay out all available cash flows as quickly as possible because stored cash does not bear interest and there is no benefit of storing cash after reentering the high state in our model. In the proof to Lemma 1 (Appendix A), we show that one debt structure that maximizes the firm's borrowing is two-period coupon debt issued at par. However, there are many debt structures that achieve this objective once the firm returns to the high state after a liquidity shock.

When the firm pledges its cash flows to new creditors, it must account for its outstanding debt obligation D_{t-1}. If v^H exceeds the present value of the outstanding debt δD_{t-1}, then the maximum amount of free cash flows that the firm can pledge to new creditors is $v^H - \delta D_{t-1}$. Conversely, if $\delta D_{t-1} > v^H$, the firm cannot borrow against its future cash flow given the high level of outstanding debt maturing at date $t+1$. Instead, the firm must have sufficient cash from past periods to repay this debt. Since cash does not bear interest, the firm must store at least $C_t = D_{t-1} - \delta^{-1} v^H$.

Lemma 1. *Consider a firm that, after a liquidity shock, reenters the high state in period $t-1$. The firm's debt capacity at date t is given by*

$$B(D_{t-1}, H) = \begin{cases} v^H - \delta D_{t-1}, & v^H \geq \delta D_{t-1} \\ \delta^{-1} v^H - D_{t-1}, & \textit{otherwise.} \end{cases} \tag{3}$$

Proof. See Appendix A. □

3.3. Refinancing in the low state

If the firm incurs a liquidity shock, its cash flow is zero, and the time when the liquidity shock is over is uncertain. In the low state, the present value of the project's expected future cash flows if the firm can survive the liquidity shock is

$$\frac{1-p}{1-\delta p} v^H.$$

Given the uncertainty about when the cash flow recovers and the cost of financial distress, the firm cannot pledge the total present value of the project's expected cash flow to investors after a liquidity shock. Thus, the challenge for the firm is to choose a financing strategy that maximizes its ability to roll over its debt after a liquidity shock. We discuss the key trade-offs of different financial strategies and a firms debt capacity here and formally derive the firm's debt capacity in the low state in Appendix B.

Consider a firm with concentrated debt that needs to roll over its entire debt. The maximum amount of debt that the firm can raise today, surviving a liquidity shock at least one period, involves taking on risky debt. Assume the firm takes on risk-free debt after a liquidity shock. The debt is risk free if and only if the firm can repay this debt even if its cash flow did not recover before the debt matures. When the firm has to refinance its new debt in the low state, the probability of recovering from the liquidity shock has not changed. Hence the firm is in the same situation as today, but with one important difference: the amount of debt that the firm has to refinance then is higher than the amount it needs to refinance today as the firm has to pay interest on the maturing debt. Thus, the financing strategy that maximizes the amount of debt, which the firm can raise right after the liquidity shock occurs, cannot involve risk-free debt.

The maximum amount the firm can raise with short-term debt at date t involves taking on risky debt that the firm can repay only if it returns to the high state in the next period. Repayment occurs with probability $(1-p)$ and the maximum amount of debt the firm can raise is

$$(1-p)v^H. \tag{4}$$

If the firm uses risky long-term debt, it increases the probability of recovering from the liquidity shock (returning to the high state) before it has to repay its new debt, which is now $(1-p^2)$. However, the face value is discounted over one additional period. The maximum amount of debt the firm can raise with long-term risky debt is

$$(1-p^2)\delta v^H. \tag{5}$$

Comparing Expressions (5) and (4), shows that the firm can roll over more debt if it uses long-term debt when

$$(1+p)\delta > 1. \tag{6}$$

This condition states that the option to wait an additional period for the firm to recover is more valuable than the interest cost of delaying the repayment of debt. If Condition (6) is violated, risky short-term debt maximizes the firm's debt capacity.

The trade off between different financing strategies is more complicated in the presence of staggered debt. With staggered debt, the firm has some outstanding debt D_{t-1} at date t, which creates additional roll-over risks. If the firm uses short-term debt to roll over the maturing debt, the outstanding debt simply reduces the maximum amount of new debt that the firm can repay next period if it recovers from the liquidity shock over the following period. Hence, the firm's debt capacity with short-term debt is

$$(1-p)(v^H - \delta D_{t-1}). \tag{7}$$

In contrast, when using long-term debt, the firm can only benefit from waiting until date $t+2$ to repay the new debt if it is able to repay its outstanding debt D_{t-1} at date $t+1$. To pay back D_{t-1}, the firm can either wait and issue debt at date $t+1$ or prefinance the outstanding debt by issuing (additional) long-term debt at date t and storing the proceeds as cash. If the firm waits to issue new debt until date $t+1$, the cash flow may not have recovered and it cannot refinance the outstanding debt and defaults. Thus, the firm is able to repay its new debt D_t only if it recovers over the next period, which occurs with probability $(1-p)$, which resembles the survival probability with short-term debt. If the firm chooses to prefinance the outstanding debt with risky long-term debt, it can repay this debt with probability $(1-p^2)$. Thus, the trade off between issuing debt at date $t+1$ and prefinancing the outstanding debt D_{t-1} resembles the trade off between short-term and long-term debt for $D_{t-1}=0$. In Appendix B.2 we show that Condition (6) makes prefinancing optimal here. Prefinancing maximizes the firm's debt capacity when it uses long-term debt if Condition (6) is satisfied. The firm's debt capacity with long-term debt and prefinancing is given by

$$(1-p^2)\delta v^H - D_{t-1} \tag{8}$$

As long as the firm can issue new debt to borrow against its future cash flows, the firm's debt capacity is given by the maximum over the Expressions (7) and (8). When Condition (6) is violated, then short-term debt dominates long-term debt. Otherwise, comparing Expressions (7) and (8) shows, there exists a critical threshold $\hat{D}$, such that the firm's debt capacity is higher when it issues short-term debt if and only if $D_{t-1} > \hat{D}$.

The trade-off between long-term and short-term debt hinges on the roll-over risks of the firm's new and outstanding debt. If the firm uses short-term debt, it defaults on the outstanding debt if the liquidity shock lasts longer than one period which dilutes the value of the outstanding debt. In contrast, long-term debt in combination with prefinancing reduces the roll-over risk of the firm's new debt but also makes the outstanding debt risk free despite the firm's liquidity problems. The value of diluting outstanding debt relative to the value of reducing the roll-over risk of new debt increases in the amount of outstanding debt. This trade-off determines the threshold $\hat{D}$.

In the remainder of our paper we assume that Condition (6) is satisfied so that long-term debt can be optimal.

4. Default Boundary, Debt Maturity Structure, and Cash-Holding Constraints

4.1. Default boundary

Whether a firm can successfully roll over its debt following a negative cash flow shock depends on its total net debt $N_t \equiv R_t + D_{t-1} - C_{t-1}$ and its debt structure. It is convenient to express the debt structure as the share of outstanding debt relative to total net debt, $\alpha_t \equiv D_{t-1}/N_t$. The ratio α_t captures the back loading of the debt structure as it measures the share of current total net debt that is due next period. $1 - \alpha_t$ is the share of net debt that the firm has to repay today. Together, N_t and α_t uniquely determine both, the firm's external funding need and its outstanding debt. Thus, we can rewrite the firm's roll over constraint (1) in the low state, substituting for $D_{t-1} = \alpha_t N_t$ and $r^L = 0$, as

$$(1 - \alpha_t)N_t \leq B(\alpha_t N_t, L) \tag{9}$$

If this condition is satisfied, the firm can roll over its maturing debt. If not, it will default. It follows that the maximum amount of net debt that the firm can roll over for a given debt structure $\hat{N}(\alpha_t)$ is implicitly defined by Condition (9) holding with equality. We call $\hat{N}(\alpha_t)$ the firm's default boundary.

In the following Proposition, we describe a firm's default boundary and the corresponding financing strategies. Figure 2 depicts the shape of the default boundary.

Proposition 1. *Consider a firm's default boundary $\hat{N}(\alpha_t)$ at date t after a liquidity shock. There exists a threshold*

$$\hat{\alpha} \equiv \left(1 - \frac{1}{(1+p)\delta}\right) \times \left(1 - (1-p)\delta\right)^{-1} \tag{10}$$

such that:

1. *For $\alpha_t \leq \hat{\alpha}$, the firm's default boundary is*

$$\hat{N} = (1 - p^2)\delta v^H.$$

To roll over its maturing debt at the default boundary, the firm must issue risky, zero coupon, long-term debt with market value $(1 - p^2)\delta v^H$ and fully prefinance its outstanding debt, storing cash $C_t = D_{t-1} = \alpha \hat{N}$.

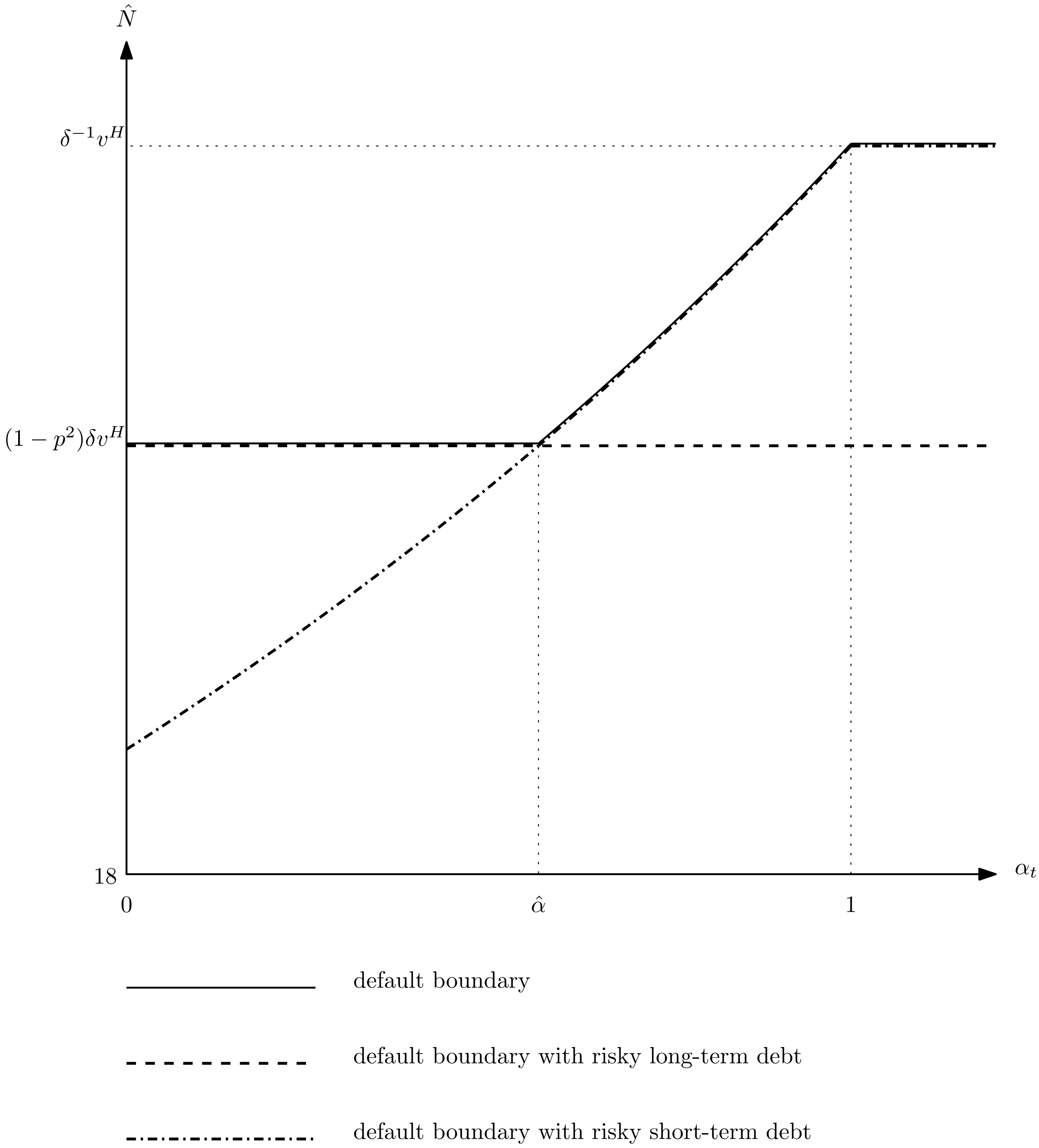

Figure 2: This figure plots the default boundary in the low state as a function of α_t. The solid line depicts the firm's default boundary described in Proposition 1, the dashed line depicts the firm's default boundary if it issues risky long-term debt, and the dashed-dotted line depicts the firm's default boundary if it issues risk short-term debt. The figure is drawn for the parameter values $r^H = 1$, $\delta = 0.96$, and $p = 0.2$.

2. *For $\alpha_t \in (\hat{\alpha}, 1)$, the firm's default boundary is*

$$\hat{N} = \frac{(1-p)v^H}{1 - \alpha_t(1-(1-p)\delta)}.$$

To roll over its maturing debt at the default boundary, the firm must issue risky short-term debt with market value $(1-p)(v^H - \delta D_{t-1})$ that it can only repay if it returns to the high state before date $t+1$. The firm must not store any cash.

3. *For $\alpha_t \geq 1$, the firm's default boundary is*

$$\hat{N} = \delta^{-1} v^H.$$

The firm must pay back its maturing debt R_t with its existing cash holdings C_{t-1} and must not issue any new debt. The firm survives the liquidity shock only if it stores its remaining cash and returns to the high state before date $t+1$.

Proof. See Appendix C.2. □

For $\alpha_t < \hat{\alpha}$ the the firm must use long-term debt and prefinance its total outstanding debt at the default boundary. Thus, for a given level of total net debt, a higher share of outstanding debt that matures at date $t+1$ does not reduce the firm's funding needs at date t in this region. As a result, the firm's default boundary does not depend on its debt structure either. It follows that the conventional argument that staggered debt benefits firms because they have to raise less debt after a liquidity shock does not apply. Thus, for low shares of outstanding debt, the benefits of staggered debt do not carry forward to highly levered firms, who are close to default and thus need to maximize their debt financing.

For $\alpha_t \in (\hat{\alpha}, 1)$, the share of outstanding debt at the default boundary becomes so high that the firm can no longer prefinance it. Instead, the firm needs to dilute it by issuing short-term debt. In this case, the default boundary increases in the share of outstanding debt because α_t reduces the amount of funds that the firm must raise and increases the value of diluting its outstanding debt.

For $\alpha_t > 1$, the firm's total net debt is lower than the outstanding debt D_{t-1} . Thus, the firm has sufficient cash to repay its maturing debt and part of its outstanding debt. In this case, the firm defaults only if it is not able to repay its outstanding debt even if it returns to the high state in period t.

4.2. Binding cash holding constraints

High cash holdings can intensify the agency conflicts in a firm because it is easier to divert cash than a firm's productive assets. Cash diversion can take many forms including perquisite consumption, self-dealing, or embezzlement. Holding cash will be particularly problematic following a liquidity shock that increases the risk of financial distress.[4]

We do not explicitly model the agency problem of holding cash. Instead, we adopt a reduced form approach and assume that following a liquidity shock, the firm's cash holdings cannot exceed some threshold $\bar{C}$. To avoid the misappropriation of funds, investors will thus refuse to finance a firm that attempts to raise funds in order store cash in excess of $\bar{C}$. Such a cash holding constraint decreases the firm's default boundary in the low state if it prevents financing strategies that maximize a firm's ability to roll over its debt. The following Lemma describes the two cases in which a cash holding constraint can be binding.

Lemma 2. *A cash holding constraint $\bar{C}$ can prevent the firm from implementing the strategies described in Proposition 1 in two cases:*

1. *It constrains the firm's ability to raise cash to prefinance its outstanding debt when*

$$\alpha_t \in \left(\bar{C}/\hat{N}(\hat{\alpha}), \hat{\alpha}\right). \tag{11}$$

2. *It constrains the level of cash a firm can hold to reduce its net debt relative to outstanding debt*

$$\alpha_t > 1 + \delta \frac{\bar{C}}{v^H} \tag{12}$$

In all other cases, the firm's debt capacity and strategy at the default boundary are as described in Proposition 1.

Proof. See Appendix C.2. □

In the following discussion, we focus on the interesting case where a cash holding constraint prevents the firm from prefinancing its outstanding debt (case 1 of Lemma 2).

4.3. Default boundary with a binding cash holding constraint

There are three different strategies that can maximize a firm's ability to roll over its debt in case 1 of Lemma 2. First, the firm can issue risky long-term debt and prefinance

[4]For recent empirical evidence on the effect of agency problems on corporate cash holdings see Dittmar and Mahrt-Smith (2007) and Nikolov and Whited (2014).

its outstanding debt up to $\bar{C}$. The remainder of the outstanding debt $D_{t-1} - \bar{C}$ is then financed by issuing additional risky debt at date $t + 1$. Second, the firm can issue risky short-term debt that dilutes the outstanding debt. Third, the firm can use the following three period financing strategy: Initially, the firm issues safe short-term debt to repay its maturing debt at date t. As a result, the firm's entire debt matures at date $t + 1$, at which date the firm issues risky long-term debt that it can repay if it returns to the high state before it needs to refinance again at date $t + 3$. This financing strategy allows the firm to reduce the risk premia when it refinances its maturing and outstanding debt (as in the case of full prefinancing). However, to implement this financing strategy, which is never optimal absent a binding cash constraint, the firm must be able to refinance its debt at date $t + 1$ in the low state.

The following Proposition describes which of these financing strategies a firm must use at the default boundary, depending on its debt structure. Figure 3 depicts the default boundary of a firm that is subject to a binding cash holding constraint.

Proposition 2. *Consider the default boundary of a firm with a cash holding constraint $\bar{C} < \hat{\alpha}\hat{N}(\hat{\alpha})$ at date t after a liquidity shock. On the interval (11) there exist two thresholds $\hat{\alpha}_1(\bar{C})$ and $\hat{\alpha}_2(\bar{C})$ such that:*

1. *For $\alpha_t \in (\bar{C}/\hat{N}(\hat{\alpha}), \hat{\alpha}_1]$, the firm's default boundary is*

$$\hat{N} = \frac{(1 - p^2)\delta v^H + p\bar{C}}{1 + p\alpha_t}.$$

   To roll over its maturing debt at the default boundary, the firm must issue risky, zero coupon, long-term debt with market value $R_t + \bar{C}$ and stores the maximum possible amount of cash $\bar{C}$.

2. *For $\alpha_t \in (\hat{\alpha}_1, \hat{\alpha}_2)$, the firm's default boundary is*

$$\hat{N} = \frac{(1 - p^2)\delta^2 v^H}{1 - \alpha_t(1 - \delta)}.$$

   To roll over its maturing debt at the default boundary, the firm must issue safe short-term debt with market value $R_t = (1 - \alpha_t)\hat{N}$ and not store any cash. At date $t + 1$ it issues risky long-term debt with market value $(1 - p^2)\delta v^H$.

3. *For $\alpha_t \in [\hat{\alpha}_2, \hat{\alpha})$, the firm's default boundary and refinancing strategy are the same as in case 2 of Proposition 1.*

Case 2 above exists if and only if $\bar{C}$ is below some threshold $\bar{C}_1$. Otherwise $\hat{\alpha}_1 = \hat{\alpha}_2$.

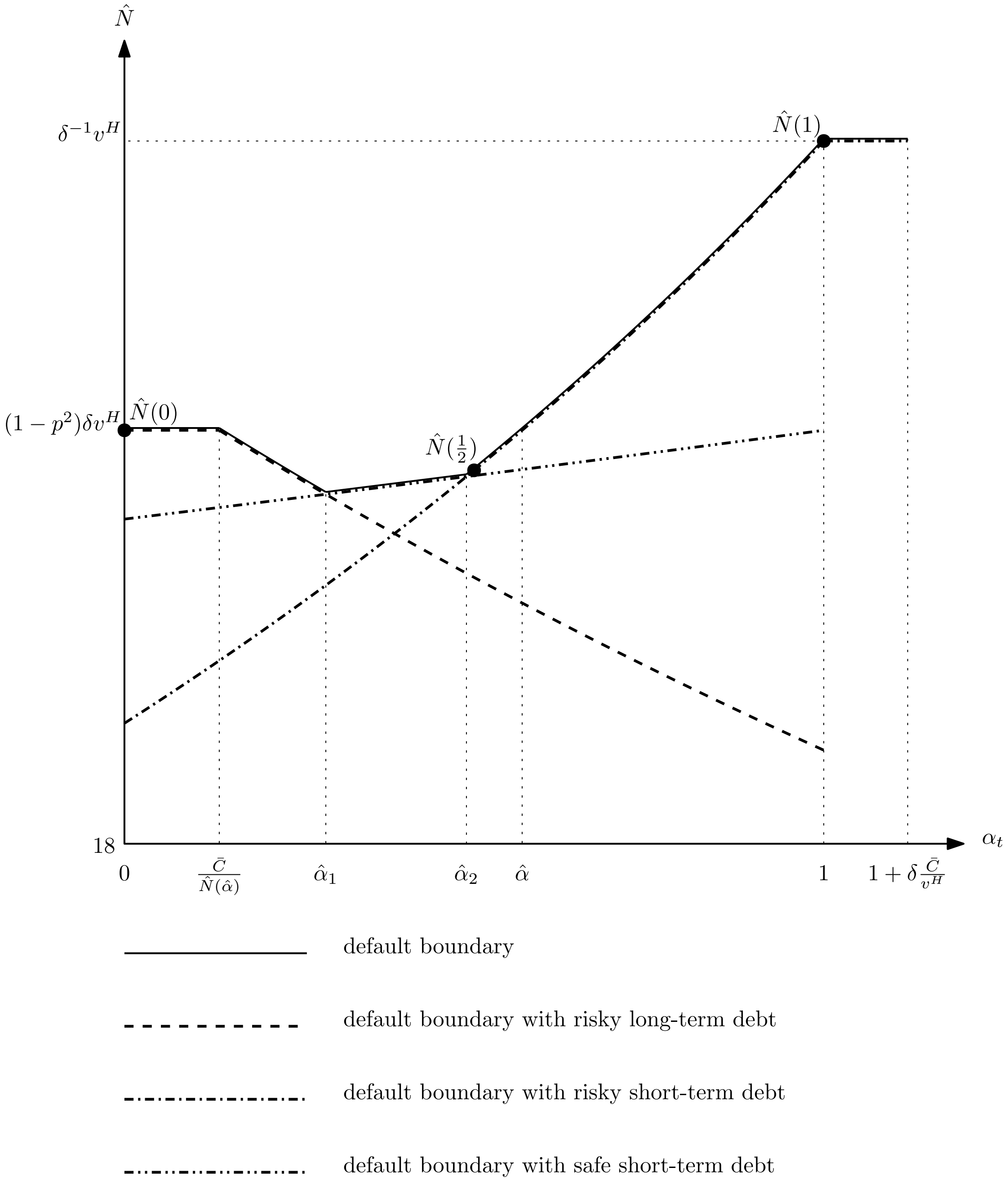

Figure 3: This figure plots the default boundary in the presence of a cash holding constraint as a function of $\alpha_t \leq 1 + \delta\frac{\bar{C}}{v^H}$. The solid line depicts the firm's overall default boundary as described by Propositions 1 and 2, the dashed line depicts the firm's default boundary if it issues risky long-term debt, the dashed-dotted line depicts the firm's default boundary if it issues risky short-term debt (for $\alpha \leq 1$), and the dash-dot-dotted line depicts the firm's default boundary if it issues safe short-term debt followed by risky long-term debt (for $\alpha \leq 1$). The figure is drawn for the parameter values $r^H = 1$, $\delta = 0.96$, $p = 0.2$, and $\bar{C} = 3$.

Proof. See Appendix C.2. □

With a binding cash constraint, the default boundary is no longer monotonic in α. Instead, the default boundary is U-shaped. Staggered debt can thus reduce the firm's ability to roll over its debt because the firm cannot use prefinancing to eliminate the rollover risk of outstanding debt. This contrasts the conventional arguments that staggered debt makes it easier for firms to repay their debt. Instead, rolling over staggered debt may require the firm to store large amounts of cash, which can cause agency problems that may ultimately prevent the firm from successful refinancing.

5. Different Financing Policies and Liquidity Shocks

5.1. Different financing policies and the default boundary

We further analyze how a firm's financing policy prior to a cash flow shock impacts its ability to roll over its debt following such a shock. We consider a firm with a constant level of net debt $N_t = N_{t-1}$ and a constant level of cash holdings $C_t = C_{t-1}$, which, in the high state, rolls over its maturing debt such that the face value and maturity of new debt equals the amount and maturity of expiring debt.

To describe a firm's financing policy, we decompose α_t into two components,

$$\alpha_t = \frac{D_{t-1}}{N_t} = \frac{D_{t-1}}{N_t + C_t} \times \left(1 + \frac{C_t}{N_t}\right) = \frac{D_{t-1}}{R_t + D_{t-1}} \times \left(1 + \frac{C_t}{N_t}\right).$$

The first component, $\gamma_t \equiv D_{t-1}/(R_t + D_{t-1})$, is the share of outstanding debt relative to total debt ($N_t + C_t = R_t + D_{t-1}$) and measures the staggering of a firm's debt structure independently of the firm's cash holdings. The second component, $\nu_t \equiv 1 + C_t/N_t$, captures the share of cash a firm holds relative to its net debt.

If the firm maintains a fully staggered debt structure, it rolls over half of its debt every period by issuing new long-term debt and hence, $\gamma_t = \frac{1}{2}$ for all t. If instead the firm maintains a concentrated debt structure using only short-term debt, then $\gamma_t = 0$ for all t. Among the debt structures we consider, these two cases are the only debt structures with constant γ_t. If the firm maintains a different debt structure, then γ_t will fluctuate between two values γ and $1 - \gamma$. For example, if the firm maintains a concentrated debt structure with long-term debt, then γ_t will fluctuate between 0 and 1, depending on whether the firm's debt matures or has been rolled over one period earlier.

Cash holdings provide liquidity whenever the firm experiences a negative cash flow shock. As a result, cash is not simply negative debt. It reduces the need for outside financing $R_{t-1} - C_t$ relative to the amount of outstanding debt D_{t-1}. Hence, when the firm holds more cash while keeping the amount of net debt constant, ν_t increases, which increases α_t. This increase in α_t holds across the firm's entire roll-over cycle unless the firm uses short-term debt only.

The following Proposition compares the impact of concentrated and fully staggered debt on a firm's ability to roll over its debt based on the results of Lemma 2 and Propositions 1 and 2.

Proposition 3. *Consider a firm with fully staggered debt* ($\alpha_t = \frac{1}{2}\nu_t$) *and compare it to a firm whose entire debt matures* ($\alpha_t = 0$)*. The amount of total net debt the firm can roll over satisfies*

$$\begin{cases} \hat{N}(\frac{1}{2}\nu_t) > \hat{N}(0) & \frac{1}{2}\nu_t > \hat{\alpha} \\ \hat{N}(\frac{1}{2}\nu_t) < \hat{N}(0) & \frac{1}{2}\nu_t \in (\bar{C}/\hat{N}(\hat{\alpha}), \hat{\alpha}) \\ \hat{N}(\frac{1}{2}\nu_t) = \hat{N}(0) & \textit{otherwise} \end{cases}$$

Comparing a firm with fully staggered debt with a firm whose entire debt is outstanding ($\alpha_t = \nu_t \geq 1$) *shows that* $\hat{N}(\frac{1}{2}\nu_t) \leq \hat{N}(\nu_t)$*.*

Proof. Follows from Propositions 1 and 2, Lemma 2, and the above discussion. □

To interpret this proposition, consider a firm that does not store any cash, which implies that $\nu_t = 1$. If the benefits of refinancing with long-term debt are sufficiently large, such that $\hat{\alpha} > \frac{1}{2}$, then staggered debt cannot increases a firm's default boundary. If the cash holding constraint is not binding, the amount of debt the firm can roll over with fully staggered deb is the same as if its entire debt were maturing. If instead the cash holding constraint is binding, then staggered debt decreases the amount of debt the firm can roll over.

In contrast, with sufficiently high cash holdings, such that $\hat{\alpha} < \frac{1}{2}$, the amount of net debt a firm can roll over following a negative cash flow shock is higher with fully staggered debt than when the firm's entire debt matures. The liquidity provided by the firm's cash holdings reduces the immediate funding need and the proportionally higher amount of outstanding debt can be diluted by issuing new risky debt.

It follows that in our model, staggered debt and cash, function as compliments in terms of firms' debt roll-over capabilities. If a firm does not store sufficient cash, then staggered debt decreases the firm's ability to withstand negative cash flow shocks. If instead the firm stores sufficient cash, then staggered debt increases its ability to withstand negative cash flow shocks.[5]

[5]Depending on the model parameters this can be true for prior cash holdings $C_t = (\gamma - 1)N_t$ that do not exceed the cash holding constraint

The last part of the Proposition concerns a firm whose entire debt is outstanding. Such a situation can occur if the firm maintains a concentrated debt structure with long-term debt and the cash flow shock occurs between its roll over dates. Since the firm does not need to refinance any maturing debt, the firm can simply wait for one period an hope that its cash flow recovers. Clearly, the amount of debt such a firm can carry through a liquidity shock without going bankrupt is maximal.

5.2. Different liquidity shocks and the default boundary

Firms may experience negative cash flow shocks of different severity. In our model, we call a cash flow shock more severe if the probability that the firm's cash flow recovers over one period $q \equiv 1 - p$ is lower.

Let $B(\alpha_t N_t, q, L)$ denote the debt capacity of a firm that experiences a negative cash flow shock with recovery probability q. The debt capacity is increasing in the recovery probability q, because a higher q decreases the risk premium for a firm's new debt. As before, a firm can roll over its net debt following a negative cash flow shock if

$$(1 - \alpha_t)N_t \leq B(\alpha_t N_t, q, L). \tag{13}$$

As in Section 3.4, the default boundary $\hat{N}(\alpha_t, q)$ is the level of net debt for which Condition 13 holds with equality. Since the debt capacity increases in q, the default boundary also increases in q. (This observation also follows directly form Propositions 1.)

Analogously, for a given level of net debt N_t and debt structure α_t, we can define a critical recovery probability $\hat{q}(N_t, \alpha_t)$ for which Condition 13 holds with equality. The critical recovery probability $\hat{q}$, characterizes a liquidity shock with the lowest recovery probability for which the firm can roll over its maturing debt. The firm can survive liquidity shocks for which the recovery probability exceeds $\hat{q}$. We call a liquidity shock with recovery probability $\hat{q}$ a critical liquidity shock.

The impact of a firm's debt structure on the critical recovery probability $\hat{q}$ corresponds to the impact of the debt structure on the firm's default boundary. Consider a debt structure α_t, a level of net debt N_t, and a recovery probability q, such that Condition (13) is binding. Suppose that the firm changes its debt structure such that Condition (13) becomes slack. In this case, the firm can either roll over a higher level of net debt for a constant recovery probability or it can roll over its existing net debt following a more severe cash flow shock with a lower recovery probability. The following Lemma formalizes this argument.

Lemma 3. *Consider two different debt structures α_t and α_t'. The default boundary $\hat{N}$ and the critical recovery probability $\hat{q}$ satisfy*

$$\hat{N}(\alpha_t', q) > \hat{N}(\alpha_t, q) \Leftrightarrow \hat{q}(\hat{N}\alpha_t, q), \alpha_t') < \hat{q}(\hat{N}(\alpha_t, q), \alpha_t).$$

Proof. See Appendix D.1. □

By construction, a firm that experiences a critical liquidity shock is at its default boundary. It follows that, for a given level of net debt N_t and debt structure α_t, we can determine a firm's refinancing strategy following a critical liquidity shock from $\hat{\alpha}(\hat{q}(N_t, \alpha_t))$, where $\hat{\alpha}(.)$ is given by Expression (10). The following Proposition is analogous to Proposition 1 and describes firms' critical liquidity shocks and refinancing strategies.

Proposition 4. *Consider a firm with a given level of net debt N_t that is* not *subject to a cash holding constraint. There exists a threshold $\tilde{\alpha}(N_t)$ such that:*

1. *For $\alpha_t \leq \tilde{\alpha}$, a critical liquidity shock has a recovery probability of*

$$\hat{q} = 1 - \sqrt{1 - \frac{N_t}{\delta v^H}}.$$

   To roll over its maturing debt following a critical liquidity shock, the firm must issue risky, zero coupon, long-term debt and fully prefinance its outstanding debt.

2. *For $\alpha_t \in (\tilde{\alpha}, 1)$, a critical liquidity shock has a recovery probability of*

$$\hat{q} = \frac{(1 - \alpha)N_t}{v^H - \alpha\delta N_t}.$$

   To roll over its maturing debt following a critical liquidity shock, the firm must issue risky debt that it can only repay if it returns to the high state over the next period.

3. *For $\alpha_t \geq 1$, the firm does not need to raise capital at date t. It does not go bankrupt at date t if it could repay its outstanding debt in the high state and there is a positive recovery probability.*

Proof. See Appendix D.2. □

For low levels of $\alpha_t < \tilde{\alpha}$, the critical recovery probability does not depend on the firm's debt structure. Hence, a higher share of outstanding debt does not allow the firm to survive more severe liquidity shocks. Conversely, for $\alpha_t \in (\tilde{\alpha}, 1)$ a higher share of outstanding debt allows the firm to survive more severe liquidity shocks. These results are driven by the same mechanisms that determine the firm's default boundary.

5.3. Different liquidity shocks and cash holding constraints

Binding cash holding constraints might also cause firms to fail following liquidity shocks that would allow unconstrained firms to roll over their debt. Consider a firm that is not subject to a binding cash holding constraint and is hit by a critical liquidity shock. From Proposition 4 it follows that this firm must prefinance its entire debt if $\alpha_t < \tilde{\alpha}(N_t)$. However, the firm cannot do so if it is subject to cash holding constraint such that

$$\alpha_t \in (\bar{C}/N_t, \tilde{\alpha}(N_t)). \tag{14}$$

As we know from Lemma 2 and Proposition 2, a cash holding constraint lowers the default boundary if it prevents the firm from optimally prefinancing its outstanding debt. Moreover the default boundary becomes non monotonous in α_t, it decreases in α_t for $\alpha_t < \hat{\alpha}_1$ and increases in α_t for $\alpha_t \geq \hat{\alpha}_1$. From Lemma 3 we know that a lower default boundary is reflected in a higher critical recovery probability $\hat{q}$. Thus, if the firm is subject to a binding cash holding constraint, it is more likely that the firm cannot roll over its debt following a liquidity shock because it is too severe.

Proposition 5. *Consider a firm with a given level of net debt that is subject to a cash holding constraint such that the interval (14) is non empty.*

1. *The critical recovery probability is non monotonous in α_t and there exists a critical threshold $\tilde{\alpha}_1(N_t)$ such that the critical recovery probability increases in α_t for $\alpha_t < \tilde{\alpha}_1$ and decreases in α_t for $\alpha_t \geq \tilde{\alpha}_1$.*[6]

2. *The critical recovery probability for $\alpha_t > 0$ is strictly higher (such that the firm can survive only less severe cash flow shocks) than if the firm has to refinance its total debt ($\alpha_t = 0$) if and only if α_t satisfies Condition (14).*

Proof. See Appendix D.3. □

[6] The threshold $\tilde{\alpha}(N_t)$ is non monotonous in N_t. The minimum recovery probability $\hat{q}$ hat allows firms to roll over their maturing debt increases in N_t. This has two effects on the threshold $\tilde{\alpha}$. First, A higher recovery probability $q \equiv 1 - p$ decreases the difference between the risk premia of short- and long-term debt, which deceases $\tilde{\alpha}$. Second, a higher recovery probability decreases the value of diluting the firm's outstanding debt, which increases $\tilde{\alpha}$.

We depict the constraints of Condition (14) in Figure 4. Area A, between the two lines, depicts the range of net debt levels N_t and debt structures α_t for which a cash holding constraint $\bar{C} = 3r^H$ increases the critical recovery probability of a firm.

Condition (14) will only be satisfied for firms with staggered debt because firms with concentrated debt do not need to prefinance their outstanding debt. Hence, if the agency costs of holding cash are sufficiently high, staggered debt can impair firms' ability to roll over their debt precisely when the firms face liquidity shocks that make rolling over their debt most difficult. Figure 4 shows that staggered debt can reduce firms' ability to withstand liquidity shocks for a considerable range of net debt levels.

6. Empirical Predictions

6.1. The Effects of Debt Structure and Corporate Governance in Financial Distress

The main empirical prediction of our model concerns firms' ability to withstand negative liquidity shocks. The probability that a firm cannot roll over its maturing debt following a liquidity shock increases in the critical recovery probability $\hat{q}$. The critical recovery probability depends on a firm's net debt N_t and its share of outstanding debt α.

The effect of a firm's share of maturing debt on its default probability depends on the agency costs of holding cash in or close to financial distress. Dittmar and Mahrt-Smith (2007) and Nikolov and Whited (2014) show empirically that the agency costs of holding cash are firm specific and depend on the quality of firms' corporate governance.

Proposition 4 describes the impact of firms' debt structures on their critical recovery probabilities for firms with strong corporate governance. These firms have low agency costs of holding cash and hence, can prefinance their debt without facing a binding cash constraint. As a result the critical recovery probability $\hat{q}$ (weakly) decreases in the amount of outstanding debt α, which reduces a firm's ability to withstand negative liquidity shocks.

Proposition 5 describes how the impact of a firm's debt structures differs for firms with weak corporate governance. These firms have high agency costs of holding cash and hence, face binding cash holding constraints that can prevent prefinancing. As a result of these constraints the critical recovery probability $\hat{q}$ is non monotonous in the share of outstanding debt α. Based on these two Propositions we state at the following empirical hypothesis.

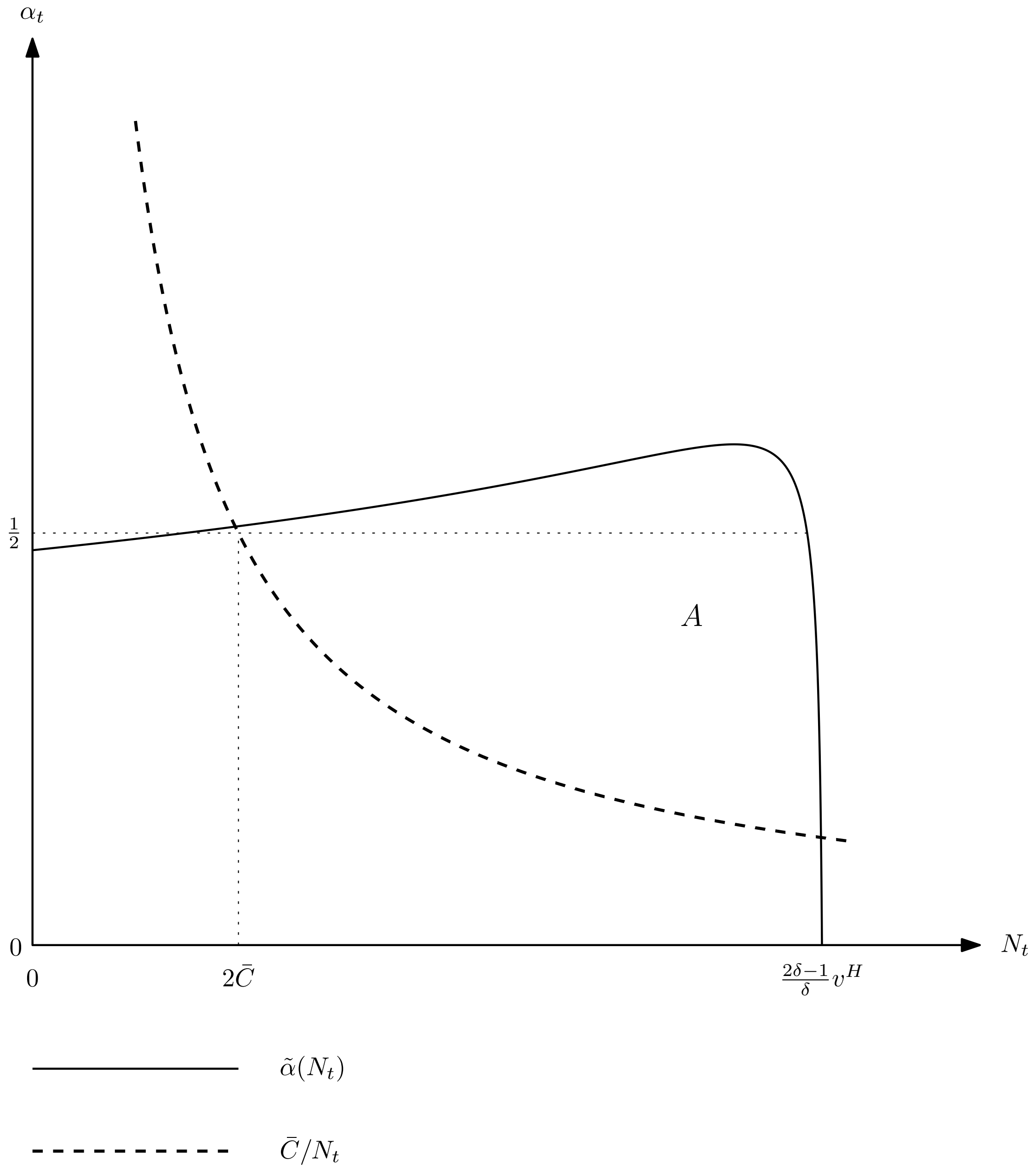

Figure 4: This figure depicts two thresholds that determine the firms refinancing strategy. The solid line plots the threshold $\tilde{\alpha}(N_t)$ below which an unconstrained firm that experiences a critical liquidity shock rolls over its maturing debt by issuing long-term debt and prefinances its outstanding debt (Expression (28) in Appendix D). The dashed line plots the threshold $\bar{C}/N_t$ above which a firm that experiences such a critical liquidity shock cannot fully prefinance its outstanding debt. Area A, between the two lines, depicts the range of net debt levels N_t and debt structures α_t for which a cash holding constraint $\bar{C}$ increases the critical recovery probability of a firm. The figure is drawn for the parameter values $r^H = 1$, $\delta = 0.96$, and $\bar{C} = 3$.

Hypothesis 1. *The effect of a firm's debt structure on its default probability following a liquidity shock depends on the strength of its corporate governance.*

1. *With strong corporate governance, the default probability decreases in the share of outstanding debt* α*.*
2. *With weak corporate governance, the default probability is non monotonous in* α*. It first increases and then decreases in the share of outstanding debt.*

A second set of predictions concerns the financing behavior of firms that become financially distressed following a liquidity shock. Propositions 1 and 2 predict that following a liquidity shock, a firm's cash holdings can increase, rather than decrease, when the firm engages in prefinancing. Whether a firms optimally engages in prefinancing depends on the amount of outstanding debt and the agency cost of holding cash, which depends on the strength of the firm's corporate governance. If the firm engages in prefinancing, it must store sufficient cash to repay its outstanding debt. Based on theses arguments we state the following hypothesis.

Hypothesis 2. *Following a liquidity shock, a firm changes its cash holdings depending on its debt structure and the strength of its corporate governance.*

1. *A firm increases its cash holdings in order to prefinance its outstanding debt if its share of outstanding debt is below some threshold* $\bar{\alpha}$*.*
2. *The prefinancing threshold* $\bar{\alpha}$ *is higher for firms with strong corporate governance than for firms with weak corporate governance.*
3. *If a firm's share of outstanding debt satisfies* $\alpha_t \leq \bar{\alpha}$*, such that the firm engages in prefinancing, then the increase in the firm's cash holdings is larger for larger amounts of outstanding debt.*

6.2. The Role of Bankruptcy Costs and Secured Debt

Prefinancing their outstanding debt allows firms to reduce roll-over risk and thereby reduce the bankruptcy risk for their newly issued debt. This reduction in bankruptcy risk can only increase a firm's debt capacity when the recovery value for new debt holders in case of bankruptcy is low enough. Thus, if there are no bankruptcy costs, then avoiding bankruptcy has no value and there is no role for prefinancing. In general, the question of staggered versus concentrated debt only matters if bankruptcy costs are sufficiently high.

If the recovery value of the firm's assets in bankruptcy is positive, the seniority structure of debt impacts a firm's ability to roll over its maturing debt. If a firm can issue new debt that is senior to its outstanding debt, the firm's ability to roll over its maturing debt using risky short-term debt increases, because its new debt is less affected by bankruptcy risk than its outstanding debt.[7] Conversely, if a firm's outstanding debt is senior to its new debt, the firm's ability to raise funds by issuing risky short-term debt decreases, because the value of outstanding debt cannot be diluted as much. Hence, if a firm's outstanding debt is senior, it becomes more likely that prefinancing maximizes a firm's ability to roll over its maturing debt.

Hypothesis 3. *If a firm's outstanding debt is more senior, then the threshold $\bar{\alpha}$ below which the firm will engage in prefinancing is higher.*

Because secured debt makes prefinancing more valuable, it also increases the effects of barriers to prefinancing. Hence, weak corporate governance that prevents firms from prefinancing their outstanding debt has a stronger on firms with large shares of secured debt.

Hypothesis 4. *A higher share of secured debt increases the probability that a firm defaults following a liquidity shock. The effect of secured debt is stronger for firms where weak corporate governance prevents prefinancing.*

6.3. Empirical Testing

It is possible to test our hypotheses by considering a sample of firms in financial distress as in Asquith et al. (1994) and splitting the sample according to the quality of firms' corporate governance. Following Dittmar and Mahrt-Smith (2007) and Nikolov and Whited (2014), possible proxies for firms' corporate governance quality are ownership concentration and indices of anti takeover defenses.

One can then test Hypothesis 1 by estimating the impact of α_t on firms' probability of default separately for both sub-samples. Similarly, one can test Hypothesis 2 by estimating the impact of α_t on changes of firms' cash holdings separately for both sub-samples. Note that the regression specifications need to account for the non-monotonous effects in Hypotheses 1 and 2.

It is possible to test Hypothesis 3, by splitting the sample into firms with high and low shares of secured debt. One can then separately estimate the effects of $\bar{\alpha}$ on changes of firms' cash holdings for firms with high and low shares of secured debt.

[7] Note that Chapter 11 of the US bankruptcy code allows firms to issue new debt that has super-priority over its existing debt. This allows a firm to cover operating expenses by diluting the claims of existing debt-holders resulting in a gamble for resurrection.

In order to test Hypothesis 4, one can first estimate the threshold below which firms with strong corporate governance start to prefinance their debt $\bar{\alpha}_s$. In a second step, one can then estimate the default probability of firms with weak corporate governance, including the share of secured debt, an indicator variable $\mathbb{I}(\alpha \leq \bar{\alpha}_s)$, and an interaction term of $\mathbb{I}(\alpha \leq \bar{\alpha}_s)$ with the share of secured debt as explanatory variables. One can then test Hypothesis 4 by verifying whether the interaction term has a significantly positive effect on the default probability of firms with weak corporate governance.

7. Conclusion

In this paper we challenge the notion that dispersed debt maturities increases firms' financial resilience against negative liquidity shocks. When firms roll over maturing debt, the price of their new debt internalizes the roll over risk associated with the firms existing outstanding debt. These roll over risk are large following a negative liquidity shock. Thus, when firms need to roll over substantial amounts of maturing debt, they need to prefinance their outstanding debt in order to eliminate the associated future roll over risks.

In order to prefinance their outstanding debt firms must raise and hold large amounts of cash. Such large cash holdings are prone to create agency costs, particularly in or close to financial distress. If these agency costs are sufficiently high then prefinancing is no longer viable, which reduces a firm's ability to roll over its maturing debt. Thus, if holding cash results in high agency costs, a dispersed maturity structure reduces firms' ability to withstand negative liquidity shocks.

A. Debt Capacity in the High State

Proof of Lemma 1. Let the firm issue two period debt with face value f and the coupon payment c at par. Issuing such debt at date t results in payment obligations $d_t(f) = c$ and $D_t(f) = c + f$. Since there is no uncertainty, the firm can issue such debt at par if

$$f = \delta c + \delta^2(c + f) \Rightarrow c(f) = \frac{1-\delta}{\delta} f.$$

For $v^H \geq \delta D_{t-1}$ consider the following financing strategy. At date t the firm issues debt with face value $f_t = v^H - \delta D_t$, which yields the first case of (3). At any date $\tau > t$ is issues debt with face value $f_\tau = \delta D_{\tau-2}$. It follows that at any date $\tau > t$ the firm needs internal funds

$$r^H = D_{\tau-2} + c(f_{\tau-1}) - f_\tau.$$

to repay its debt and thus pays out its cash flows at the earliest possible dates.

If $\delta D_{t-1} > v^H$, the firm stores cash $C_t = D_{t-1} - \delta^{-1} v^H$, which must stem from past periods, and sets $f_t = 0$, which yields the second case of (3). It then continues with the above strategy starting at date $t + 1$. □

B. Debt Capacity in the Low State

In this section we derive a firm's debt capacity in the low state. We do so for an arbitrary cash holding constraint $\bar{C}$. The unconstrained case corresponds to $\bar{C} \to \infty$. We assume that Condition (6) holds.

Consider a firm that issues debt in the low state at date t. The state of the firm's project can take the following paths over the next 2 periods:[8]

$t+1$	$t+2$
L	L
L	H
H	H

These three possible paths are clearly ranked in the cash flows they generate. If the firm survives only on the future path $\{H, H\}$, both d_t and D_t will only be paid back with probability $(1 - p)$. Because the recovery rate in bankruptcy is zero, the market value of the firm's new debt claims is

$$(1 - p)(\delta d_t + \delta^2 D_t) \tag{15}$$

If the firm survives on the future paths $\{L, H\}$ and $\{H, H\}$, d_t is safe while D_t gets paid with probability $(1 - p^2)$. Hence, the market value of the firm's new debt claims is

$$\delta d_t + (1 - p^2)\delta^2 D_t. \tag{16}$$

If the the firms survives on all three paths, the new debt is safe and has market value

$$\delta d_t + \delta^2 D_t.$$

We denote the firm's debt capacity if it chooses a financing strategy with which it

[8] The path $\{H, L\}$ is not possible because H is an absorbing state.

survives if and only if it reaches the high state over the next n periods by $B_n(D_{t-1}, L)$. If the firm is constraint in the amount of cash it can hold, a debt capacity $B_n(D_{t-1}, L)$ is well defined if and only if there exists initial values $D_{t-2}, d_{t-1}, D_{t-1}, C_{t-1}$ such that the firm can survive for n periods in the low state without storing $C > \bar{C}$. It follows that for every level of outstanding debt, the overall debt capacity

$$B(D_{t-1}, L) = \max_{n \in \mathcal{A}(D_{t-1})} B_n(D_{t-1}, L),$$

where $\mathcal{A}(D_{t-1}) = \{n \mid B_n(D_{t-1}, L) \text{ is well defined}\}$.

We now establish several Lemmata that describe the debt capacities $B_n(D_{t-1}, L)$. Together these expressions then determine the debt capacity $B(D_{t-1}, L)$

B.1. Debt capacity if the firm can survive in the low state for one period

Lemma 4. *Consider a firm's ability to repay maturing debt obligations at date t after a liquidity shock. Assume that the firm issues debt such that it only survives on the future path $\{H, H\}$.*

The firm's debt capacity is given by

$$B_1(D_{t-1}, L) = \min\left\{(1-p)(v^H - \delta D_{t-1}),\ \delta^{-1}(v^H - \delta D_{t-1})\right\}. \tag{17}$$

if $\bar{C} \geq -B_1(D_{t-1}, H)$. If $v - \delta D_{t-1} < 0$, the firm cannot issue debt at date t; it needs to store cash $D_{t-1} - \delta^{-1} v^H$, which must stem from past periods. If $v - \delta D_{t-1} > 0$, the firm can issue debt at date t. To utilize its debt capacity the firm must issue risky debt with market value $\max\{(1-p)(v^H - \delta D_{t-1}), 0\}$ *that it repays with probability* $(1-p)$.

Proof. If the firm only survives on the future path $\{H, H\}$, it can roll over its debt at date $t+1$ in the high state if

$$d_t + D_{t-1} \leq r^H \tau + C_t + B(D_t, H). \tag{18}$$

The market value of the firm's new debt is then given by (15). Hence, the firms debt capacity $B_1(D_{t-1}, L)$ at date t is determined by the following optimization problem

$$\max_{d_t, D_t, C_t} (1-p)(\delta d_t + \delta^2 D_t) - C_t \tag{19}$$

subject to 18, $C_t, d_t, D_t \geq 0$, and $C_t \leq \bar{C}$.

The objective (19) is increasing in d_t and D_t and decreasing in C_t. Since $B(D_t, H)$ is decreasing in D_t the constraint (18) must be binding. Substituting for $B(D_t, H)$ in (18) and solving for D_t yields

$$\begin{aligned} D_t &= B^{-1}(d_t + D_{t-1} - r^H - C_t, H) \\ &= \min\{\delta^{-1}(v^H - d_t - D_{t-1} + r^H + C_t), \delta^{-1}v^H - d_t - D_{t-1} + r^H\tau + C_t\} \\ &= \delta^{-1}\min\{v^H - (d_t + D_{t-1} - r^H - C_t), v^H - \delta(d_t + D_{t-1} - r^H - C_t)\}. \end{aligned}$$

Substituting into the objective function yields

$$(1-p)\delta(d_t + \min\{v^H - (d_t + D_{t-1} - r^H - C_t), v^H - \delta(d_t + D_{t-1} - r^H - C_t)\}) - C_t.$$

Taking the derivative with respect to C_t yields

$$\begin{cases} (1-p)\delta^2 - 1 & d_t + D_{t-1} - r^H - C_t < 0 \\ (1-p)\delta - 1 & d_t + D_{t-1} - r^H - C_t > 0 \end{cases}$$

Since both expressions are negative it follows that either $C_t \geq 0$ or $D_t \geq 0$ must be binding. Hence, the firms stores cash

$$C_t = \max\{0, -v^H + (d_t + D_{t-1} - r^H)\}.$$

Substituting into the objective function yields

$$\begin{aligned} (1-p)\delta(d_t + \max\{\min\{v^H - (d_t + D_{t-1} - r^H), v^H - \delta(d_t + D_{t-1} - r^H)\}, 0\}) \\ - \max\{0, -v^H + (d_t + D_{t-1} - r^H)\}. \end{aligned}$$

Taking the derivative with respect to d_t yields

$$\begin{cases} (1-p)\delta(1-\delta) & d_t + D_{t-1} - r^H < 0 \\ 0 & d_t + D_{t-1} - r^H \in (0, v^H) \\ (1-p)\delta - 1 & d_t + D_{t-1} - r^H > v^H \end{cases}$$

Because $(1-p)\delta(1-\delta) > 0$, $(1-p)\delta - 1 < 0$, and $d_t \geq 0$ it follows that

$$d_t \in \begin{cases} [\max\{r^H - D_{t-1}, 0\}, v^H + r^H - D_{t-1}] & v^H + r^H - D_{t-1} > 0 \\ \{0\} & v^H + r^H - D_{t-1} \leq 0 \end{cases}$$

Substituting for d_t and C_t in $B^{-1}(d_t + D_{t-1} - r^H - C_t)$ yields

$$D_t = \begin{cases} \delta^{-1}(v^H - (d_t + D_{t-1} - r^H)) & v^H + r^H - D_{t-1} > 0 \\ 0 & v^H + r^H - D_{t-1} \leq 0 \end{cases}$$

Substituting into the objective yields

$$B_1(D_{t-1}, L) = \begin{cases} (1-p)\delta(v^H + r^H - D_{t-1}) \\ v^H + r^H - D_{t-1} \end{cases}$$

Observing that $v^H + r^H = \delta^{-1} v^H$ yields (17).

The Condition $\bar{C} \geq -B_1(D_{t-1}, H)$ ensures that $C_t = \max\{0, -v^H + (D_{t-1} - r^H)\} \leq \bar{C}$. □

B.2. Debt capacity if the firm can survive in the low state for two periods

Lemma 5. *Consider a firm's ability to repay maturing debt obligations at date t after a liquidity shock when it cannot store more cash than $\bar{C}$. Assume that the firm issues debt such that it only survives on the future paths $\{L, H\}$ and $\{H, H\}$ and that (6) holds.*

The firm's debt capacity is given by

$$B_2(D_{t-1}, L) = \min\{(1-p^2)\delta v^H - D_{t-1}, (1-p^2)\delta(v^H - (1-p)^{-1}(D_{t-1} - \bar{C})) - \bar{C}\}. \quad (20)$$

if $\bar{C} > -(1-p)v^H + D_{t-1}$. To utilize its debt capacity the firm must issue risky, zero coupon, long-term debt that it repays with probability $(1-p^2)$. If $D_{t-1} \leq \bar{C}$ the firm must issue debt with market value $(1-p^2)\delta v^H$ and store cash D_{t-1}. If $D_{t-1} > \bar{C}$ the firm must issue debt with market value $(1-p^2)\delta(v^H - (1-p)^{-1}(D_{t-1} - \bar{C}))$ and store cash $\bar{C}$.

Proof. To survive on the future path $\{L, H\}$, the firms must be able to repay any debt that matures at date $t+1$ in the low state. The firm only survive if it reaches the high state date $t+2$. Hence, its debt capacity in the low state at date $t+1$ is $B_1(D_t, L)$ and the relevant roll over constraint is given by

$$d_t + D_{t-1} - C_t \leq B_1(D_t, L). \quad (21)$$

At date t, the market value of debt is then given by (16). Hence the firm's debt capacity

$B_2(D_{t-1}, L)$ at date t is determined by the following optimization problem

$$\max_{d_t, D_t, C_t} \delta d_t + (1-p^2)\delta^2 D_t - C_t \tag{22}$$

subject to (21), $C_t, d_t, D_t \geq 0$, and .

The objective (22) is increasing in d_t and D_t and decreasing in C_t. Since $B_1(D_t, L)$ is decreasing in D_t the constraint (21) must be binding. Substituting for $B_1(D_t, L)$ in (21) and solving for D_t yields

$$\begin{aligned} D_t &= B_1^{-1}(d_t + D_{t-1} - C_t, L) \\ &= \min\{\delta^{-1} v^H - (1-p)^{-1}\delta^{-1}(d_t + D_{t-1} - C_t), \delta^{-1} v^H - (d_t + D_{t-1} - C_t)\} \\ &= \delta^{-1} \min\{v^H - (1-p)^{-1}(d_t + D_{t-1} - C_t), v^H - \delta(d_t + D_{t-1} - C_t)\}. \end{aligned}$$

Substituting into the objective function yields

$$\delta d_t + (1-p^2)\delta \min\{v^H - (1-p)^{-1}(d_t + D_{t-1} - C_t), v^H - \delta(d_t + D_{t-1} - C_t)\} - C_t.$$

Taking the derivative with respect to C_t yields

$$\begin{cases} (1+p)\delta - 1 & d_t + D_{t-1} - C_t < 0 \\ (1-p^2)\delta^2 - 1 & d_t + D_{t-1} - C_t > 0 \end{cases}$$

Since $(1+p)\delta - 1 > 0$ and $(1-p^2)\delta^2 - 1 < 0$ it follows that

$$C_t = \min\{d_t + D_{t-1}, \bar{C}\}.$$

Substituting into the objective function yields

$$\delta d_t + (1-p^2)\delta(v^H - (1-p)^{-1}\max\{0, d_t + D_{t-1} - \bar{C}\}) - \min\{d_t + D_{t-1}, \bar{C}\}.$$

Taking the derivative with respect to d_t yields

$$\begin{cases} \delta - 1 & d_t + D_{t-1} < \bar{C} \\ -\delta p & d_t + D_{t-1} > \bar{C} \end{cases}$$

Since both expression are negative it follows that $d_t = 0$.

Substituting for d_t and C_t into $B_1^{-1}(d_t + D_{t-1} - r^H - C_t, L)$ yields

$$D_t = \begin{cases} \delta^{-1} v^H & D_{t-1} \leq \bar{C} \\ \delta^{-1}(v^H - (1-p)^{-1}(D_{t-1} - \bar{C})) & D_{t-1} > \bar{C} \end{cases}$$

Substituting into the objective function yields

$$B_2(D_{t-1}, L) = \begin{cases} (1-p^2)\delta v^H - D_{t-1} & D_{t-1} \leq \bar{C} \\ (1-p^2)\delta(v^H - (1-p)^{-1}(D_{t-1} - \bar{C})) - \bar{C} & D_{t-1} > \bar{C} \end{cases}$$

Because $(1+p)\delta > 1$ the debt capacity can be written as

$$B_2(D_{t-1}, L) = \min\{(1-p^2)\delta v^H - D_{t-1}, (1-p^2)\delta(v^H - (1-p)^{-1}(D_{t-1} - \bar{C})) - \bar{C}\}.$$

The Condition $v^H - (1-p)^{-1}(D_{t-1} - \bar{C}) > 0$ ensures that $D_t \geq 0$. □

B.3. Debt capacity if the firm can survive in the low state for three periods

Lemma 6. *Consider a firm's ability to repay maturing debt obligations at date t after a liquidity shock when it cannot store more cash than $\bar{C}$. Assume that the firm issues debt such that it survives if and only if it reaches the high state over the next three periods and that (6) holds.*

The firm's debt capacity is given by

$$B_3(D_{t-1}, L) = \min\{\delta(B_2(0, L) - D_{t-1}), B_2(0, L) - D_{t-1}\} \tag{23}$$

if $\bar{C} \geq -B_2(D_{t-1}, L)$. If $B_2(0, L) - D_{t-1} < 0$, the firm cannot issue debt at date t; it needs to store cash $D_{t-1} - B_2(0, L)$, which must stem from past periods. If $B_2(0, L) - D_{t-1} > 0$, the firm can issue debt at date t. To utilize its debt capacity the firm must issue safe short-term debt with market value $\max\{\delta(B_2(0, L) - D_{t-1}), 0\}$.

Proof. A firm can survive for at three periods if at date $t+1$ in the low state it can survive for another two periods:

$$d_t + D_{t-1} - C_t \leq B_2(D_t, L). \tag{24}$$

The new debt such a firm issues is safe. Hence the firm's debt capacity $B_3(D_{t-1}, L)$ at date t is determined by the following optimization problem

$$B_3(D_{t-1}, L) = \max_{d_t, D_t, C_{\hat{t}}} \delta d_t + \delta^2 D_t - C_{\hat{t}}$$

subject to 24, $C_t, d_t, D_{\hat{t}} \geq 0$, and $C_t \leq \bar{C}$.

The objective is increasing in d_t and D_t and decreasing in C_t. Since $B_2(D_t, L)$ is decreasing in D_t the constraint (21) must be binding. Substituting for $B_2(D_t, L)$ in (21) and solving for D_t yields

$$\begin{aligned} D_t =& B_2^{-1}(d_t + D_{t-1} - C_t, L) \\ =& \min\{(1-p^2)\delta v^H - (d_t + D_{t-1} - C_t), \\ & (1-p)v^H + (1-(1+p)^{-1}\delta^{-1})\bar{C} - (1+p)^{-1}\delta^{-1}(d_t + D_{t-1} - C_t)\} \end{aligned}$$

Substituting into the objective yields

$$\begin{aligned} &\delta d_t + \delta^2 \min\{(1-p^2)\delta v^H - (d_t + D_{t-1} - C_t), (1-p)v^H \\ &\qquad + (1-(1+p)^{-1}\delta^{-1})\bar{C} - (1+p)^{-1}\delta^{-1}(d_t + D_{t-1} - C_t)\} - C_t \end{aligned}$$

Taking the derivative with respect to C_t yields

$$\begin{cases} \delta^2 - 1 & d_t + D_{t-1} - C_t < \dfrac{p(1-p^2)\delta}{(1+p)\delta - 1} v^H - \bar{C} \\ \delta(1+p^{\cdot})^{-1} - 1 & d_t + D_{t-1} - C_t > \dfrac{p(1-p^2)\delta}{(1+p)\delta - 1} v^H - \bar{C} \end{cases}$$

Since both expressions are negative it follows that either $C_t \geq 0$ or $D_t \geq 0$ must be binding. From $(1+p)\delta > 1$ it follows that

$$B_2^{-1}(d_t + D_{t-1} - C_t, L) = 0 \Leftrightarrow (1-p^2)\delta v^H - (d_t + D_{t-1} - C_t) = 0$$

which implies that

$$C_t = \max\{0, -(1-p^2)\delta v^H + d_t + D_{t-1}\}.$$

Substituting into the objective yields

$$\delta d_t + \delta^2 \max\{(1-p^2)\delta v^H - (d_t + D_{t-1}), 0\} - \max\{0, -(1-p^2)\delta v^H + d_t + D_{t-1}\}.$$

Taking the derivative with respect to d_t yields

$$\begin{cases} \delta(1-\delta) & d_t + D_{t-1} < (1-p^2)\delta v^H \\ \delta - 1 & d_t + D_{t-1} > (1-p^2)\delta v^H \end{cases}$$

Since $\delta(1-\delta) > 0$ and $\delta - 1 < 0$ it follows that

$$d_t = \max\{(1-p^2)\delta v^H - D_{t-1}, 0\}$$

which implies that $D_t = 0$. Substituting into the objective then yields

$$\begin{aligned} B_3(D_{t-1}, L) &= \min\{\delta((1-p^2)\delta v^H - D_{t-1}), (1-p^2)\delta v^H - D_{t-1}\} \\ &= \min\{\delta B_2(0, L) - D_{t-1}, B_2(0, L) - D_{t-1}\} \end{aligned}$$

□

B.4. Debt capacity if the firm can survive in the low state for $n > 3$ periods

Lemma 7. *Consider a firm's ability to repay maturing debt obligations at date t after a liquidity shock when it cannot store more cash than $\bar{C}$. Assume that the firm issues debt such that it survives if and only if it reaches the high state over the next $n > 3$* periods and that *(6) holds.*

The firm's debt capacity is given by

$$B_n(D_{t-1}, L) = \min\{\delta^{n-2} B_2(0, L) - \delta D_{t-1}, \delta^{n-3} B_2(0, L) - D_{t-1}\} \tag{25}$$

if $\bar{C} \geq -B_{n-1}(D_{t-1}, L)$. If $\delta^{n-2} B_2(0, L) - \delta D_{t-1} < 0$, the firm cannot issue debt at date t; it needs to store cash $D_{t-1} - \delta^{n-3} B_2(0, L)$, which must stem from past periods. If $\delta^{n-2} B_2(0, L) - \delta D_{t-1} > 0$, the firm can issue debt at date t. To utilize its debt capacity the firm must issue safe debt with market value $\max\{\delta^{n-2} B_2(0, L) - \delta D_{t-1}, 0\}$.

Proof. A firm can survive for at least $n > 0$ periods if at date $t + 1$ it can survive for at least $n - 1$ periods:

$$d_t + D_{t-1} - C_t \leq B_{n-1}(D_t, L) \tag{26}$$

The new debt such a firm issues is safe. Hence the debt capacity of a firm that issues debt at date t and survives for $n > 3$ periods in the low state is given by

$$B_n(D_{t-1}, L) = \max_{d_t, D_t, C_t} \delta d_t + \delta^2 D_t - C_t \tag{27}$$

subject to (26), $C_t, d_t, D_t \geq 0$, and $C_t \leq \bar{C}$.

Suppose that for $n \geq 3$ the debt capacity is given by (25) which is consistent with Lemma 6.

For $n > 3$, the objective (27) is increasing in d_t and D_t and decreasing in C_t. Since $B_{n-1}(D_t, L)$ is decreasing in D_t the constraint (26) must be binding. Substituting for $B_{n-1}(D_t, L)$ in (26) and solving for D_t yields

$$\begin{aligned} D_t &= B_{n-1}^{-1}(d_t + D_{t-1} - C_t, L) \\ &= \min\{\delta^{n-4} B_2(0, L) - \delta^{-1}(d_t + D_{t-1} - C_t), \delta^{n-4} B_2(0, L) - (d_t + D_{t-1} - C_t)\} \end{aligned}$$

Substituting into the objective function yields

$$\delta d_t + \delta^2 \min\{\delta^{n-4} B_2(0, L) - \delta^{-1}(d_t + D_{t-1} - C_t), \delta^{n-4} B_2(0, L) - (d_t + D_{t-1} - C_t)\} - C_t$$

Taking the derivative with respect to C_t yields

$$\begin{cases} \delta - 1 & d_t + D_{t-1} - C_t < 0 \\ \delta^2 - 1 & d_t + D_{t-1} - C_t > 0 \end{cases}$$

Since both expressions are negative it follows that either $C_t \geq 0$ or $D_t \geq 0$ must be binding. Inspection of $B_{n-1}^{-1}(., L)$ shows that

$$B_{n-1}^{-1}(d_t + D_{t-1} - C_t, L) = 0 \Leftrightarrow \delta^{n-4} B_2(0, L) - \delta^{-1}(d_t + D_{t-1} - C_t) = 0$$

which implies that

$$C_t = \max\{0, -\delta^{n-3} B_2(0, L) + d_t + D_{t-1}\}.$$

Substituting into the objective yields

$$\delta d_t + \delta^2 \max\{\delta^{n-4} B_2(0, L) - \delta^{-1}(d_t + D_{t-1}), 0\} - \max\{0, -\delta^{n-3} B_2(0, L) + d_t + D_{t-1}\}.$$

Taking the derivative with respect to d_t yields

$$\begin{cases} 0 & d_t < \delta^{n-3} B_2(0, L) - D_{t-1} \\ \delta - 1 & d_t > \delta^{n-3} B_2(0, L) - D_{t-1} \end{cases}$$

It follows that

$$d_t \in \begin{cases} (0, \delta^{n-3} B_2(0,L) - D_{t-1}) & \delta^{n-3} B_2(0,L) - D_{t-1} > 0 \\ \{0\} & \text{otherwise} \end{cases}$$

Substituting for d_t and C_t in $B^{-1}(d_t + D_{t-1} - r^H - C_t)$ yields

$$D_t = \begin{cases} \delta^{n-4} B_2(0,L) - \delta^{-1}(d_t + D_{t-1}) & \delta^{n-3} B_2(0,L) - D_{t-1} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Substituting into the objective yields

$$\delta^2 \max\{\delta^{n-4} B_2(0,L) - \delta^{-1} D_{t-1}, 0\} - \max\{0, -\delta^{n-3} B_2(0,L) + d_t + D_{t-1}\}.$$

which can be rewritten as Expression (25). The Condition $\bar{C} \geq -B_{n-1}(D_{t-1}, L)$ ensures that $C_t < \bar{C}$. Because for $n = 3$ Expressions (25) and (23) coincide the Lemma follows by perfect induction. □

C. Default Boundaries

C.1. The firm's default boundary as a function of its outstanding debt D_{t-1}

From Condition (1) it follows that a firms default boundary in the low state is given by

$$B(D_{t-1}, L) + D_{t-1}.$$

Hence, base on the results in Appendix B we can describe a firm's default boundary as function of the firms outstanding debt D_{t-1}.

Proposition 6. *Consider a firm's ability to repay its maturing and outstanding debt at date t after a liquidity shock. There exist several thresholds $\hat{D}_i(\bar{C})$ such that*

1. *For $D_{t-1} \leq \hat{D}_1(\bar{C})$, the firm maximizes its ability to roll over debt by issuing risky, zero coupon, long-term debt with market value $(1-p^2)\delta v^H$ and fully prefinancing its outstanding debt by storing cash D_{t-1}. The default boundary is*

$$(1-p^2)\delta v^H.$$

2. *For $D_{t-1} \in (\hat{D}_1(\bar{C}), \hat{D}_2(\bar{C})]$, the firm maximizes its ability to roll over debt by prefinancing its outstanding debt up to $\bar{C}$ and by issuing risky, zero coupon, long-*

term debt with market value $(1-p^2)\delta v^H - (1-p)D_{t-1} + p\bar{C}$*. The default boundary is*

$$(1-p^2)\delta v^H - p(D_{t-1} - \bar{C}).$$

3. *For* $D_{t-1} \in (\hat{D}_2(\bar{C}), \hat{D}_3(\bar{C}))$*, the firm maximizes its ability to roll over debt by issuing safe, short-term debt with market value* $(1-p^2)\delta^2 v^H - \delta D_{t-1}$ *and not storing any cash. A date* $t+1$ *it issues risk long-term debt with market value* $(1-p^2)\delta v^H$*. The default boundary is*

$$(1-p^2)\delta^2 v^H + (1-\delta)D_{t-1}.$$

4. *For* $D_{t-1} \in [\hat{D}_3(\bar{C}), \delta^{-1}v^H)$*, the firm maximizes its ability to roll over debt if it issues risky debt with market value* $(1-p)(v^H - \delta D_{t-1})$ *and does not store any cash. The firm can issue long-term or short-term debt. The default boundary is*

$$(1-p)v^H + (1-(1-p)\delta)D_{t-1}.$$

5. *For* $D_{t-1} \in [\delta^{-1}v^H, \delta^{-1}v^H + \bar{C}]$*, the firm's default boundary is*

$$\delta^{-1}v^H.$$

   If the firm's default boundary is binding, it can survive the liquidity shock with positive probability only if it has sufficient cash from past periods to store $D_{t-1} - \delta^{-1}v^H$ *and repay its maturing debt.*

6. *For* $D_{t-1} > \delta^{-1}v^H + \bar{C}$*, the firm can never repay its outstanding debt.*

There exist two thresholds $\bar{C}_1$ *and* $\hat{D}$ *and such that*

1. *For* $\bar{C} < \bar{C}_1$*,* $\hat{D}_1 = \bar{C} < \hat{D}_2 < \hat{D}_3 < \delta^{-1}v^H$ *and hence, all cases above exist.*
2. *For* $\bar{C} \in [\bar{C}_1, \hat{D})$*,* $\bar{C} < \hat{D}_2 = \hat{D}_3 < \delta^{-1}v^H$ *and hence, Case 3 above does not exist.*
3. *For* $\bar{C} > \hat{D}$*,* $\hat{D}_1 = \hat{D}_2 = \hat{D}_3 = \hat{D}$ *and hence, Cases 2 and 3 above do not exist.*
4. *For* $\bar{C} \to \infty$*, Case 6 above does not exist.*

Proof. Consider the debt capacities $B_n(D_{t-1}, L)$ derived in Lemmata 4-7. Inspection of $B_n(D_{t-1}, L)$ in Appendix B yields the following observations.

1. $B_1(D_{t-1}, L) > B_n(D_{t-1}, L)$ for $D_{t-1} \geq \delta^{-1}v^H$ and all $n \geq 2$.

2. $B_2(0, L) > B_n(0, L)$ for all $n \neq 2$.

3. $B_3(D_{t-1}, L) > B_n(D_{t-1}, L)$ for all $n \geq 4$ and D_{t-1}.

4. $B_1(D_{t-1}, L) + D_{t-1}$ is increasing in D_{t-1}.

5. $B_2(D_{t-1}, L) + D_{t-1}$ is decreasing in D_{t-1}.

6. $B_3(D_{t-1}, L) + D_{t-1}$ is increasing in D_{t-1}.

7. $\frac{\partial}{\partial D_{t-1}} B_3(D_{t-1}, L) \leq \frac{\partial}{\partial D_{t-1}} B_1(D_{t-1}, L)$ for all D_{t-1}.

Let $\check{D}_{ij}$ be defined by $B_i(\check{D}_{ij}, L) = B_j(\check{D}_{ij}, L)$ and let $\hat{D}_1 = \min\{\bar{C}, \check{D}_{12}\}$, $\hat{D}_2 = \min\{\check{D}_{12}, \check{D}_{13}\}$, and $\hat{D}_3 = \max\{\check{D}_{23}, \check{D}_{12}\}$. The above observations then yield the following results:

1. For $D_{t-1} \leq \hat{D}_2$, it follows from Observations (2, 4-6) that the firm's debt capacity and behavior at the default boundary are described by Lemma 5, which yields cases 1 and 2 of Proposition 6.

2. For $D_{t-1} \in (\hat{D}_2, \hat{D}_3)$ it follows from Observations (3,5-7) that the firm's debt capacity and behavior at the default boundary are described by Lemma 7 for $n = 3$, which yields cases 3 of Proposition 6.

3. For $D_{t-1} \geq D_{23}$ it follows from Observations (1,4,5,7) that the firm's debt capacity and behavior at the default boundary are described by Lemma 4, which yields cases 4 and 5 of Proposition 6.

4. For $D_{t-1} > \delta^{-1} v^H + \bar{C}$ the firms needs cash $D_{t-1} - \delta^{-1} v^H$ in the good state. Case 6 of Proposition 6 follows because the firms cannot store so much cash.

The thresholds $\bar{C}_1$ and $\hat{D}$ are given by

$$\bar{C}_1 \equiv (1-p)\frac{(1+p)(2\delta^2 - \delta) - 1}{(1+p)\delta - 1} v^H$$
$$\hat{D} \equiv \frac{(1+p)\delta - 1}{(1-p)^{-1} - \delta} v^H$$

One can show that $\bar{C}_1 > 0$ if and only if $(1+p)\delta(2\delta - 1) > 1$ and $\hat{D} > 0$ if and only if (6) holds. Using the expressions from Lemmata 4-7 one can also show the following:

1. For $\bar{C} < \bar{C}_1$, $\bar{C} < \check{D}_{12} < \check{D}_{13} < \check{D}_{23}$. Hence, all parts of Proposition 6 exist.

2. For $\bar{C} \in [\bar{C}_1, \hat{D})$, $\bar{C} < \check{D}_{13} < \check{D}_{12}$ and $\check{D}_{23} < \check{D}_{13}$. Hence, Case 3 of Proposition 6 does not exist.

3. For $\bar{C} \geq \hat{D}$, $\check{D}_{23} < \check{D}_{12} < \bar{C}, \check{D}_{13}$. and $\hat{D}_1 = \hat{D}_2 = \hat{D}_3 = \hat{D}$. Hence, the Cases 2 and 3 of Proposition 6 do not exist.

4. For $\bar{C} \to \infty$ case 6 of Proposition 6 does not exist because the firm can always store sufficient cash to repay its maturing debt.

□

C.2. The firm's default boundary as a function of its debt structure α_t

Proof of Propositions 1 and 2 and Lemma 2. Proposition 6 describes the default boundary as a function of the firm's outstanding debt $\bar{N}(D_{t-1})$. Propositions 1 and 2 follow from solving $\hat{N} = \bar{N}(\alpha\hat{N})$. The thresholds $\hat{\alpha}$, $\bar{C}/\hat{N}(\hat{\alpha})$, $\hat{\alpha}_1$, and $\hat{\alpha}_2$ are as follows:

$$\begin{aligned}
\hat{\alpha} &= \frac{\hat{D}}{B_1(\hat{D}, L) + \hat{D}} \\
\frac{\bar{C}}{\hat{N}(\hat{\alpha})} &= \frac{\bar{C}}{B_2(0, L)} \\
\hat{\alpha}_1 &= \frac{\hat{D}_2}{B_2(\hat{D}_2, L) + \hat{D}_2} \\
\hat{\alpha}_2 &= \frac{\hat{D}_3}{B_1(\hat{D}_3, L) + \hat{D}_3}
\end{aligned}$$

The first case of Lemma 2 also follows directly from Proposition 6. The second case follows from the fact when Condition (12) holds the firms can only repay its outstanding debt when $N_t \leq \frac{\bar{C}}{\alpha_t}$ because it cannot store more cash. Note that in this case $\hat{N}(\alpha_t) = \frac{\bar{C}}{\alpha_t} < \hat{N}(1)$ and Condition (9) is slack. □

D. Critical Liquidity Shocks

D.1. The relation between $\hat{N}$ and $\hat{p}$.

Proof of Lemma 3. From the definition of $\hat{q}$ and $\hat{N}$ it follows that $q = \hat{q}(\hat{N}\alpha_t, q), \alpha_t)$.

If $\hat{q}(\hat{N}(\alpha_t, q), \alpha_t') < \hat{q}(\hat{N}(\alpha_t, q), \alpha_t)$ if follows from $\frac{\partial}{\partial q} B(\alpha_t N_t, L, q) > 0$ that Condition (13) is slack for $N_t = \hat{N}(\alpha_t, q)$, $q = \hat{q}(\hat{N}(\alpha_t, q), \alpha_t)$ and $\alpha_t = \alpha_t'$. The ($\Leftarrow$) part then follows from $\frac{\partial}{\partial N} B(\alpha_t N_t, L, q) < 0$.

The proof of the ($\Rightarrow$) part is analogous. □

D.2. Refinancing after critical liquidity shocks

Proof of Proposition 4. For any given N_t and α_t the debt capacity of an unconstrained firm is given by $B(.) = \max\{B_1(.), B_2(.)\}$. It follows that for $N_t \leq \delta v^H$

$$\hat{q} = \min\{1 - \sqrt{1 - \frac{N_t}{\delta v^H}}, 1 - \frac{v^H - (1 - \alpha_t(1-\delta))N_t}{v^H - \delta\alpha N_t}\}$$

The first term of the maximization corresponds to prefinancing and follows from Expression (20). The second term corresponds to debt that can be repaid if the cash flow recovers over one period and follows from Expression (17). It follows that a firm that experiences a critical liquidity shock will prefinance its outstanding debt if and only if

$$\sqrt{1 - \frac{N_t}{\delta v^H}} \geq \frac{v^H - (1 - \alpha_t(1-\delta))N_t}{v^H - \delta\alpha N_t}$$

Solving the above expression as an equality yields the (unique) threshold

$$\tilde{\alpha}(N_t) = \frac{v^H\left(1 - \sqrt{1 - \frac{N_t}{\delta v^H}}\right) - N_t}{\delta N_t\left(1 - \sqrt{1 - \frac{N_t}{\delta v^H}}\right) - N_t}. \tag{28}$$

Parts 1 and 2 of the Proposition then follow from

$$\frac{\partial}{\partial \alpha_t}\frac{v^H - (1 - \alpha_t(1-\delta))N_t}{v^H - \delta\alpha_t N_t} > 0 \text{ for } v^H > \delta N_t$$

and some simple algebra.

Part 3 follows from the fact that for α_t the firm does not need to issue any new debt because $R_t < C_{t-1}$. □

D.3. Debt Structure and Critical Liquidity Shocks.

Proof of Proposition 5. On the interval (14) if follows from Proposition 2 that[9]

$$\hat{q}(N_t, \alpha_t) = \min\left\{1 - \frac{\sqrt{(\bar{C} - \alpha_t N_t)^2 + 4\delta v^H(\delta v^H - N_t)} + \bar{C} - \alpha_t N_t}{2\delta v^H},\right.$$
$$\left. 1 - \frac{\sqrt{(1-\delta)\alpha_t N_t + \delta^2 v^H - N_t}}{\delta\sqrt{v^H}}, \frac{(1-\alpha)N_t}{v^H - \alpha\delta N_t}\right\} \tag{29}$$

[9]The other branches of the underlying quadratic equations correspond to $p < 0$.

Since outside the interval (14) the firm is not constrained by a cash holding constraint the critical recovery probability is described by Proposition 4. It is then easy to check that when Condition (14) holds $\hat{q}(N_t, \alpha_t) > \hat{q}(N_t, 0)$.

It is easy to check that (i) for $\alpha_t = \bar{C}/N_t < \tilde{\alpha}(N_t)$, the first branch of Expression (29) is strictly smaller than the two other branches and (ii) for $\alpha_t = \tilde{\alpha}(N_t) > \bar{C}/N_t$ the third branch of Expression (29) is strictly smaller than the two other branches.

Inspection shows that the first branch of Expression 29 is increasing in α_t and the two other branches are deceasing in in α_t. Moreover, it follows from Proposition 4 that for $\alpha_t < \bar{C}/N_t$, $\hat{q}(N_t, \alpha_t)$ is constant in α_t and for $\alpha_t > \tilde{\alpha}(N_t)$, $\hat{q}(N_t, \alpha_t)$ is decreasing in α_t. Hence there exists a unique critical threshold $\tilde{\alpha}_1(N_t)$ as described in the Proposition. □

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