Mach, Thirring & Lense, Gödel
- getting dizzy in space-time

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Talk given at the

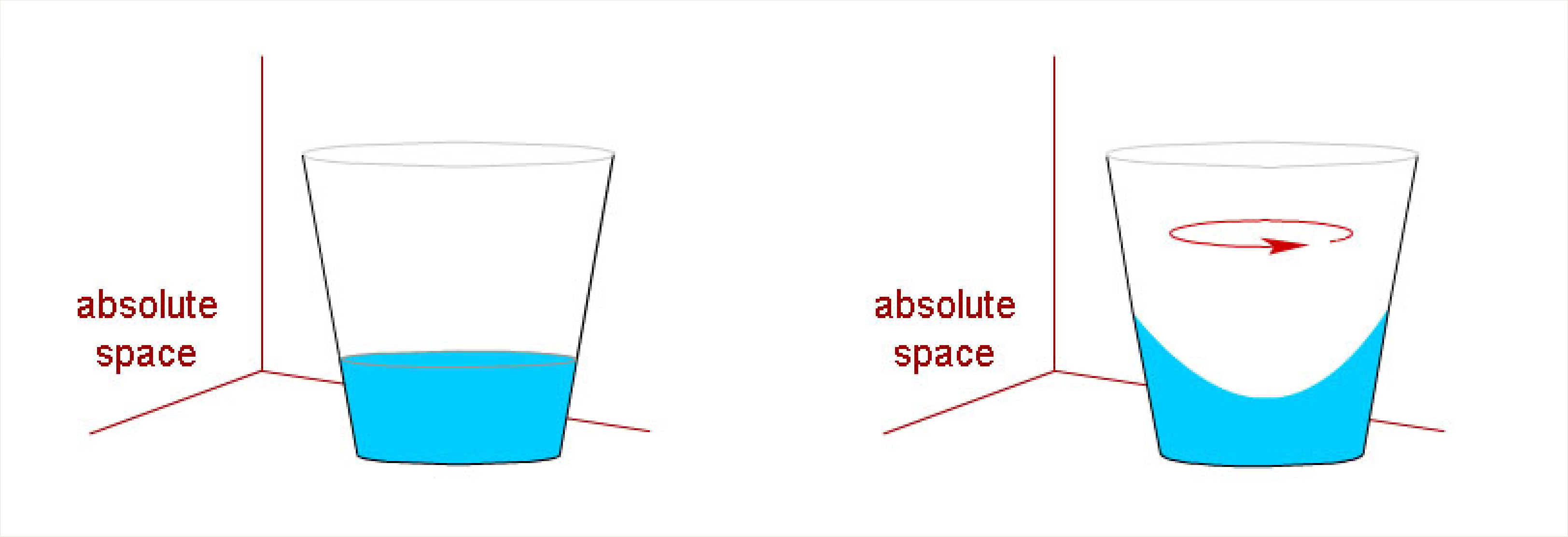
International Symposium on Kurt Gödel's Scientific Heritage

Brno University of Technology Centre

Brno, April 26, 2006

Isaac Newton, 1687

- Inertia is a phenomenon the relates the motion of bodies to absolute space.
- Rotation with respect to absolute space gives rise to centrifugal forces, as illustrated by the “bucket experiment“:



Ernst Mach, 1883

- E. Mach: *Die Mechanik in ihrer Entwicklung – historisch kritisch dargestellt*

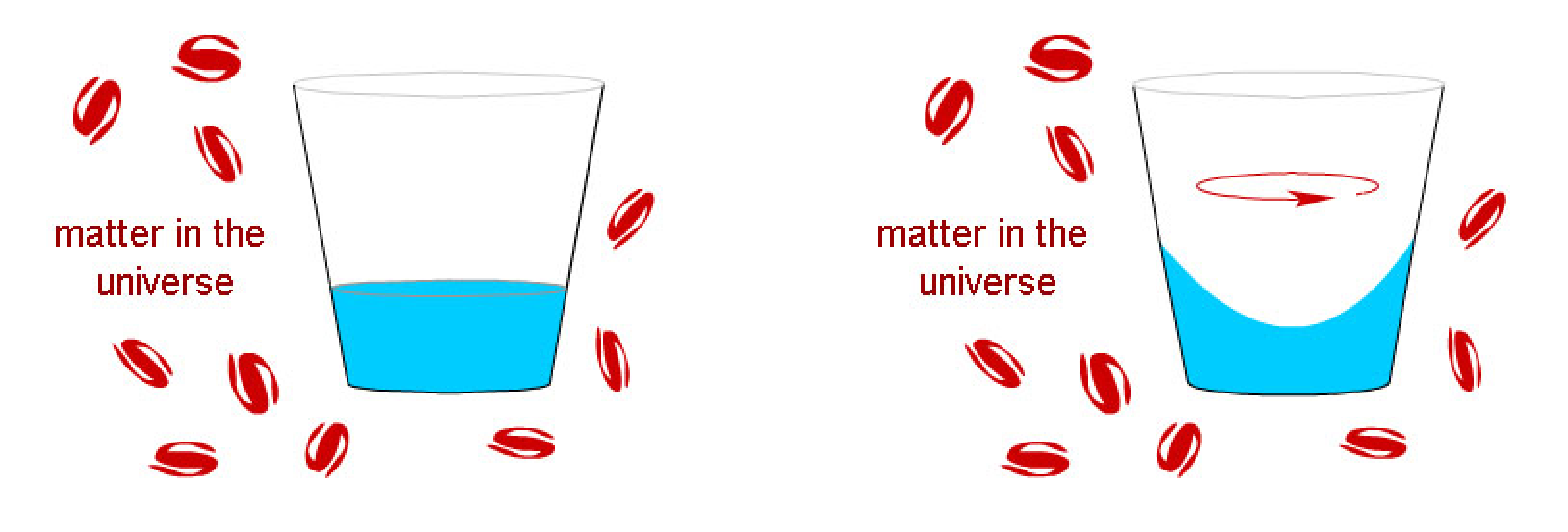
  Leipzig (1883)

- E. Mach: *The Science of Mechanics: A Critical and Historical Account of Its Development*

  Translated by Thomas J. McCormack, Opening Court Publishing Co., La Salle, IL (1942)

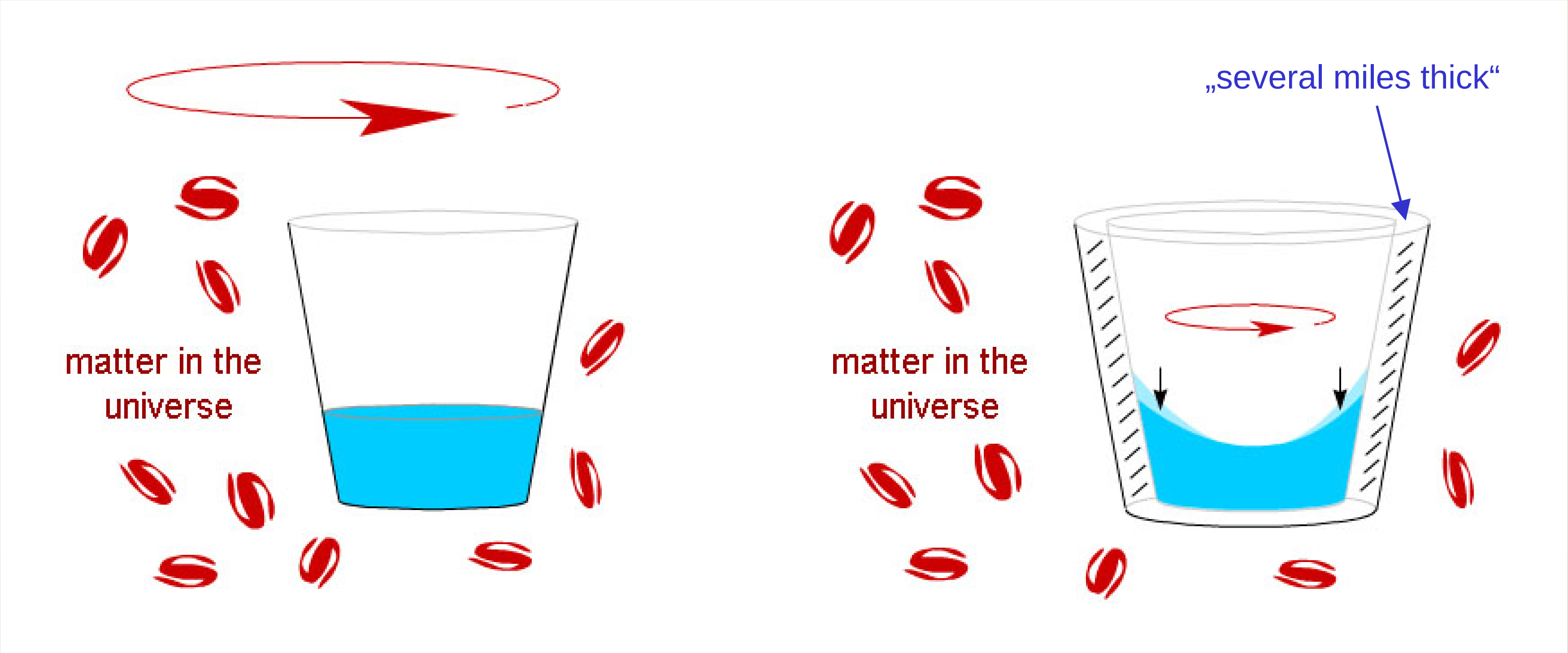
Ernst Mach, 1883

- There is no absolute space.
- Inertia is a phenomenon the relates the motion of bodies to the motion of *all* matter in the universe ("Mach's Principle").



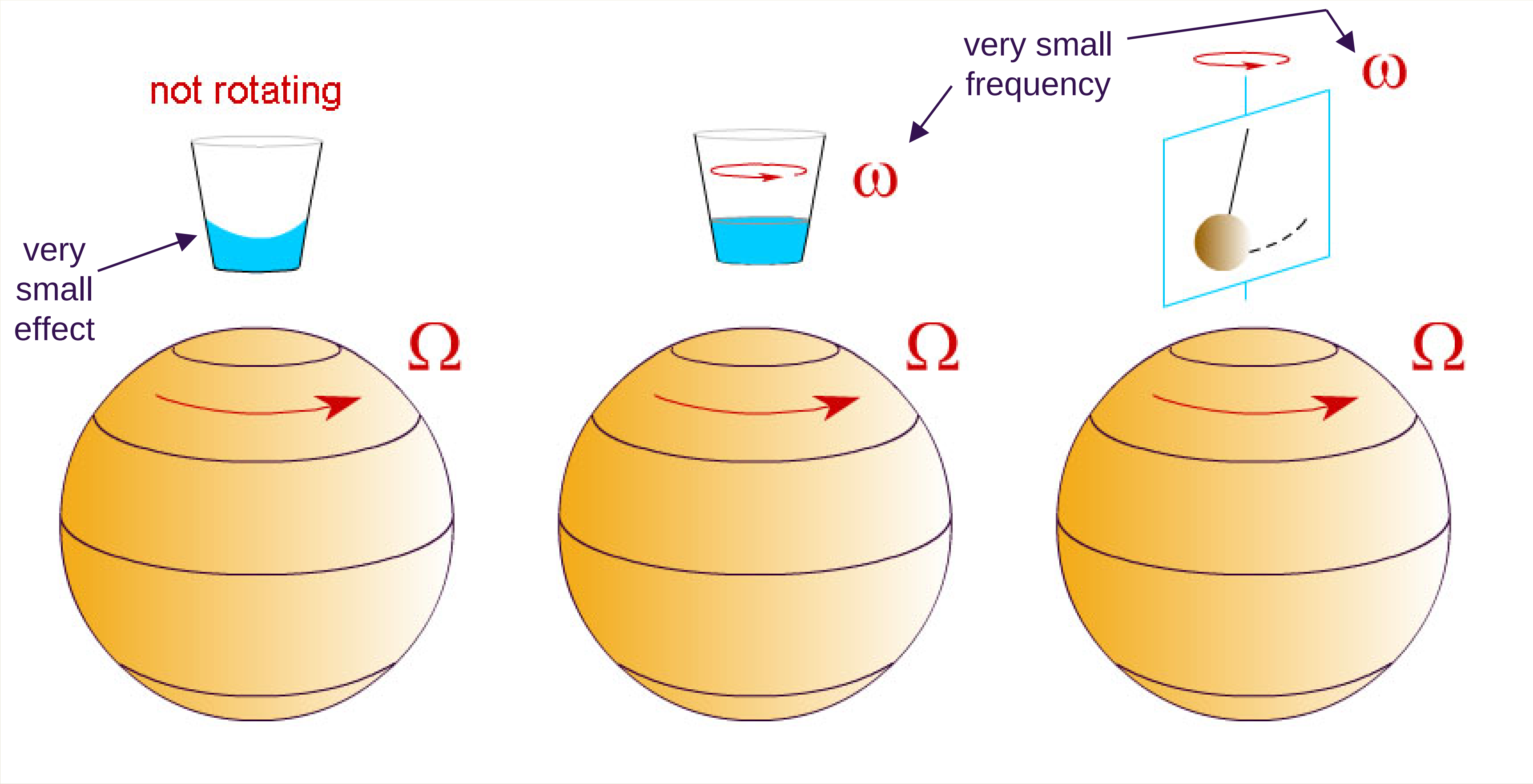
Ernst Mach, 1883

- A simultaneous rotation of *all* the matter in the universe is unobservable.
- The rotation of a *part* of the universe affects the behaviour of inertial frames.



Machian effects

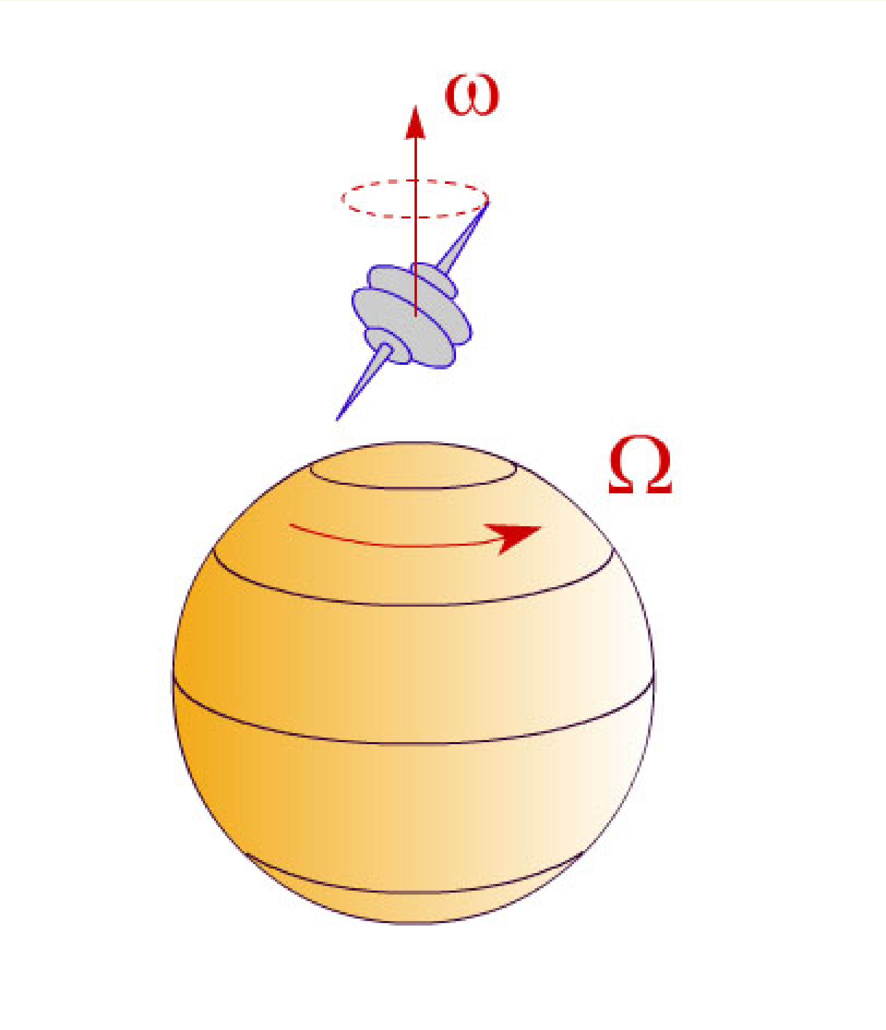
→ The rotation of the earth should “drag“ (local) inertial frames.



ω will later be called Thirring-Lense frequency.

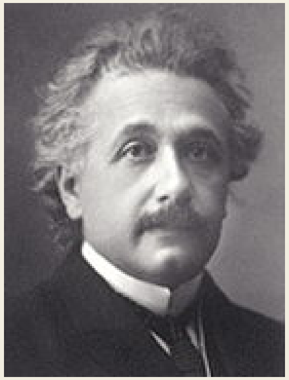
Gyroscopes

More convenient than water buckets are torque-free gyroscopes...



Dragging = precession of gyroscope axes

Albert Einstein, 1915



The general theory of relativity:

- Gravity is identified with the geometry of space-time.

Space-time metric: $$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

- Matter curves space-time.

Einstein's field equations: $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- The free motion of a (small) body in a given gravitational field is such that its proper time is maximal.

Geodesic equation: $$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\lambda\nu} \frac{dx^\lambda}{ds} \frac{dx^\nu}{ds} = 0$$

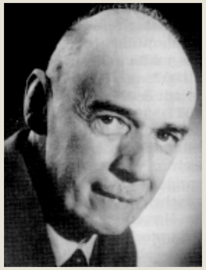
Hans Thirring und Josef Lense, 1918

- Newtonian gravity does *not* predict Machian effects.
- General relativity *does*:
  - H. Thirring: *Über die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie*

    Phys. Zeitschr. 19, 33 (1918)
  - H. Thirring: *Berichtigung zu meiner Arbeit „Über die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie“*

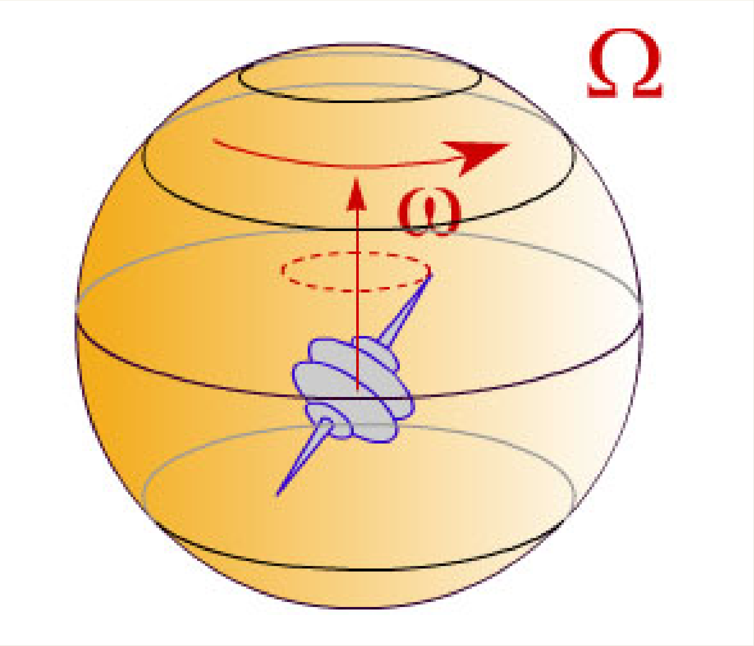
    Phys. Zeitschr. 22, 19 (1921)
  - J. Lense und H. Thirring: *Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Relativitätstheorie*

    Phys. Zeitschr. 19, 156 (1918)

Rotating matter shell – interior region

- The interior of a rotating spherical matter shell is (approximately) an inertial frame that is dragged, i.e. *rotates* with respect to the exterior region:



M = mass of the sphere
R = radius of the sphere

$$\frac{\omega}{\Omega} = \frac{4}{3}\frac{GM}{c^2 R} \equiv \frac{2}{3}\frac{R_S}{R}$$

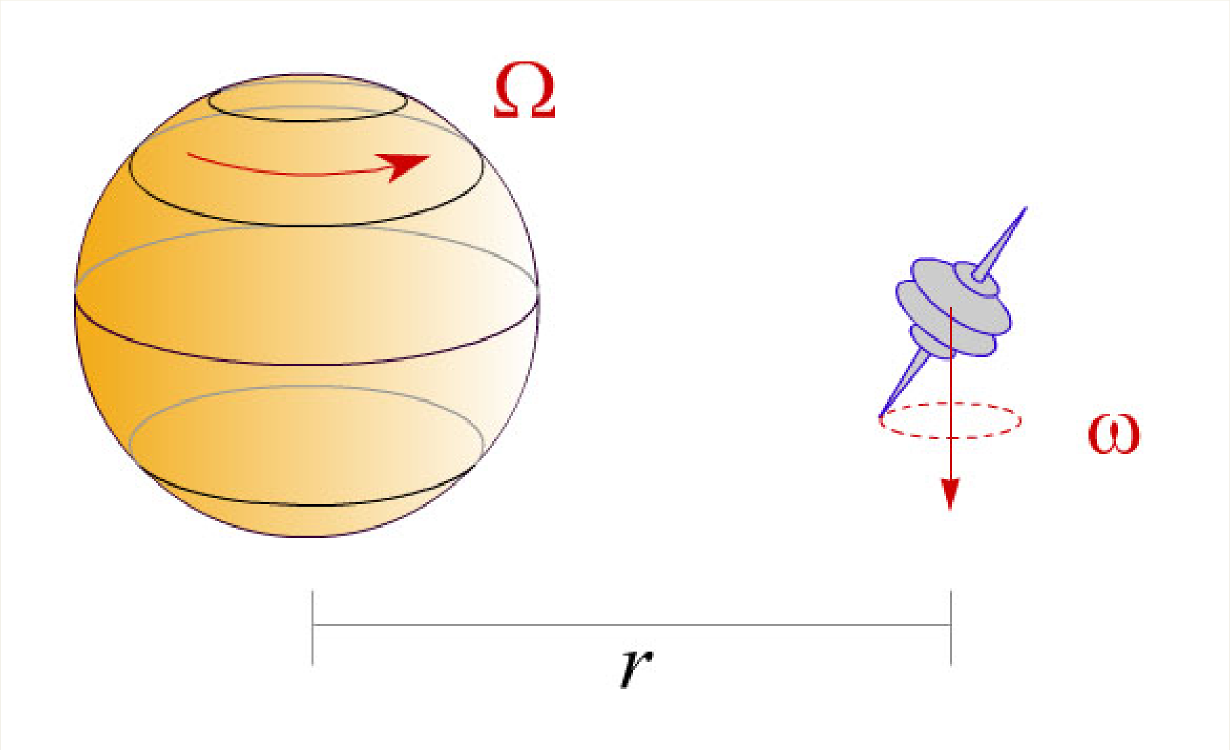
(valid in the weak field approximation = linearized theory)

Rotating matter shell – exterior region

- Dragging effects outside the shell:

In the equatorial plane:

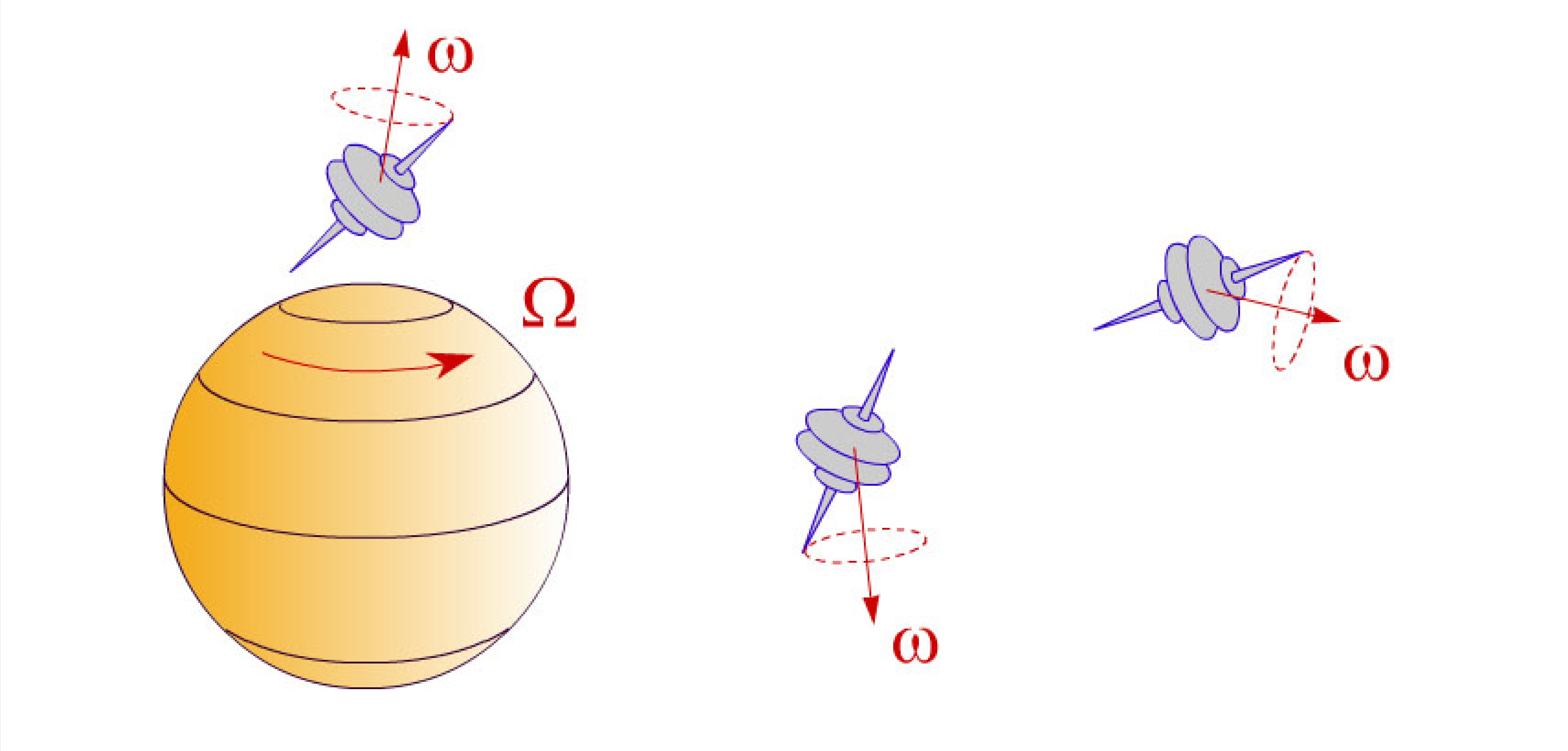
$$\frac{\omega}{\Omega} = -\frac{2}{3}\frac{GM}{c^2 R}\left(\frac{R}{r}\right)^3$$



→ The magnitude of Machian effects decreases with distance like $1/r^3$.

Rotating planet or star

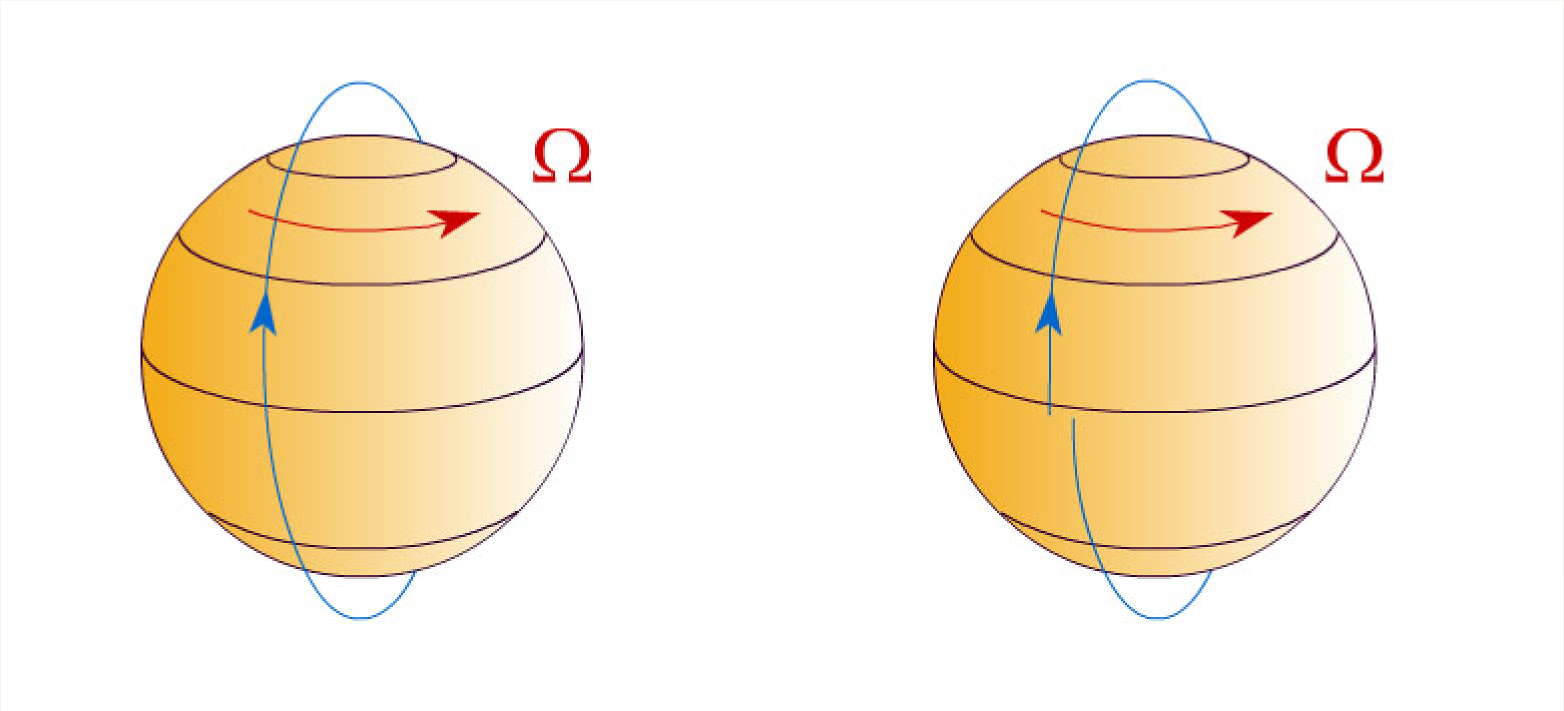
- Dragging effects near a massive rotating sphere:



$$\vec{\omega} \equiv \vec{\omega}(\vec{x})$$

Satellite orbits

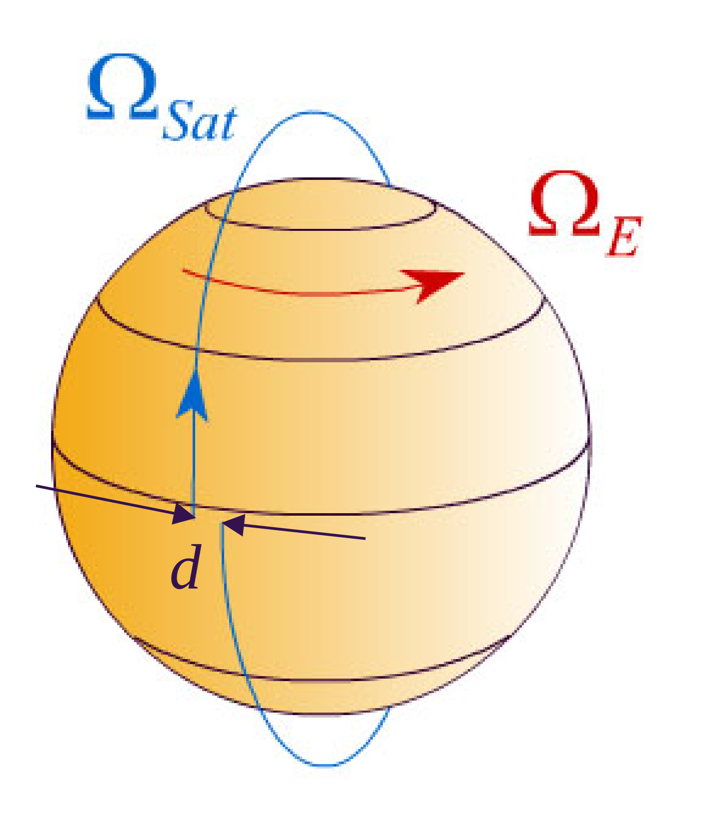
- Dragging of the orbital plane:



Newtonian gravity General relativity

Satellite orbits

- Magnitude of the effect:



Circular orbit of radius r:

$$d = \frac{4\pi}{5}\frac{\Omega_E}{\Omega_{Sat}}\frac{R_S R^2}{r^2}$$

Earth satellite with close orbits:

$$d = 0.13 \text{ cm} \quad (R_S = 0.886 \text{ cm})$$

Angular frequency of the orbital plane:

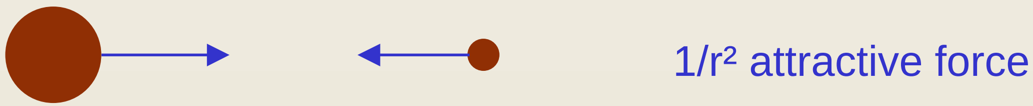
0.26 arc-seconds/year

Skip theory

The role of Machian effects in general relativity

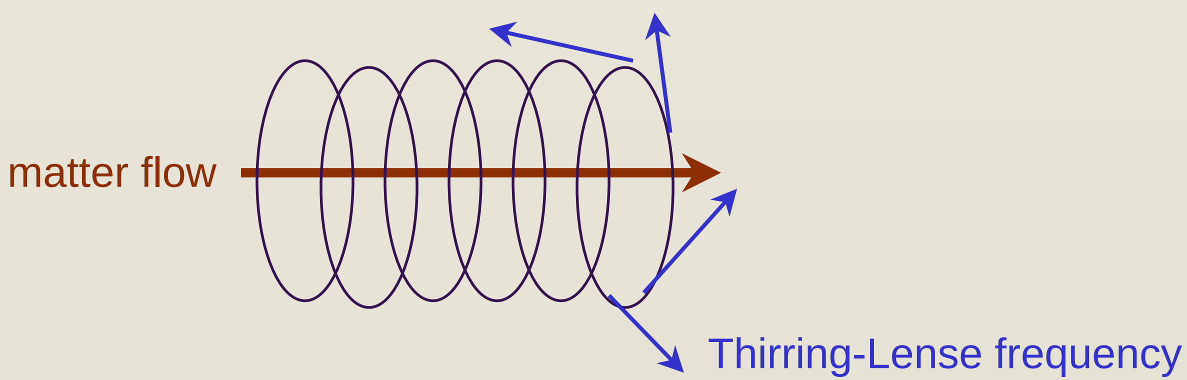
- Useful analogy that applies for stationary (weak) gravitational fields:

"Newtonian" part of the gravitational field → "electric" behaviour:



Rotating body: Both behaviours apply!

"Machian" part of the gravitational field → "magnetic" behaviour (sometimes called "gravimagnetism"):



Computation of Machian effects for weak fields

$$ds^2 = -f(dt - g_i\,dx^i)^2 + \gamma_{ij}\,dx^i dx^j$$

"electric" component (f)

"magnetic" components (g_i)

stationarity

$$\frac{\partial f}{\partial t} = \frac{\partial g_i}{\partial t} = 0$$

Einstein's field equations

Geodesic equation

linearized theory

slow motion

$$\vec{\nabla} \times \vec{G} = 0 \qquad \vec{\nabla} \cdot \vec{\omega} = 0$$

$$\vec{\nabla} \cdot \vec{G} = -4\pi\rho \qquad \vec{\nabla} \times \vec{\omega} = 8\pi\vec{j}$$

Newton's potential:

$$G_i = -\frac{1}{2}\partial_i f$$

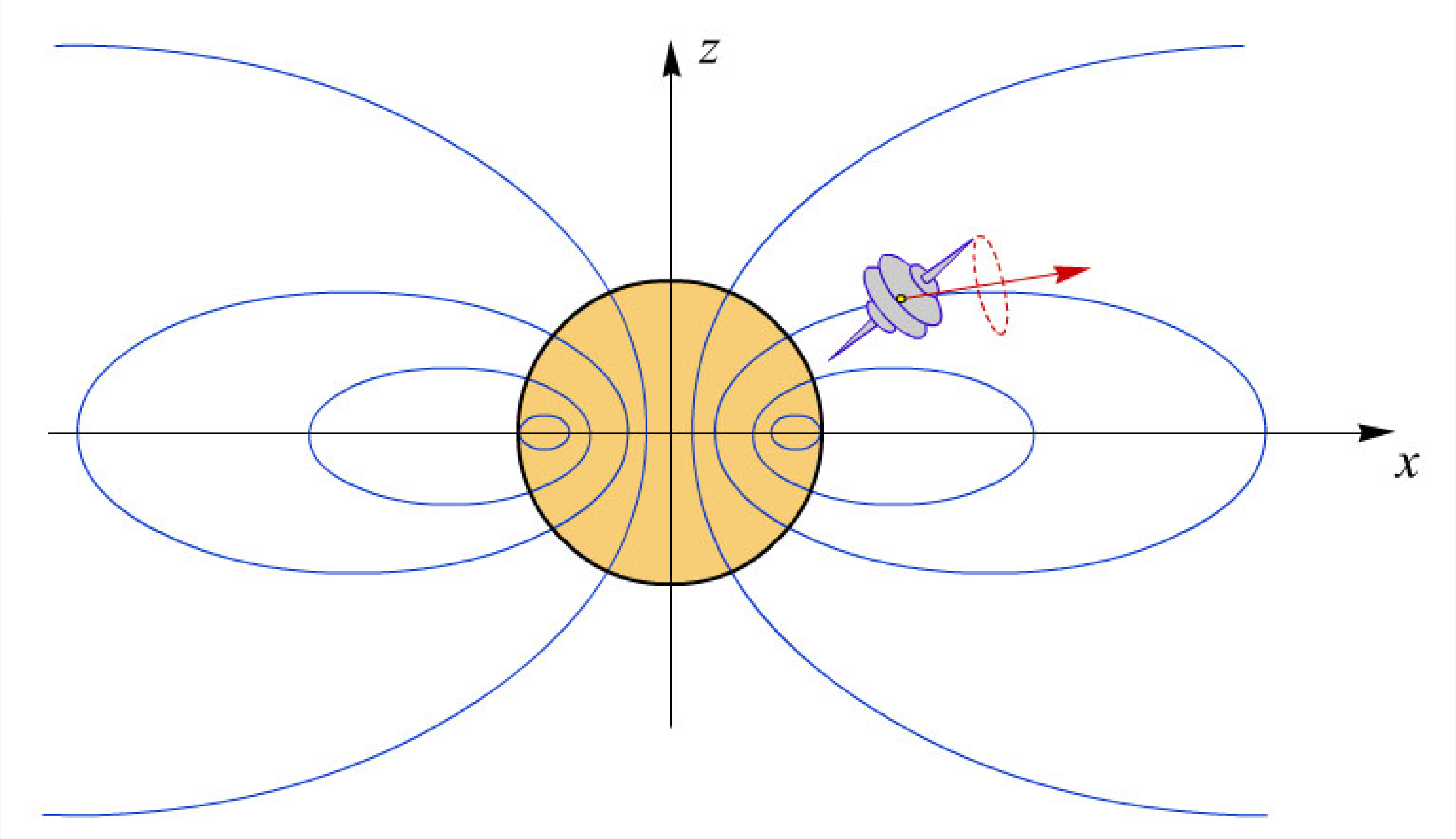
Thirring-Lense frequency:

$$\omega_i = -\frac{1}{2}\varepsilon_{ijk}\partial_j g_k$$

$$\omega^\mu = -\frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\xi_\nu \nabla_\rho \xi_\sigma$$

$$m\frac{d\vec{u}}{dt} = m\left(\vec{G} - 2\vec{u} \times \vec{\omega}\right)$$

Rotating charge distribution/rotating matter



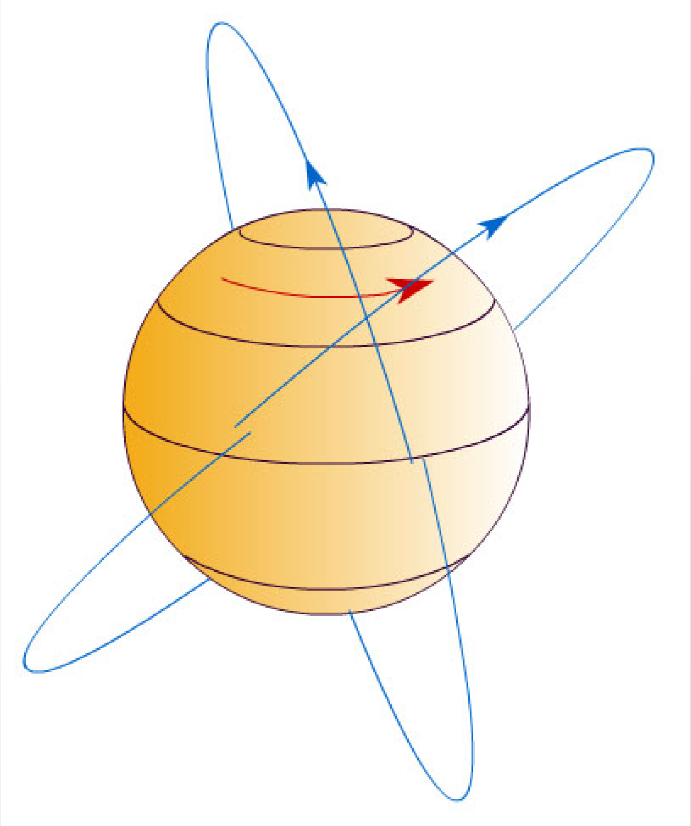
Does the Thirring-Lense effect exist in nature?

- Evaluation of LAGEOS satellite data

→ Observed value = $99\% \pm 5\%$ of the predicted value

- Gravity Probe B, 2004-6

→ Expectation for 2006: Confirmation within $\pm 1\%$



Skip project details

Does the Thirring-Lense effect exist in nature?

- George Pugh (1959), Leonard Schiff (1960)

  Suggestion of a precision experiment using a gyroscope in a satellite

- I. Ciufolini, E. Pavlis, F. Chieppa, E. Fernandes-Vieira and J. Perez-Mercader: *Test of general relativity and measurement of the Lense-Thirring effect with two Earch satellites*

  Science, 279, 2100 (27 March 1998)

  Measurement of the orbital effect to 30% accuracy, using satellite data (preliminary confirmation)

- I. Ciufolini and E. C. Pavlis: *A confirmation of the general relativistic prediction of the Lense-Thirring effect*

  Nature, 431, 958 (21 October 2004)

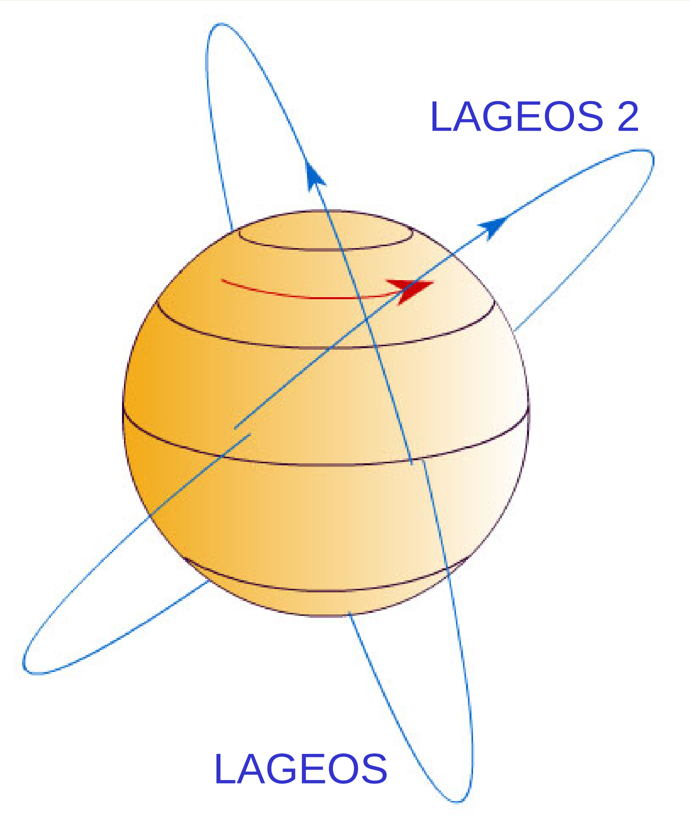
  Confirmation of the orbital effect to 6% accuracy, using satellite data

- Gravity Probe B, 2004-6

  Expected confirmation of gyroscope dragging to 1% accuracy

Ciufolini *et. al.*, 1998

- 2 satellites LAGEOS (NASA, launched 1976) and LAGEOS 2 (NASA + ASI, launched 1992)
- Original goal: precise determination of the Earth‘s gravitational field
- Major semi-axes:
  $a_1 = 12270 \text{ km}, \; a_2 = 12210 \text{ km}$
- Excentricities:
  $\varepsilon_1 = 0.004 \text{ km}, \; \varepsilon_2 = 0.014$
- Diameter: 60 cm, Mass: 406 kg
- Position measurement by reflexion of laser pulses
  (accurate up to some mm!)
- Evaluation of 4 years position data
- Main difficulty: deviations from spherical symmetry of the Earth‘s gravity field



Ciufolini *et. al.*, 1998

- The perturbations by the shape of the Earth are much larger than the expected dragging effect, hence they must be taken into account! Model of the Earth‘s gravitational field: EGM-96
- Further perturbations were accounted for:
  - Perturbation on the satellite motion by the pressure of the sun light
  - Perturbation on the satellite motion by residual air resistance
  - Variations of the Earth‘s angular velocity (tides!)
  - Variations in the positions of the poles
  - Movement of the ground station by continental drift
  - Gravitative perturbations induced by moon, sun and planets
- Clever choice of observables in order to compensate for uncertainties in EGM-96 and to separate „Machian“ from „Newtonian“ causes for the precession of orbital planes

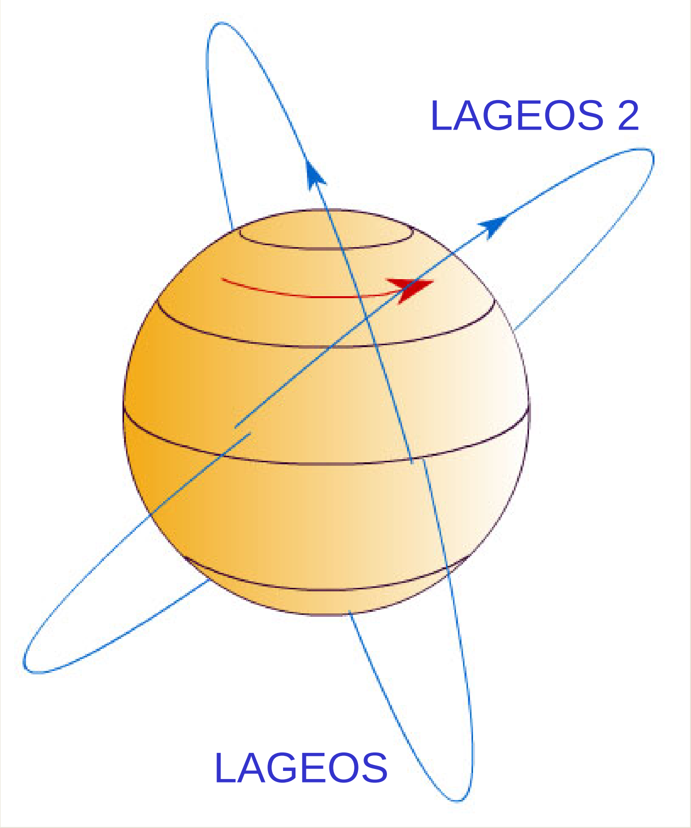
→ Observed value =
$110\% \pm 20\%$ of the predicted value

preliminary confirmation

Ciufolini *et. al.*, 2004

- LAGEOS und LAGEOS 2
- Improved model of the Earth‘s gravitational field: EIGEN-GRACE02S
- Evaluation of 11 years position data
- Improved choice of observables (combination of the nodes of both satellites)

→ Observed value =
$99\% \pm 5\%$ of the predicted value



Gravity Probe B

- Satellite based experiment, NASA und Stanford University
- Goal: direct measurement of the dragging (precession) of gyroscopes‘ axes by the Thirring-Lense effect (Thirring-Schiff-effect)

- 4 gyroscopes with quartz rotors: the roundest objects ever made!
- Launch: 20 April 2004
- Flight altitude: 400 miles
- Orbital plane: Earth‘s center + north pole + IM Pegasi (guide star) → Launch window: 1 Second!
- Proper motion of the guide star IM Pegasi: 35 mas/yr
  - Same order as the Thirring-Lense-Effekt!
  - Since 1997 measurements to 0.1 mas/yr accuracy (using microwave VLBI by comparison with quasars that lie nearby to the star on the sky)

Gravity Probe B

- Expectation for 2006: Measurement of the Thirring-Lense frequency with an accuracy of $\pm 1\%$
- Web site:

  http://einstein.stanford.edu/

Kurt Gödel, 1949

- K. Gödel: *An example of a new type of cosmological solution of Einstein's field equations of gravitation*

  Rev. Mod. Phys. 21, 447 – 450 (1949)

Kurt Gödel, 1949

- The field equations of general relativity admit a cosmological model (the “Gödel universe“) exhibiting some remarkable properties:
  - The source of the gravitational field is a perfect fluid with

    $$\rho = p$$

    or, equivalently, pressureless dust + a (negative) cosmological constant.
  - It is completely singularity-free and geodesically complete.
  - It is homogeneous and stationary (but not static).
  - Nearby observers, both at rest with respect to matter, rotate with respect to each other.
  - It contains closed timelike curves.

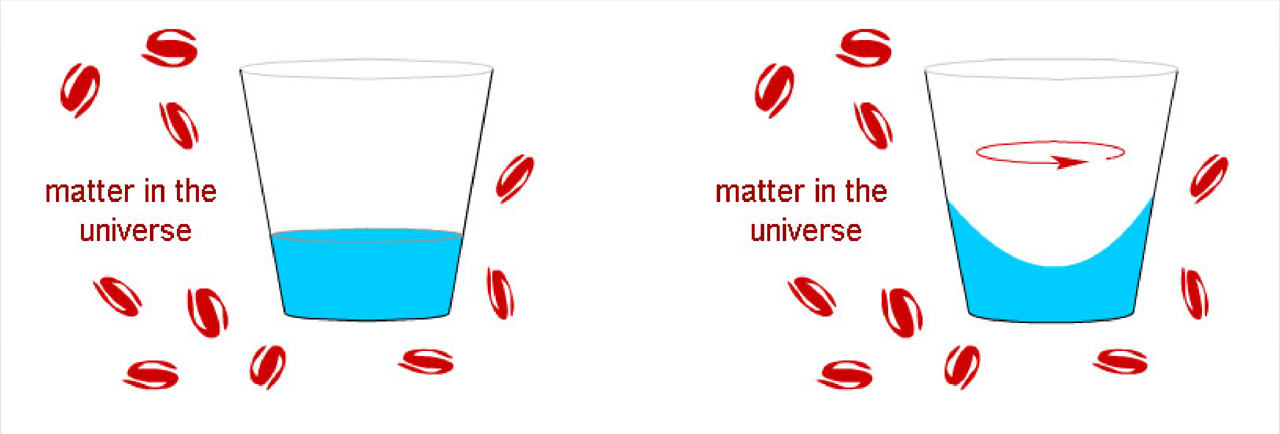
$$ds^2 = \frac{1}{2\omega^2}\left(-\left(dt + e^x dz\right)^2 + dx^2 + dy^2 + \frac{1}{2}e^{2x}dz^2\right)$$

Kurt Gödel, 1949

- Nearby observers, both at rest with respect to matter, rotate with respect to each other:
  - Any observer (at rest with respect to matter) who always looks towards a particular nearby observer gets dizzy.
  - Any observer (at rest with respect to matter) who orients himself along a fixed direction of his local inertial frame (such that he will *not* get dizzy) sees all nearby observers rotating around him with angular velocity ω.
    Hence, in this sense, local inertial frames rotate with respect to each other.
  - Due to the existence of an axis of rotation for every such observer, space-time is not isotropic.
  - However recall: Space-time is homogeneous, hence there is no “axis of rotation of the universe“.

Kurt Gödel, 1949

- Does the Gödel universe confirm or contradict Mach‘s Principle?
  - It confirms Mach‘s principle, because inertia (the notion of local inertial frames) is tied to the global distribution and motion (relative „rotation“) of matter.
  - It contradicts Mach‘s principle, because local inertial frame rotate with respect to each other, while the universe *as a whole* does *not* rotate around some particular axis.



Kurt Gödel, 1949

- Are there rotational effects of this type in our universe?
  - If our universe was a Gödelian one, we would expect

$$\omega < 3\times 10^{-4} \text{ arc-seconds/century}$$

  - Observation of planet orbits:

$$\omega < 0.1 \text{ arc-seconds/century}$$

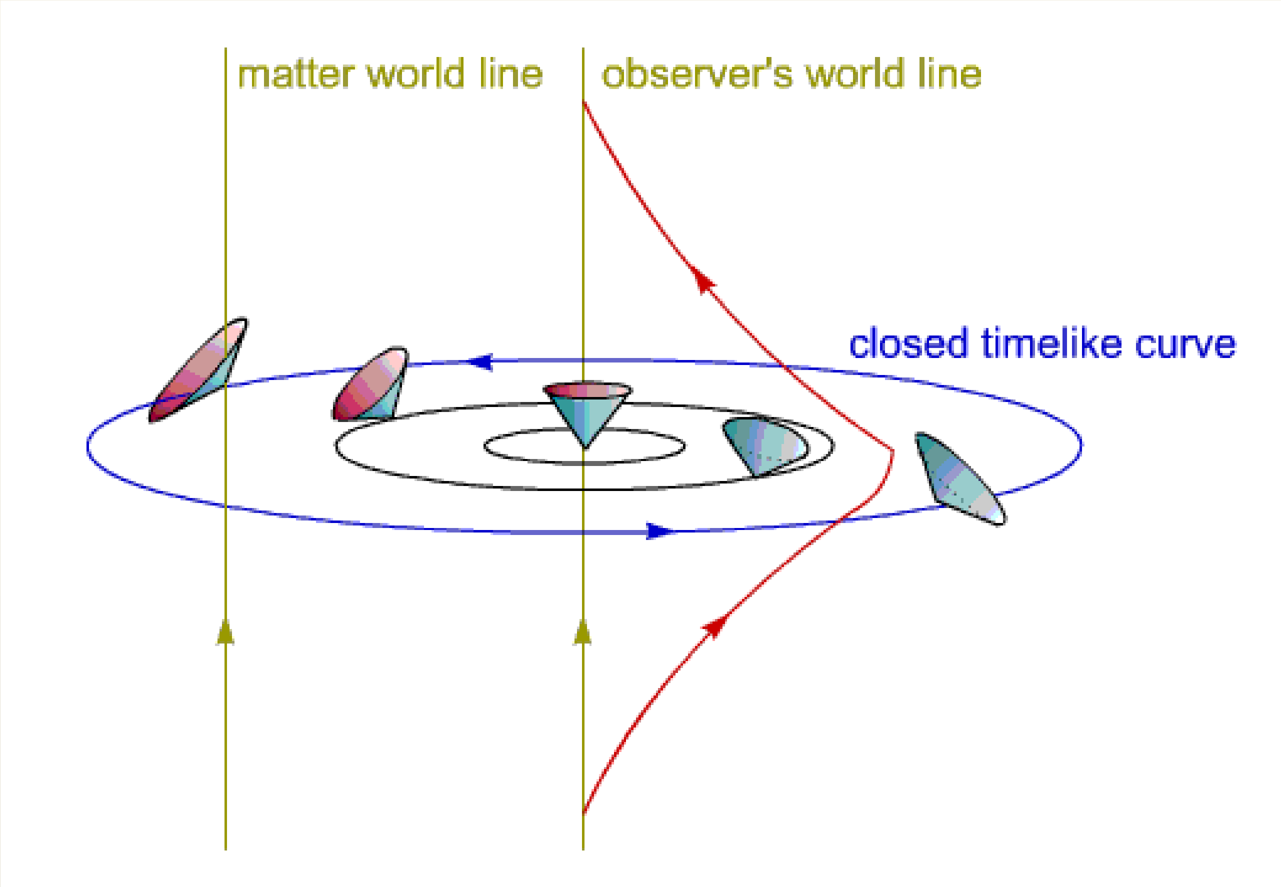
  - Observation of the microwave background radiation (using the COBE data and an expanding generalization of Gödel‘s universe):

$$\omega < 10^{-9} \text{ arc-seconds/century}$$

  - Suggestion to use quantum gyroscopes

Kurt Gödel, 1949

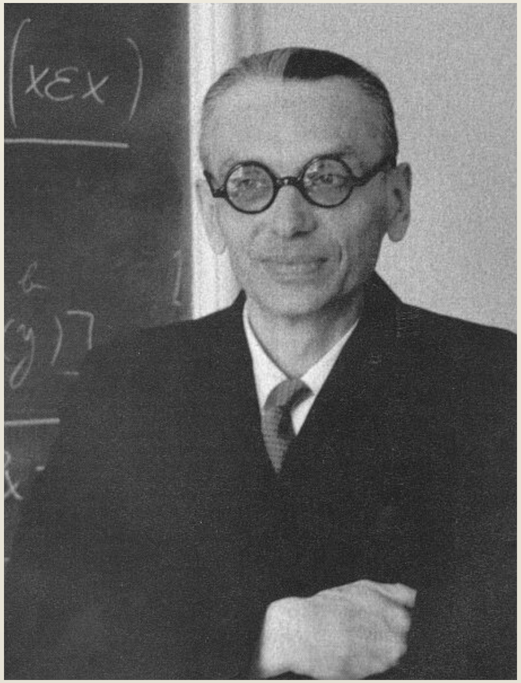
- Closed timelike curves:



(frame adopted to a particular observer, one coordinate being suppressed)

Cosmology after Gödel

- Deeper mathematical classification of cosmological solutions
- Development of more realistic cosmological models
- Discussion about Mach‘s Principle



Thank you...

... for your attention!

This presentation may be found under

http://homepage.univie.ac.at/franz.embacher/Rel/Goedel/