The didactical significance of interactive animations

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Abstract:

A picture says more than thousand words. An interactive animation says more than thousand pictures. By using interactive animations in mathematics education new patterns of perception may be activated and presumably lead to new forms of the inner representation of mathematical issues. This conjecture is illustrated in terms of some examples.

Zur didaktischen Funktion interaktiver Animationen: Ein Bild sagt mehr als tausend Worte. Eine interaktive Animationen sagt mehr als tausend Bilder. Mit Hilfe dynamischer Diagramme können dem Mathematikunterricht neue Wahrnehmungsformen erschlossen werden, die vermutlich zu neuen Formen der "inneren Verankerung" mathematischer Sachverhalte führen. Diese These wird anhand einiger Beispiele illustriert.

An interactive animation says more than thousand pictures

When skipping through mathematical learning resources on the web, one frequently encounters tiny learning units, sometime simply called “applets”, equipped with sliders (scroll bars) or other functionalities such that some user activity is necessary, visualizing and animating mathematical issues of all sorts. To begin with, let us look at the interactive animation

→ On the definition of the derivative
http://www.univie.ac.at/future.media/moe/galerie/diff1/diff1.html#ableitung

as a first example. It visualises and emphasises the fact that the derivative of a (differentiable) function is a again a function, and it allows to get acquainted with the appropriate vocabulary (such as the tangent slope at some point, local extremal points, and the like) at the level of *ideas* and *concepts* rather than on the level of *computation* – a goal that is difficult to achieve by means of static pictures.

New ways of perception and understanding

Graphical means (mathematical diagrams) have now been supporting the formation of the "inner representation" of mathematical issues by learners since thousands of years. Besides factors like pre-knowledge and the conscious reflection of mathematical notions, the perception of graphical diagrams of all sorts is of utmost importance in the learning process. It is certainly not a coincidence that the word *image* denotes *picture* and *imagination* at the same time, and that we talk about "inner pictures" we form about mathematical issues.

My hypothesis that shall be illustrated by some examples is the following: The value interactive animations is often underestimated. Their use in mathematics education may, in a way analogous to static diagrams, lead to the formation of "inner animations", whereby the perception is strengthened by the additional factor of the user *action* necessary to handle a tool of this type.

From the notion of the number to the partial derivative

A number of interactive animations may be found at the platform ***mathe online***:

- Galerie: http://www.mathe-online.at/galerie.html
- English (gallery): http://www.univie.ac.at/future.media/moe/galerie.html
- User generated material: http://www.mathe-online.at/materialien/ (available only in German)

What most of these learning units have in common is that the they deal with restricted topics, mainly in connection with the introduction of a mathematical notion or an interrelation between different representations of a mathematical object. Their goal is to support the process of formation of mathematical notions, in particular when visualisation and a certain type of user action is possible and reasonable. In some cases the topics covered are deliberately kept "small" in order to focus the attention to certain aspects of a mathematical structure. Hence, interactive animations do not compete with mathematical tools such as spreadsheet calculation, dynamical geometry and computer algebra, but rather coexist with them.

Interactive animations may be used as standalone learning units, as well as embedded in project-like "learning paths" (*Lernpfade*) [1], such as in

- http://www.mathe-online.at/lernpfade/
- http://www.austromath.at/medienvielfalt/

The latter of these two web sites presents material generated in the framework of the project *Medienvielfalt im Mathematikunterricht* (Diversity of media in mathematics teaching) [2], the subject of some other talks and a workshop at this conference.

I now present an example of a more or less connected series of animations that covers 8 years of mathematics education, and that has been derived from the DynaGraph concept (the English translations of the titles are put in square brackets):

→ Der Zahlenstrahl [The number beam]
http://www.mathe-online.at/materialien/Franz.Embacher/files/zahlenstrahl/zahlenstrahl.html

→ Zahlen und die Zahlengerade [Numbers and the number beam]
http://www.mathe-online.at/materialien/Franz.Embacher/files/zahlen/zahlen.html

→ Rechenoperationen [Operations with numbers]
http://www.mathe-online.at/materialien/Franz.Embacher/files/rechenoperationen/rechenoperationen.html

→ Potenzen [Powers]
http://www.mathe-online.at/materialien/Franz.Embacher/files/potenzen/potenzen.html

→ Der Mittelwert am Zahlenstrahl [The mean value (average) on the number beam]
http://www.mathe-online.at/materialien/Franz.Embacher/files/mittelwertZstr/mittelwertZstr.html

→ Was ist der Mittelwert? [What is the mean value (average)?]
http://www.mathe-online.at/materialien/Franz.Embacher/files/mittelwert/mittelwert.html

→ Lineare Gleichung [Linear equation]
http://www.mathe-online.at/materialien/Franz.Embacher/files/lineareGleichung/lineareGleichung.html

→ Funktionale Abhängigkeiten verstehen (DynaGraph) [Unterstanding functional dependencies]
http://www.mathe-online.at/galerie/fun1/fun1.html#FunktAbh

→ Ableitungen messen [Measuring derivatives]
http://www.mathe-online.at/galerie/diff1/diff1.html#ablMess

→ Partielle Ableitungen messen [Measuring partial derivatives]
http://www.mathe-online.at/galerie/partdiff/partdiff.html#partAblMess

→ Lineare Abbildung [Linear mapping]
http://www.mathe-online.at/galerie/linalg/linalg.html#lineareAbbildung

Instead of giving a thorough description of what these animations show and how they work, the reader is asked to have a look at them right now.

In that they visualise what happens when “the value of a variable changes”, they support thinking in terms of alternative representations of number operations and functions. What is crucial when utilizing them in mathematics education is that they are given to the students together with exercises supporting their role as *tools for understanding*. (They all have inbuilt exercises that may be used, if appropriate). They open the students the possibility to *operate* with mathematical structures in a new way, to perform new kinds of “experiments” and to give algebraic statements a new meaning.

In addition, some of the built-in exercises are of a special character: They may only be solved when some basic understanding has already been achieved. From the point of view of experts, these problems are *very simple*, their solution requiring only a moment’s thought and a slider move. The according animations may therefore serve as *indicators* showing whether the underlying notions are in fact “very simple”

for the students, or whether the words of the exercise have been understood at all. (In this sense, they are tools serving teachers' needs).

The learning units listed above contradict the position that animations are only suited for *young* learners and *very basic* content. (The last two animations are in fact beyond the horizon of school mathematics!)

Working with interactive animations in the way suggested here necessitates changes in the traditional way of introducing (and exploring) concepts in mathematics courses and thus leads to changes in the flavour not only of the learning process, but of the mathematical content itself!

It should be stressed that at a *later* stage in the learning process, the transition to a more powerful tool, such as computer algebra, supporting application, problem solving and mathematical proof at a more sophisticated level, may be in place.

More examples

When looking at the details, there are many different types of interactive animations. As an example, the

→ Excel-Plotter
http://www.mathe-online.at/

shows the influence of parameter values on function graphs, while the learning unit

→ Folgen und Zahlengerade [Sequences and the number line]
http://www.mathe-online.at/galerie/grenz/grenz.html#folgenz

visualises the notions of convergence and divergence of numerical series (and it gives a feeling of „how fast" a series converges).

Concluding, let me state that interactive animations are powerful tools that are capable of enriching mathematical imagination and thought of learners of all ages.

References

[1] Franz Embacher, *Das Konzept der Lernpfade in der Mathematik-Ausbildung*, Vortrag gehalten am Institut für Wissenschaft und Kunst (im Rahmen der Vortragsreihe Internet – Forschung - Lehre), Wien, 7. Juni 2004.
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[2] Franz Embacher, *Medienvielfalt im Mathematikunterricht*, zwei Vorträge gehalten im Rahmen des Lehrgangs Mathematikunterricht HEUTE, am Pädagogischen Institut Salzburg, 13. April 2005, und im Rahmen des Akademielehrgangs eLearning-Didaktik, Altmünster, 21. April 2005.
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The links used in this paper may be found on the web under the address http://homepage.univie.ac.at/franz.embacher/MatheDidaktik/DES-TIME-Dresden-20.-23.7.2006/