

## DPPs

A **d**escending **p**lane **p**artition of size  $n$  is an array of successively intended rows filled with positive integers less or equal than  $n$  such that

- ▶ the number of entries in a row is less than the largest entry in that row,
- ▶ the entries are weakly decreasing along rows and strictly decreasing along columns.

## ASMs

An **a**lternating **s**ign **m**atrix of size  $n$  is an  $n \times n$  matrix with entries  $-1, 0, 1$  such that

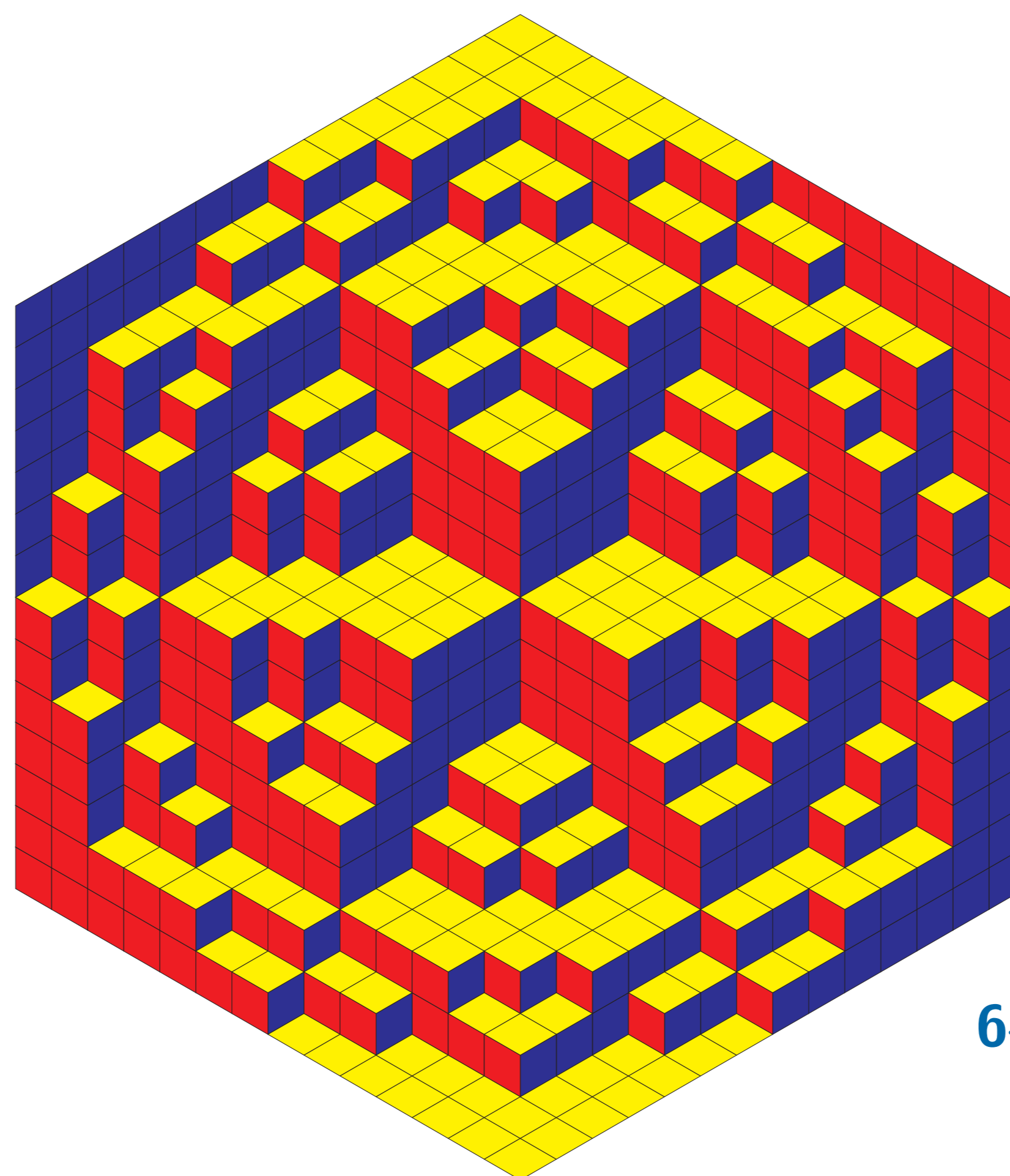
- ▶ the non-zero entries alternate in all rows and columns,
- ▶ all column- and row-sums are 1.

### DPP

```

7 5 5 4 3 1
 4 4 2
   2
  
```

### TSSCPP

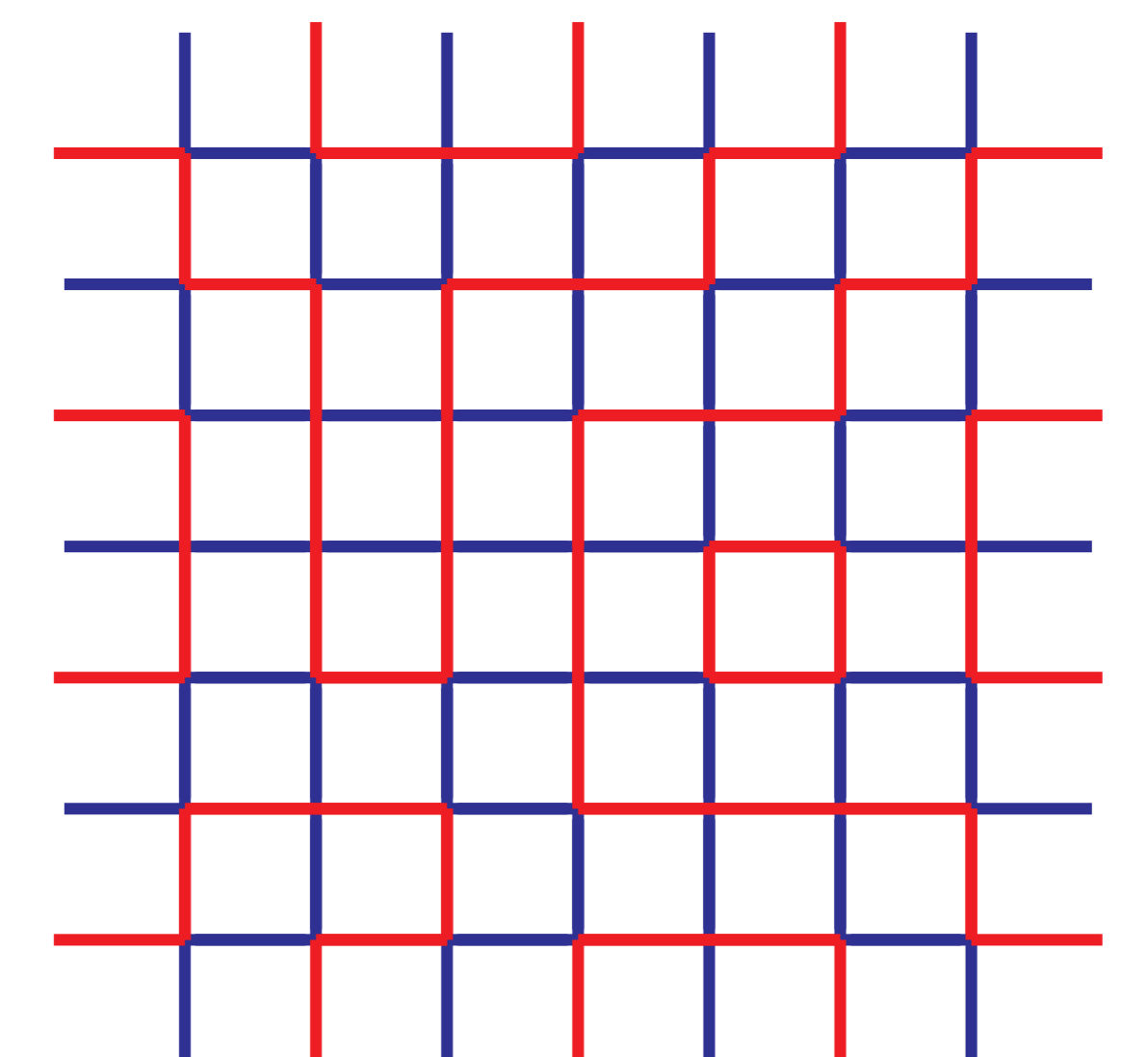


### AST

```

0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 1 0 0 0 0 0 -1 1
0 1 0 -1 0 1 0 0 0 0
0 0 0 0 0 0 0 1
1 0 0 -1 1
1 0 0
1
  
```

### FPL

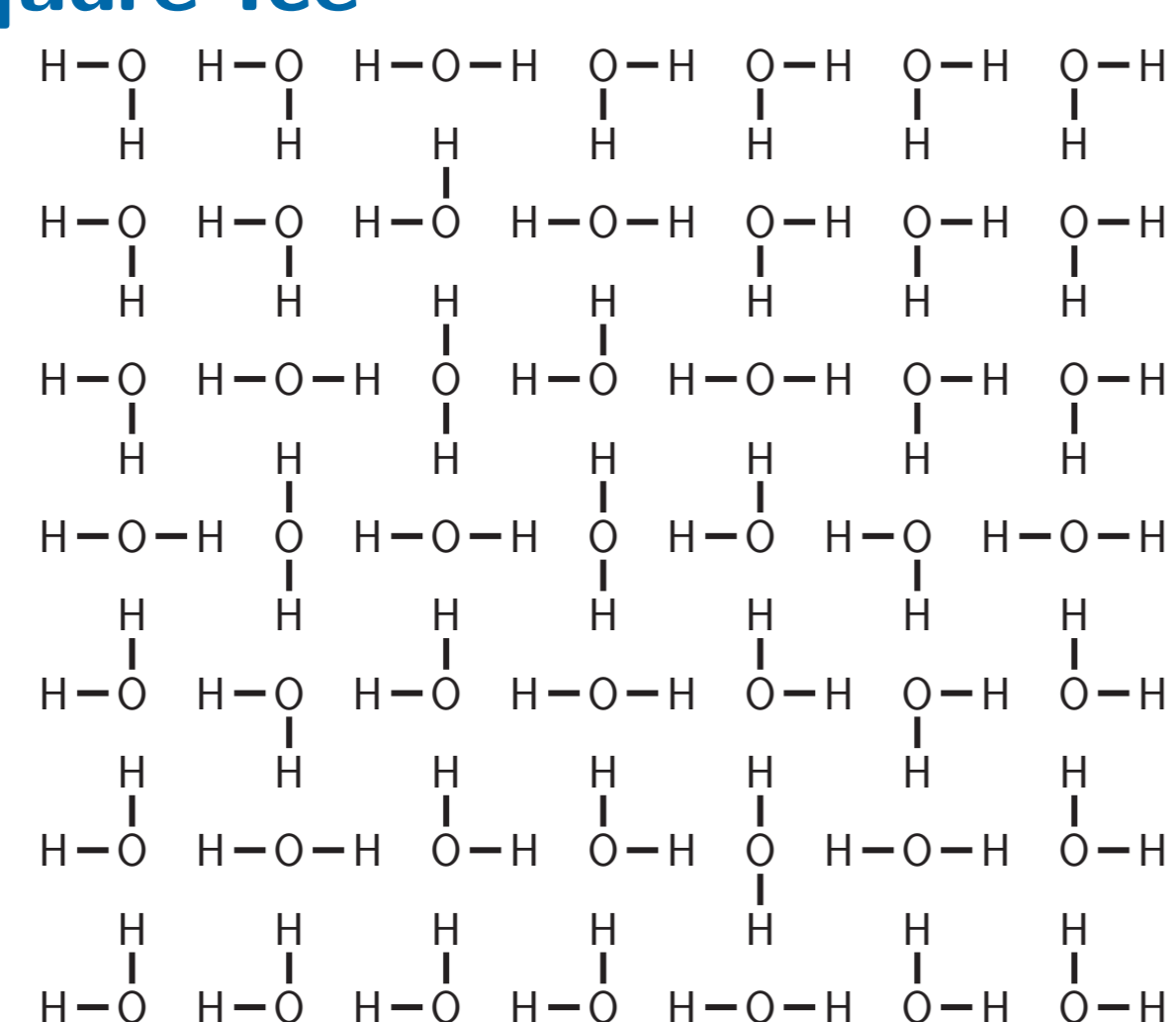


### ASM

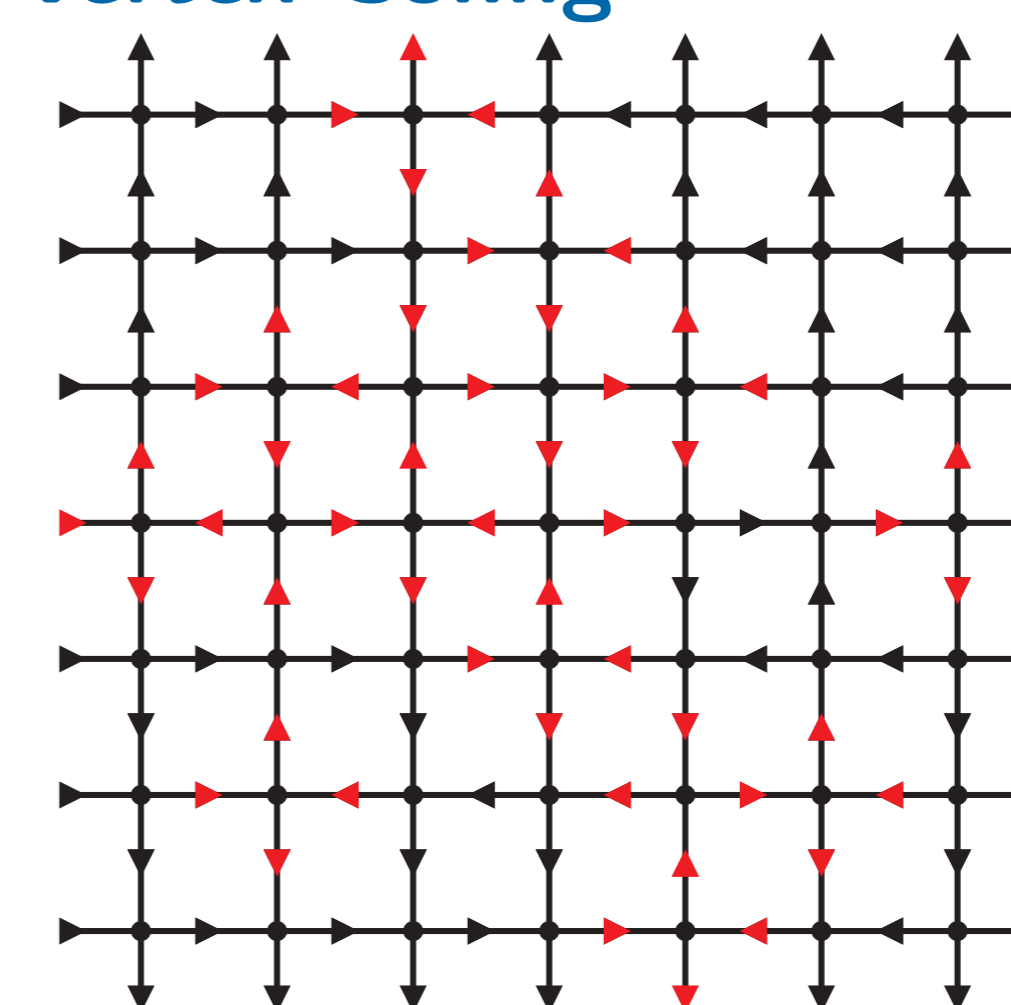
```

0 0 1 0 0 0 0
0 0 0 1 0 0 0
0 1 -1 0 1 0 0
1 -1 1 -1 0 0 1
0 0 0 1 0 0 0
0 1 0 0 -1 1 0
0 0 0 0 1 0 0
  
```

### Square Ice



### 6-vertex Config



## Square ice

A **s**quare **i**ce of size  $n$  is an arrangement of  $n^2$  water molecules, where the O-atoms are placed on an  $n \times n$  grid with interlacing H-atoms and H-atoms at the left and right boundary.

## TSSCPP

A **t**otally **s**ymmetric **s**elf **c**omplementary **p**lane **p**artition of size  $n$  is filling of a  $2n \times 2n \times 2n$  box with unit-cubes such that

- ▶ the configuration stays invariant under reflection and interchanging the axes X,Y,Z,
- ▶ the "empty part" of the  $2n \times 2n \times 2n$  box forms the same configuration.

## What's known

- ▶ For  $n$  a positive integer and one of the above families, the number of objects of size  $n$  is  $\prod_{i=0}^{n-1} \frac{(3i+1)!}{(n+i)!}$ .
- ▶ We can relate ASMs, FPLs, square ice and six-vertex configurations.
- ▶ Explicite formulas for certain refined enumerations are known.

## What's open

- ▶ Relate all different families.
- ▶ Transfer existing generalisations of one type to the others.