





Florian Aigner¹

Faculty of Mathematics, University of Vienna, Austria ¹ Supported by the Austrian Science Foundation FWF, START grant Y463.

DPPs

A descending plane partition of size *n* is an array of successively intended rows filled with positive integers less or equal than *n* such that

- ▶ the number of entries in a row is less than the largest entry in that row,
- the entries are weakly decreasing along rows and strictly decreasing along columns.

ASTs

An alternating sign triangle of size *n* is a triangular configuration of *n* centred rows with entries -1, 0, 1 such that

- \blacktriangleright the *i*-th row counted from the bottom has 2i 1 elements,
- ▶ the non-zero entries alternate in all rows and columns,

▶ all row-sums are 1,

▶ the topmost non-zero entry is 1 for all columns.

ASMs

ASM

0

An alternating sign matrix of size n is an $n \times n$ matrix with entries -1, 0, 1

FPLs

A fully packed loop of size n is an edge-colouring of the $n \times n$ grid with *n* external edges such that

such that

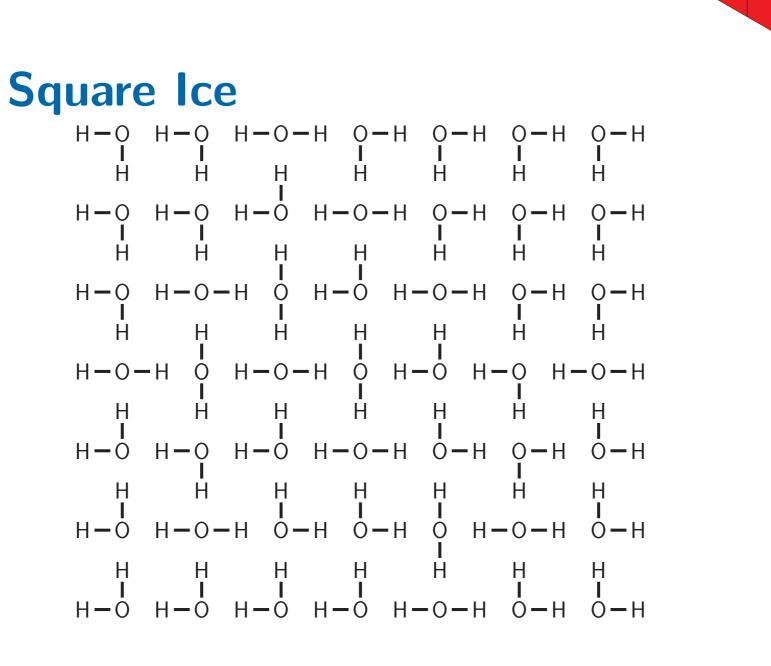
- ▶ the non-zero entries alternate in all rows and columns,
- ▶ all column- and row-sums are 1.

- ever vertex has two incident red and two incident blue coloured edges,
- ► the external edges are coloured alternately starting with the top left external edge being red.

AST DPP 0 -1 1**TSSCPP** 3 1 5 $1 \quad 0 \quad -1 \quad 0 \quad 1 \quad 0$ 0 0 5 4 4 4 2 $0 \ 0 \ 0 \ 1$ 0 $1 \quad 0 \quad 0 \quad -1 \quad 1$ 2 1 0 0 **FPL** 0 0 0 0 $1 \ -1 \ 1 \ -1 \ 0 \ 1$ $0 \quad 0 \quad -1 \quad 1 \quad 0$

 \cap

 $-1 \ 0$

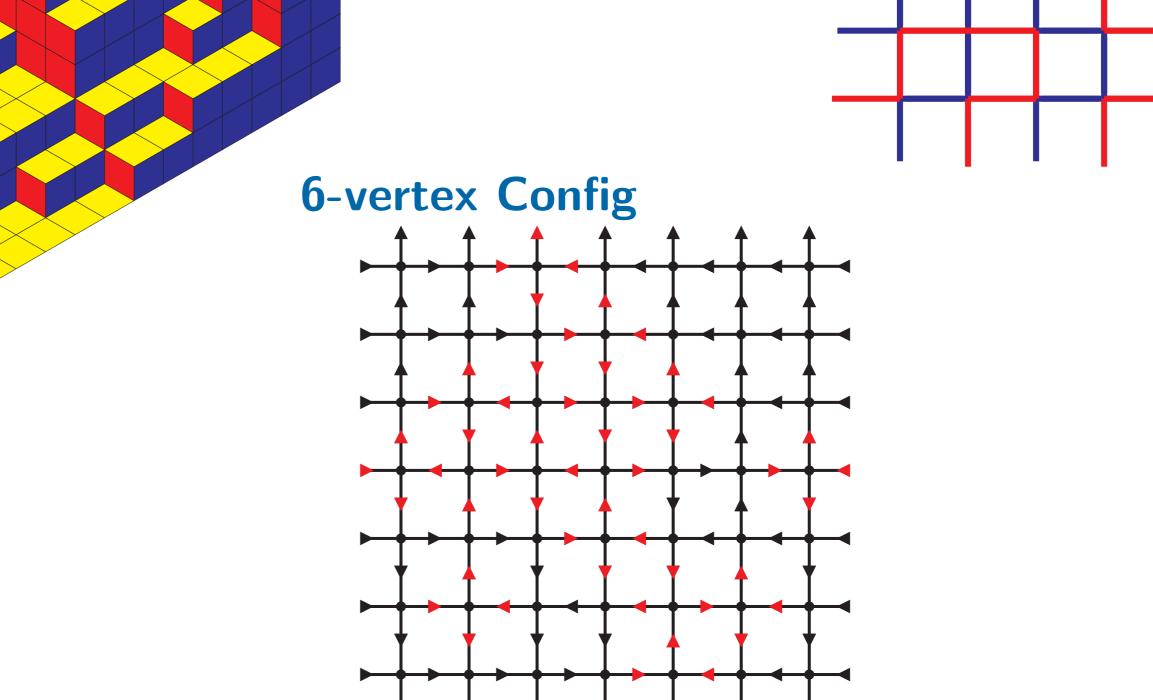


Square ice

A square ice of size *n* is an arrangement of n^2 water molecules, where the O-atoms are placed on an $n \times n$ grid with interlacing H-atoms and H-atoms at the left and right boundary.

TSSCPP

A totally symmetric self complementary plane partition of size *n* is filling of a $2n \times 2n \times 2n$ box with unit-cubes such that



Six-vertex configuration

A six-vertex configuration of size n is an $n \times n$ grid with n external edges together with an orientation of the edges such that

- every vertex has two edges pointing to it and two edges pointing
- the configuration stays invariant under reflection and interchanging the axes X,Y,Z,
- the "empty part" of the $2n \times 2n \times 2n$ box forms the same configuration.

away,

the left and right boundary edges are orientated inward and the bottom and top boundary are orientated outward.

What's known

- ► For *n* a positive integer and one of the above families, the number of objects of size n is $\prod_{i=0}^{n-1} \frac{(3i+1)!}{(n+i)!}.$
- ► We can relate ASMs, FPLs, square ice and six-vertex configurations.
- Explicite formulas for certain refined enumerations are known.

What's open

- Relate all different families.
- Transfer existing generalisations of one type to the others.