Constraining Exotic Interactions

Filip Ficek and Dmitry Budker**

Beyond-the-standard-model interactions mediated by an exchange of virtual "new" bosons result in a finite set of possible effective interaction potentials between standard-model particles such as electrons and nucleons. The classification of such potentials is discussed and recent experiments searching for such exotic interactions at spatial scales from sub-nanometers to tens of thousand kilometers are briefly reviewed.

1. Introduction

Modern physics acknowledges the existence of four fundamental interactions—strong, weak, electromagnetic, and gravitational. They vary in strengths and ranges, and for different physical systems some of them may be more important than the others (e.g., strong interactions inside baryons or gravitational interactions in the galaxy). In spite of the fact that there is no direct proof of existence of any other fundamental interaction (although there are many observations suggesting their existence, as we discussed in the next section), there is in principle no argument ruling out such a possibility. Instead, one can only constrain strengths of such hypothetical interactions using precise experimental measurements.

In this short article, which is intended as a brief introduction rather than a comprehensive review (see ref. [1] for a review on results of searches for exotic interactions based on the techniques of atomic, molecular, and optical physics), we present the basic ideas underlying searches for hypothetical interactions called "exotic interactions," as they may be present in extensions of the standard model. We begin with a presentation of a nonrelativistic framework used to deal with fundamental interactions carried by spin-0 and spin-1 bosons at low-energy scales and then we explore some of the systems used to give such constraints at various scales.

F. Ficek
Institute of Physics
Jagiellonian University
Łojasiewicza 11, 30-348, Kraków, Poland
E-mail: filip.ficek@uj.edu.pl
Prof. D. Budker
Helmholtz Institute Mainz
Johannes Gutenberg University
55099 Mainz, Germany
E-mail: budker@uni-mainz.de
Prof. D. Budker
Department of Physics
University of California at Berkeley
Berkeley, CA 94720-7300, USA
Prof. D. Budker
Nuclear Science Division
Lawrence Berkeley National Laboratory
Berkeley, CA 94720, USA

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2. Exotic Potentials

At this moment, we know that every fundamental interaction, except gravity for which a satisfactory quantum theory is not yet known, is carried by some interacting bosons—photons for electromagnetic, gluons for strong, and $Z^0/W^{\pm}$ bosons for weak interactions. We suspect that exotic interactions also would be carried by some, yet undiscovered bosons. Many modern physics puzzles, such as the nature of dark matter[2] and dark energy,[3,4] the strong-CP problem,[5] or the hierarchy problem,[6] may be explained by beyond standard model theories predicting the existence of such new bosons. The examples include axions,[7–12] familons,[13,14] majorons,[15,16] new spin-0 or spin-1 gravitons,[17–20] Kaluza–Klein zero modes in string theory,[21] paraphotons,[22–24] and new Z' bosons.[25–27] Despite the different nature of all these particles and the reasons the corresponding models were proposed, interactions they carry may be described within one, general framework introduced by Moody and Wilczek[5] and expanded by Dobrescu and Mocioiu.[28] We follow the lines of ref. [28] in order to introduce this framework.

Let us consider an interaction between two fermion particles mediated by a light boson with mass m_0, as shown in **Figure 1**. The particle 1 with initial momentum $p_{1,i}$ interacts with the particle 2 having initial momentum $p_{2,i}$. The interaction is carried by a boson with momentum q and as a result, the two particles carry out momenta $p_{1,f}$ and $p_{2,f}$, respectively. Let us consider this event in a center of mass of this system. Then due to the energy-momentum conservation, all the information about the kinematics of the collision is contained in the two momenta, $p_{1,i}$ and $p_{1,f}$. We are interested in low-energy interactions, as the higher-order relativistic corrections are negligible at atomic and larger scales which are our points of interest. Then, the rest mass dominates the particle energy and we may consider just the spatial parts $\mathbf{p}_{1,i}$ and $\mathbf{p}_{1,f}$ of the momenta $p_{1,f}$ and $p_{2,f}$. We construct out of them the mean momentum of one of the particles $\mathbf{P}$ and the difference in initial and final momenta for this particle $\mathbf{q}$ (which are equal in magnitudes to the respective quantities for the second particle)

$$\mathbf{P} = \frac{1}{2}(\mathbf{p}_{1,i} + \mathbf{p}_{1,f}) \tag{1}$$

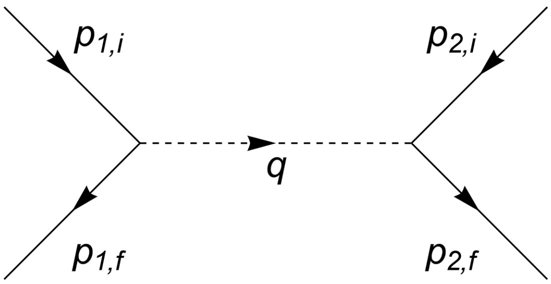

Figure 1. A Feynman diagram of two interacting fermions.

$$\mathbf{q} = \mathbf{p}_{1,i} - \mathbf{p}_{1,f} \tag{2}$$

If spins of the particles are $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively, then all the information about the collision is carried by four vectors: $\mathbf{P}$, $\mathbf{q}$, $\mathbf{s}_1$, and $\mathbf{s}_2$. Dobrescu and Mocioiu showed,[28] that any scalar constructed from these vectors can be presented as a linear combination of only 16 base scalars $\mathcal{O}_i$ with coefficients depending on $\mathbf{P}^2$ and $\mathbf{q}^2$. We consider them as interaction potentials written in momentum space. One of them does not include any of spins, that is, is algebraically equivalent to 1. Interactions described by this potential are usually called fifth forces[29] and are often considered in a context of modifications of Newtonian gravity.[30] Interactions coming from the other 15 potentials are spin-dependent and they divide into two groups: ones that do not include $\mathbf{P}$, called velocity-independent or static, and ones that include $\mathbf{P}$, called velocity-dependent.

Potentials in momentum space can be easily converted to a position space. As an example, we may consider a potential labeled in ref. [28] as $\mathcal{O}_3$. In momentum space we may write it as

$$\mathcal{O}_3 = \frac{1}{m_e^2}(\mathbf{s}_1 \cdot \mathbf{q})(\mathbf{s}_2 \cdot \mathbf{q}) \tag{3}$$

where the factor containing the electron mass m_e is introduced for dimensional reasons. It may be rewritten into position space by performing a Fourier transform with an appropriate propagator $\mathcal{P}$

$$\mathcal{V}_i = -\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \mathcal{P}\mathcal{O}_i \tag{4}$$

We are considering the exchange presented in Figure 1 within a Lorentz invariant quantum field theory, which fixes the form of the propagator to $\mathcal{P} = -\frac{1}{\mathbf{q}^2 + m_0^2}$.[28] In principle, other forms are possible, for example, coming from the exchange of two bosons instead of one or from Lorentz-symmetry violation. In the Lorentz-invariant, single-boson-exchange framework, we get an exotic potential of the form

$$\mathcal{V}_3 = -\frac{1}{4\pi}\left[\mathbf{s}_1 \cdot \mathbf{s}_2 \left(\frac{1}{\lambda r^2} + \frac{1}{r^3} + \frac{4\pi}{3}\delta^3(r)\right) - (\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})\left(\frac{1}{\lambda^2 r^3} + \frac{3}{\lambda r^4} + \frac{3}{r^5}\right)\right] e^{-r/\lambda} \tag{5}$$

where $\mathbf{r}$ is a vector connecting the interacting particles, r is its length (distance between the particles), and $\lambda = \hbar/m_0c$ is the Compton length of the interaction-mediating boson. We have included here a sign correction recently introduced by Daido and Takahashi.[31]

Filip Ficek is a Ph.D. student in theoretical physics at Jagiellonian University, Kraków, Poland. His research interests focus on general relativity and mathematical physics, and he is also involved in the search for constraints on "beyond-standard-model" interactions, in collaboration with the Budker Group in Mainz, Germany.

Dmitry Budker obtained his Ph.D. in physics from UC Berkeley, CA, USA in 1993. He is currently the leader of the Matter-Antimatter Asymmetry section of the Helmholtz Institute, Johannes Gutenberg University, Mainz, Germany and Professor of the Graduate School at UC Berkeley. His research interests are in experimental tests of fundamental symmetries, magnetic resonance, the laboratory search for dark matter, atomic spectroscopy, and precision electromagnetic measurements.

To obtain a final form of the exotic potential in position space, we need to give it a proper dimension by inserting an overall constant. Dimensional analysis yields $\hbar^3/m_e^2c$, where $\hbar$ is the reduced Planck constant and c is the speed of light, as a correct combination. Additionally, we put a dimensionless coupling constant f_3^{12} which represents the strength of this interaction and may depend on interacting particles (hence the index 12 referring to particle 1 and particle 2). The coupling coefficients f_3^{12} (often written as $g_3^1 g_3^2/4\pi\hbar c$) are determined by experimental searches. In the end we get

$$V_3 = -f_3^{12}\frac{\hbar^3}{4\pi m_e^2 c}\left[\mathbf{s}_1 \cdot \mathbf{s}_2 \left(\frac{1}{\lambda r^2} + \frac{1}{r^3} + \frac{4\pi}{3}\delta^3(r)\right) - (\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})\left(\frac{1}{\lambda^2 r^3} + \frac{3}{\lambda r^4} + \frac{3}{r^5}\right)\right] e^{-r/\lambda} \tag{6}$$

This potential, usually called a pseudovector dipole–dipole potential, was for the first time considered by Moody and Wilczek in ref. [5] and may be associated, for example, with an exchange of an axion. Another often considered dipole–dipole potential comes from an exchange of an axial-vector particle and has the form of

$$V_2 = f_2^{12}\frac{\hbar c}{\pi}(\mathbf{s}_1 \cdot \mathbf{s}_2)\frac{e^{-r/\lambda}}{r} \tag{7}$$

The procedure that gave us Equation (5) may be repeated for the remaining 15 scalars, although one should be cautious when dealing with velocity-dependent potentials. After performing the Fourier transformation for these operators, Dobrescu and Mocioiu[28] kept vectors $\mathbf{P}$ as variables instead of changing them into operators related to gradient. As realized by M. G. Kozlov,[32] this gives the potentials in some kind of mixed representation, which may be used at the laboratory scale (as considered in ref. [28]), but is not suitable for atomic scales. As an example, one may take the momentum space operator

$$\mathcal{O}_8 = \frac{1}{m_e^2}(\mathbf{s}_1 \cdot \mathbf{P})(\mathbf{s}_2 \cdot \mathbf{P}) \tag{8}$$

and perform Fourier transformation obtaining[28]

$$\mathcal{V}_8 = \frac{1}{4\pi r}(\mathbf{s}_1 \cdot \mathbf{P})(\mathbf{s}_2 \cdot \mathbf{P})e^{-r/\lambda} \tag{9}$$

In order to get correct position space forms of velocity-dependent potentials, one needs to perform an additional antisymmetrization, as described by Ficek et al.[32] Then Equation (9) transforms to

$$V_8 = -\, f_8^{12}\frac{\hbar^3}{4\pi m_e^2 c}\left\{\mathbf{s}_1 \cdot \left(\frac{m_1}{m_1+m_2}\nabla_2 - \frac{m_2}{m_1+m_2}\nabla_1\right)\right.$$
$$\left.\left\{\mathbf{s}_2 \cdot \left(\frac{m_1}{m_1+m_2}\nabla_2 - \frac{m_2}{m_1+m_2}\nabla_1\right), e^{-r/\lambda} r\right\}\right\} \tag{10}$$

where $\{\cdot,\cdot\}$ denotes anticommutator. The full list of potentials can be found in ref. [1].

Let us point out that potentials obtained by the described method come from very general principles, so it is worth considering whether all of them have some physical interpretation. Recently Fadeev et.al.[33] performed an alternative construction of exotic potentials. One may start from the most general Lorentz-invariant Lagrangian describing interactions between standard-model fermions and spin-0 or spin-1 bosons. For example, in a scalar sector such Lagrangian has the form

$$\mathcal{L}_\phi = \phi\bar{\psi}\left(g_\psi^s + i\gamma_5 g_\psi^p\right)\psi \tag{11}$$

where ψ is a fermionic field, ϕ is a scalar field, γ_5 is a Dirac matrix, and g_ψ^s, g_ψ^p parameterize interaction strengths (the first one applies to P-even, hence s as scalar, and the second to P-odd interactions, hence p, as pseudoscalar). Similar terms may be written for massless and massive spin-1 particles giving six parameters in total. Investigating the vertices of the Feynman diagram presented in Figure 1 with this general Lagrangian yields potentials that can be considered in a nonrelativistic limit. These limits happen to be linear combinations of the potentials obtained by Dobrescu and Mocioiu, although not all 16 of them are present. This suggests that some of the $\mathcal{O}_i$ scalars have no physical significance. The additional result coming from this alternative approach is the fact that not all of the dimensionless coupling constants $f_i^{\psi\psi}$ are independent—they can be expressed as combinations of g_ψ^s, g_ψ^p, and the remaining four parameters mentioned above.

Every exotic potential presented by Dobrescu and Mocioiu[28] contains exponential factor $\exp(-r/\lambda)$ suppressing the interaction at scales higher than the Compton wavelength λ of the mediating boson. This means, that the boson mass m_0 determines the characteristic scale of interaction and, as an effect, investigating different physical systems may give us constraints on exotic interactions carried by bosons with different masses. In the next section, we review some of physical systems yielding constraints at different mass scales.

3. Methods

In this section, we present several experimental methods used to obtain constraints on exotic interactions. The common idea behind them all consists of performing an experiment and then comparing its results with standard (e.g., QED based) theoretical predictions in order to find any deviations or at least determine the uncertainties to which the agreement between theory and observations can be established. The difference between experimental results and theoretical predictions gives us a window where some additional exotic interactions may fit (the narrower the window, the more stringent the final constraints). By calculating the influence of hypothetical exotic interactions on the results of experiments, we may search for them, and either find something or obtain constraints on the appropriate coupling constants. We show how this procedure works in a particular case in the next section.

As mentioned at the end of the last section, experiments performed at different scales are affected by forces mediated by bosons with different masses. Because of this, we need to utilize experiments working at various scales to properly investigate the exotic interaction parameter space. Also some experimental setups may be sensitive to different kinds of exotic interactions, such as spin-dependent or velocity-dependent, while others are not, which highlights the need for large diversity of investigated systems.

In the following sections, we discuss four experimental methods yielding constraints on spin-dependent exotic interactions at various scales, from nanometers up to thousands of kilometers (or from 1 keV down to 10^{-12} eV, equivalently). We present the constraints on coupling constants for axial-vector and pseudoscalar dipole–dipole interactions between electrons ($|f_2^{ee}|$ and $|f_3^{ee}|$, respectively) where possible. More detailed descriptions of the results together with limits on other potentials may be found in the cited references. Apart from the experimental techniques presented below, there is a variety of others, for example, trapped ions experiments,[34] molecular spectroscopy,[35,36] measurements of the spin precession of atomic gases.[37,38] There are also new ideas such as, for example, a scanning tunneling microscopy.[39] A comprehensive review of searches of exotic interactions with atomic and molecular experiments can be found in ref. [1]. Also, one can find a list of constraints on some coupling constants at different scales in *The Review of Particle Physics*.[40]

Let us also mention that the strength of exotic forces is constrained not only with earthbound experiments. Such hypothetical interactions would influence many astrophysical processes and their impact should be visible in astronomical observations. By comparing the predictions regarding such observables as red-giant cooling rate[41,42] or strength of neutrino flux from supernovae[43] with observational evidence, it is possible to put strong limits on the possible new interactions. However, one has to keep in mind, that some of these results heavily rely on the astrophysical models employed.

3.1. Atomic Spectroscopy

Precision levels achieved by modern atomic spectroscopy and QED-based theoretical calculations, together with a good agreement between them, let us obtain stringent constraints on exotic interactions at the atomic scale. The diversity of exotic atoms permits us to search for exotic interactions strengths between various particles. The existing results include limits on interactions at the atomic scale between two electrons (from helium[32]), electron and positon (from positronium[34]), electron and antimuon (from muonium[44,45]), or electron and antiproton (from antiprotonic helium[46]). Details regarding searches for exotic interactions vary from system to system, so, as an example, in the remaining part of this section we focus on limits on exotic interactions between electrons coming from helium fine structure, as described by Ficek et al.[32]

Let us investigate the $n = 2$ state (where n is the principal quantum number) of orthohelium. It consists of a metastable state 2^3S and a triplet of 2^3P states. Transition energies between these states have been precisely measured.[47,48] These frequencies may be compared with QED-based calculations[49,50] (whose precision is lower than that of experimental data) to reveal that they agree within the uncertainties. It suggests that possible exotic interactions must fit in these uncertainties. Let us focus on one of the transitions. We want to define a quantity characterising the level of agreement between theory and experiment taking into account the uncertainties, called ΔE from now on. If we denote by μ the mean difference between its theoretical and experimental frequencies and also we define $\sigma = \sqrt{\sigma_{\rm th}^2 + \sigma_{\rm exp}^2}$ (where $\sigma_{\rm th}$ and $\sigma_{\rm exp}$ are theoretical and experimental uncertainties, respectively), we may introduce ΔE as a number such that (cf. Equations (A1) and (A2) of ref. [32] where, apart from the typo in Equation (A2), an equivalent definition of ΔE is given)

$$\int_{-\Delta E}^{\Delta E} \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} dx = 0.9 \tag{12}$$

This number may be calculated for every transition and can be interpreted as a maximal possible energy shift caused by exotic interactions for this transition (at 90% acceptance level). Let us now point out that we can factor out the coupling constant f_i^{ee} from every exotic potential V_i getting $V_i = f_i^{ee} U_i$, where U_i is a well-defined operator. We now consider a transition between states A and B, characterized by electron wavefunctions $|\psi_A\rangle$ and $\psi_B\rangle$, respectively. Exotic potential V_i shifts energy of the state A by $\langle\psi_A|V_i|\psi_A\rangle = f_i^{ee}\langle\psi_A|U_i|\psi_A\rangle$, and analogously for the state B. It means, that the total change in frequency for a transition A$\leftrightarrow$B caused by potential V_i is $|\langle\psi_A|V_i|\psi_A\rangle - \langle\psi_B|V_i|\psi_B\rangle|$. This quantity cannot be larger than ΔE. Connecting all these information, we arrive at the final expression

$$|f_i^{ee}| \le \frac{\Delta E}{|\langle\psi_A|U_i|\psi_A\rangle - \langle\psi_B|U_i|\psi_B\rangle|} \tag{13}$$

By performing these steps for appropriate transitions within helium fine structure, one can obtain limits $|f_2^{ee}| \le 10^{-9}$ and $|f_3^{ee}| \le 3 \times 10^{-8}$ at the scale of 1 nm (1 keV).[32]

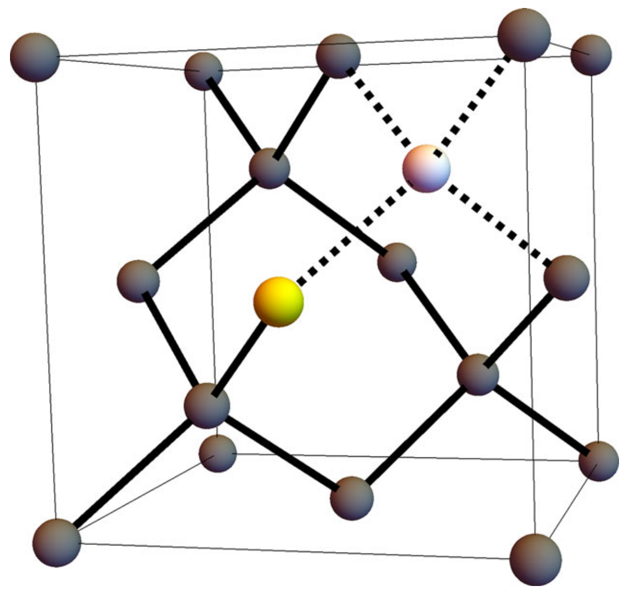

Figure 2. Atomic structure of a nitrogen-vacancy center in diamond. The gray balls are carbon atoms, the yellow ball is a nitrogen atom, while the white ball symbolises the vacancy.

3.2. Nitrogen-Vacancy Centers in Diamond

Nitrogen-vacancy (NV) centers are point defects in diamond structure. These occur when a pair of neighboring carbon atoms are substituted with a single nitrogen atom and a vacancy (**Figure 2**). They have broad applications in quantum information, metrology, and nanotechnology.[51] They can also be used to measure magnetic fields with nanoscale resolution.[52] Because spin-dependent exotic interactions couple to the matter in a way similar to a magnetic field, this last property, together with the possibility of isolation of magnetic noise, suggests that NV centers may be used to search for exotic interactions.[53,54]

In two recently published articles, Xing Rong and his collaborators described how they used a single NV center as a quantum sensor to constrain axion-mediated monopole–dipole interactions between electron and nucleon[53] and vector-mediated dipole–dipole interactions between electrons.[54] Results obtained for the latter interaction allowed to obtain a limit for a dimensionless coupling constant $|f_2^{ee}| \le 5.7 \times 10^{-19}$ at the scale of 500 μm (2.5 meV).

3.3. Torsion Balance

In the original paper by Moody and Wilczek,[5] the authors proposed constraining exotic interactions with the use of techniques of experimental gravity, specifically, precise torsion-balance measurements. Such experiments are based on ideas similar to the ones behind the famous Cavendish experiment,[55] shown schematically in **Figure 3**. Basically, two 0.73 kg lead spheres (inside $ABCD$ boxes) were attached to the opposite ends of horizontally suspended, 1.8 m wooden rod m and located 23 cm away from two 158 kg lead spheres W acting as weak sources of gravitational attraction. The rod with the small balls twists to the angle

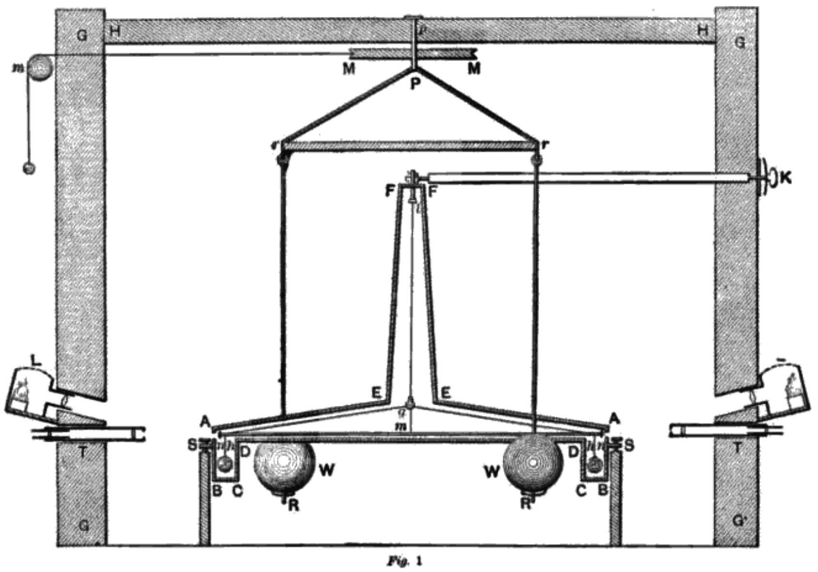

Figure 3. Torsion balance used in the original Cavendish experiment.[55]

where a torque coming from the aforementioned gravitational force is balanced by the torque exerted by the spring. This system, initially used to find Earth's mean density[56] (which could be converted to the value of gravitational constant), after some changes may search for deviations from Newton's inverse-square law. Such deviations could come from yet undiscovered forces, rather then being connected to the nature of gravity. They would be results of spin-independent fifth-forces, as both source and detector in this setup are unpolarized, and such experiments may yield constraints on their strengths.[30,57–59]

After further modifications to the torsion balance, it is also possible to search for constraints on spin-dependent exotic forces. Such setup must not only contain polarized test bodies and sources, but also should be shielded from any external magnetic fields. Examples of such apparatus come from the Eöt-Wash Group at the University of Washington, where they were used to constrain various types of spin-dependent interactions, including CP-violating forces,[60] axion mediated forces,[61] and spin–spin interactions between electrons at various length scales.[62,63] The most recent results come from a system utilizing a 4 cm wide ring containing 20 magnetized segments of alternating high and low spin-density materials.[62] This setup allows for a great reduction of an influence of external magnetic fields, while keeping sensitivity to exotic spin-dependent forces thanks to variations in spin density. The results coming from these experiments yield constraints on the coupling constants being $|f_2^{ee}| \leq 5.1 \times 10^{-40}$ and $|f_3^{ee}| \leq 1.4 \times 10^{-17}$ at the scale of 40 mm (30 μeV).

3.4. Geoelectrons

The methods described in the two previous sections rely on experimental setups, where both the "source" of the exotic force and the "detector" sensitive to this force are situated in a laboratory. One may use another approach, where the "source" is located outside the laboratory. As an example of realization of this idea, we discuss the use of geoelectrons, that is, polarized electrons within the Earth.

The authors of refs. [64,65] constrained several spin-dependent and velocity-dependent potentials at planetary scales by comparing results of local Lorentz-invariance searches[66,67] with an electron spin density map constructed by the authors. With the use of recent advances in fields such as geophysics, seismology, or mineral physics, it was possible to model temperature, magnetic field, and density of unpaired electrons within the Earth, and to ultimately obtain a complete map of electron spin density. Then, appropriate integrations over the whole planet volume yielded estimates for the possible influences of exotic interactions coming from geoelectrons. Finally, by comparing these estimates and the experimental data, it was possible to obtain stringent constraints on various coupling constants,[64,65] such as $|f_2^{ee}| \leq 5.7 \times 10^{-47}$ at the scale of 10 000 km (1.2×10^{-12} eV).

4. Summary and Outlook

In this brief paper, we provided a glimpse of the theory underlying the ongoing searches for exotic interactions and gave several examples of searches spanning a broad range of spatial scales, from the atomic subnanometer scale all the way to the planetary scale of tens of thousand kilometers. Such experiments provide a powerful way to look for physics beyond the standard model. At the same time, they constitute an indirect search for possible components of dark matter and dark energy. There are all indications that we will see significant improvement in the sensitivity of these methods in the coming years via a combination of improvements in the sensitivity of the experiments combined (where necessary) with higher-accuracy theory.

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Conflict of Interest

The authors declare no conflict of interest.

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