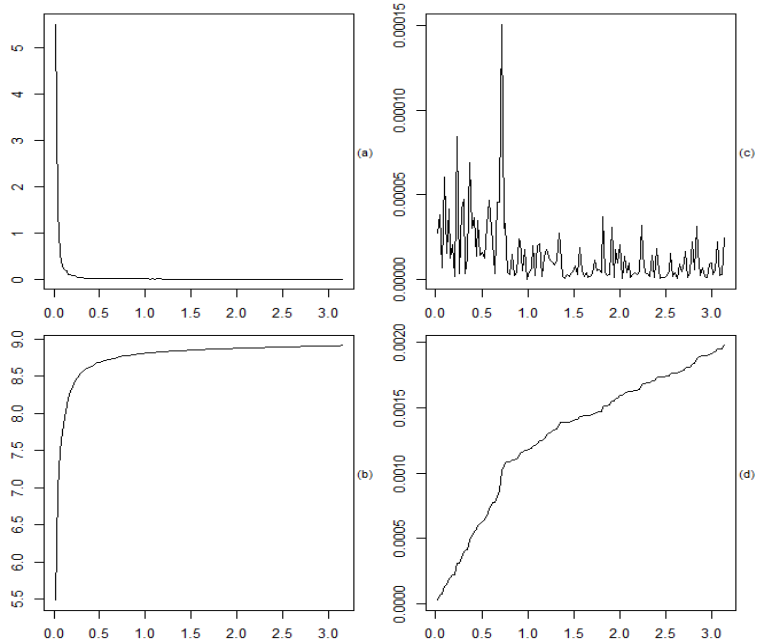


Figure 2.1 (a) Periodogram of quarterly log U.S. GDP
(b) Cumulative periodogram of quarterly log U.S. GDP
(c) Periodogram of first differences of quarterly log U.S. GDP
(d) Cumulative periodogram of first differences

Table 2.1: Testing the differenced log GDP for white noise

| $j$ | $j$'th frequency | Sum of periodogram at $j$ lowest frequencies times $4\pi/s^2$ | 0.95-quantile of $\chi^2(2\times j)$ | p-value |
|---|---|---|---|---|
| 10 | 0.2309995 | 42.65310 | 31.41043 | 0.002271 |
| 20 | 0.4619989 | 82.30326 | 55.75848 | 0.000094 |
| 30 | 0.6929984 | 117.95660 | 79.08194 | 0.000012 |
| 40 | 0.9239978 | 158.67368 | 101.87947 | 0.000000 |
| 50 | 1.1549973 | 171.99232 | 124.34211 | 0.000010 |

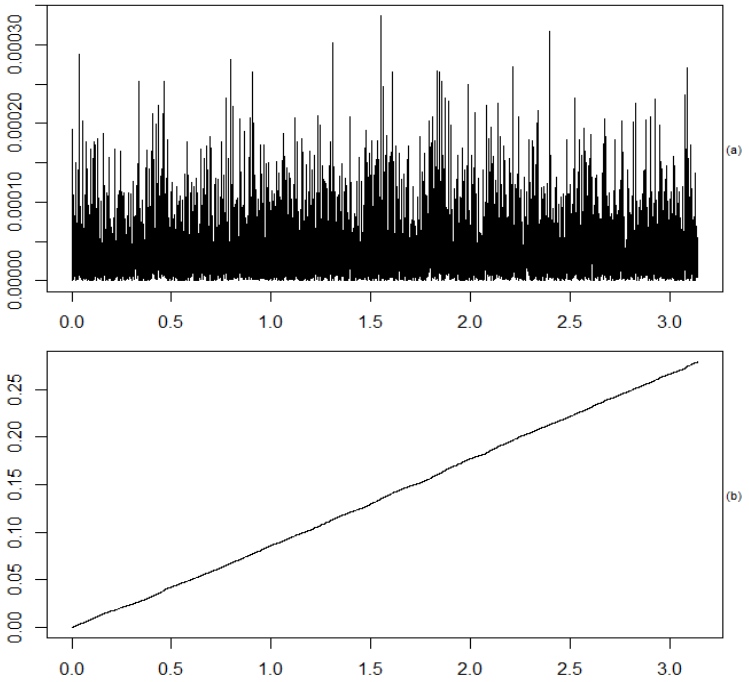


Figure 2.2 (a) Periodogram of daily log returns of IBM
(b) Cumulative periodogram of daily log returns of IBM

Table 2.2: Testing the daily log returns of IBM for white noise

| $j$ | $j$'th frequency | Sum of periodogram at $j$ lowest frequencies times $4\pi/s^2$ | 0.95-quantile of $\chi^2(2\times j)$ | p-value |
|---|---|---|---|---|
| 1000 | 0.4636353 | 1908.567 | 2105.154 | 0.92754 |
| 2000 | 0.9272706 | 3836.035 | 4148.248 | 0.96794 |
| 3000 | 1.3909058 | 5842.042 | 6181.315 | 0.92629 |
| 4000 | 1.8545411 | 7888.392 | 8209.190 | 0.81089 |
| 5000 | 2.3181764 | 9998.937 | 10233.749 | 0.50112 |

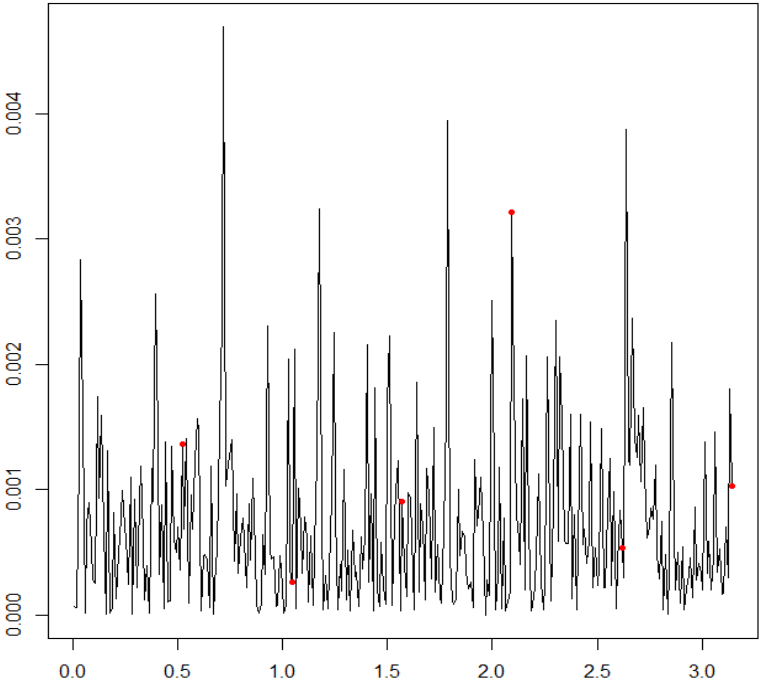


Figure 2.3: Periodogram of monthly log returns of IBM
(Dec1962-Nov2015, red circles: seasonal frequencies)

Table 2.3: Testing the monthly log returns of IBM for seasonal patterns

| $j$ | Sum of periodogram ordinates at first $j$ seasonal frequencies times $4\pi/s^2$ | Degrees of freedom | p-value |
|---|---|---|---|
| 1 | 3.707617 | 2 | 0.15664 |
| 3 | 6.880388 | 6 | 0.33205 |
| 6 | 18.4504 | 11 | 0.07170 |

Periodogram of y:

```
h <- spec.pgram(y,taper=0,detrend=FALSE,fast=FALSE,
                                              plot=FALSE)
fr <- 2*pi*h$freq; pg <- h$spec/(2*pi)
```

or

```
m <- floor(n/2); fr <- (2*pi/n)*(1:m)
ft <- fft(y)[2:(m+1)]  # only 1,2,…,m
                       # excl. 0,m+1,m+2,...,n-1
pg <- (1/(2*pi*n))*(Mod(ft))^2
```

Figure 2.3: see *Spectral analysis*, p. 1

Tables:

Test statistic excl. $\pi$: see *Spectral analysis*, p. 6a

Test statistic incl. $\pi$: see *Spectral analysis*, p. 6b