

Homological Algebra

Dietrich Burde

Lecture Notes 2022

Contents

Chapter 1. Introduction	1
Chapter 2. Rings and modules	3
2.1. Definition of ring and module	3
2.2. Actions on rings and modules	5
2.3. Free, projective, injective and flat modules	8
Chapter 3. Categories and functors	17
3.1. Categories	17
3.2. Abelian categories	26
Chapter 4. Resolutions and derived functors	29
4.1. Projective and injective resolutions	29
4.2. Homology and homotopy	31
4.3. The fundamental theorem of homological algebra	33
4.4. The long exact sequence in homology	36
4.5. The functors Tor and Ext	40
4.6. Double complexes	42
4.7. The Yoneda Ext functor	46
Chapter 5. Homology and cohomology of groups	53
5.1. Functorial definition of group homology and cohomology	53
5.2. The bar resolution	56
5.3. Group cohomology by explicit coboundary map	59
5.4. The zeroth cohomology group	62
5.5. The first cohomology group	62
5.6. Inflation, restriction and the cup product	66
Bibliography	79