

# When is the sky the limit? Multi-dimensional room acoustic analysis of auditoria: The height dimension of concert halls and opera houses

Klaus-Hendrik Lorenz-Kierakiewitz<sup>1</sup>, Christoph Reuter<sup>2</sup>  
Benjamin Pfändner<sup>3</sup>, Chris Erkal<sup>1</sup>

<sup>1</sup> Peutz Consult GmbH, D-40599 Düsseldorf, E-Mail: khl@peutz.de

<sup>2</sup> SInES, Musicology Department, University of Vienna, A-1090 Vienna, E-Mail: christoph.reuter@univie.ac.at

<sup>3</sup> Peutz Consult GmbH, D-90433 Nürnberg, E-Mail: bp@peutz.de

## Abstract

In 2017, a novel methodology for the analysis of the room acoustic properties of various room types, including concert halls and opera houses, was proposed. This methodology employs a multi-dimensional approach, combining rapid visualisations of parameter values in up to six dimensions, such as room volume, seating capacity, room width, reverberation time, clarity and strength. Over the past eight years, this method has undergone continuous development and refinement, with the underlying database being expanded successively to currently 550 data sets ([1], [2], [3], [4]).

Combined with the ever-increasing capabilities of computer-based data analysis packages, interesting results have been obtained regarding the acoustic quality of rooms, particularly of rehearsal, concert and opera halls, by correlating room acoustic parameters with subjective preference ratings, which are available for many of these halls. However, the dimension of room height has not yet been included in the analyses. The aim of this paper is to complement and complete the results obtained by including the room height in the analysis of the acoustic quality of concert halls and opera houses.

## Introduction

This analysis method has been used in the previous studies to show that rooms with different uses form specific clusters based on their acoustic values in three- or multi-dimensional visualisations of the parameter space. In these type clusters, rooms which are rated as particularly good in terms of audibility have characteristic focal points (centroids) and value ranges [4]. This method can be used to investigate in more detail the ranges of characteristic values within which concert and opera halls are considered to be particularly good in terms of acoustics and audibility, in order to draw general conclusions about the objective requirements for concert and opera hall acoustics.

In principle, these questions and some results are presented in [4] for concert halls and [5] for opera houses, but the height dimension has not yet been taken into account in these evaluations. However, this will be the case in the evaluations presented in this contribution.

## Remarks on audibility

The generally recognised necessary prerequisite for good audibility appropriate to the intended use of the room, is a value of the reverberation time  $T$  in an appropriate range.

The previous studies have shown that another necessary condition for good audibility is that the value of strength  $G$  has to be in an appropriate range for the use as well.

In addition to these two necessary conditions, there are also sufficient conditions for good audibility, namely that the values of the other room acoustic parameters, such as EDT, clarity, BQI and  $AL_{\text{cons}}$ , are also within a suitable range for the use of the room.

It can be implicitly deduced from this, that a room in one fixed current state cannot be as good for one use as it is for the other, unless the values of the room acoustic parameters, and thus also the architectural details, can be varied in the room.

## Influence of the consultant

Design parameters that theoretically can be influenced include the room volume ( $V$ ), the geometric shape of the room, the room width ( $W$ ), the room height ( $H$ ), the maximum distance from the edge of the stage, the absorption ( $A$ ) by the seating capacity (number of seats), as well as the fine geometry and the reflection and scattering characteristics of the boundary surfaces.

In practice, these design parameters should ideally be guided by acoustic advice from the very first sketch. This would be the ideal situation to strive for, especially when high demands are placed on the desired audibility – unfortunately, the reality all too often is somewhat different.

However, especially at an early stage of design, for planning purposes it is very useful and effective in terms of room acoustic quality to be able to compare global data with that of other halls that have been rated as good, to intervene directly if necessary and to easily visualise the acoustic effects of measures.

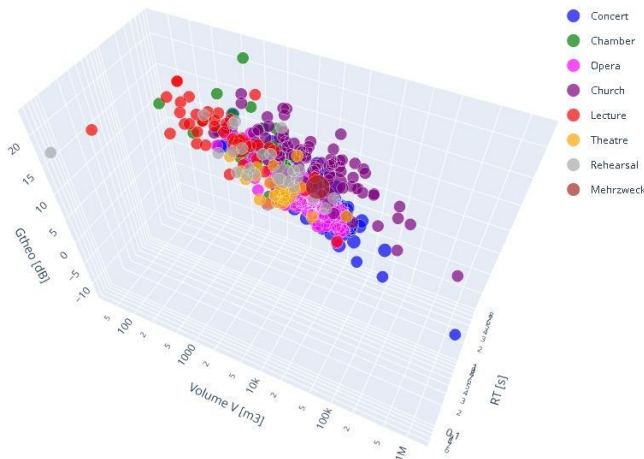
## Database of the study

When the method of 3+ dimensional analysis of room acoustic parameter values was presented at the DAGA 2017 [1], the analysed database consisted of only 104 data sets (or N-tuples in parameter space), with each N-tuple representing a room or room state. In the meantime, this database was expanded to 550 data sets, including 190 data sets of concert halls (150 with height data) and 130 data sets on the room acoustics of opera houses (60 with height data), including more than 200 data sets from own room acoustic measurements. All other data were taken from the relevant literature, in particular [7] and [8].

For this contribution, the data sets have been supplemented with height data per room found in the literature or in sectional drawings.

Figure 1 shows a view of the 550 data sets in the database currently available in a  $T_{\text{occ},3} - V - G_{\text{theo}}$  plot [2].

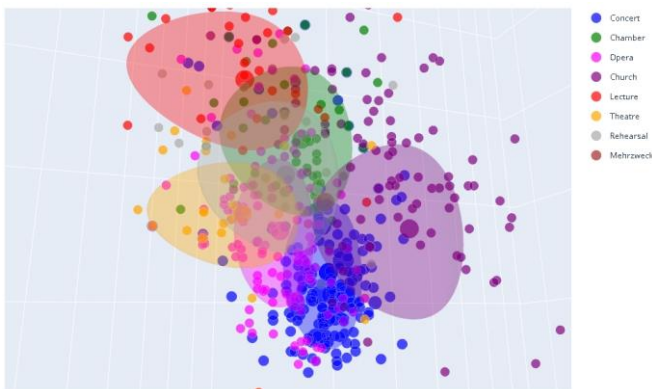
The marker colour indicates the type of room use resp. type:



**Figure 1:** View of the 550 data sets currently available in a T-V- $G_{theo}$  plot. Marker colours: room type (**concert halls**, **chamber music halls**, **opera houses**, **churches**, **theatres**, **lecture halls**, **rehearsal halls**).

Using this approach, it can be shown that the different types of room usages clearly form clusters in characteristic areas of parameter space, with the values of the centroids forming a separate centre for each room type, see fig. 2.

The centroids are surrounded by characteristic areas of the parameter space defined by the standard deviations  $\pm\sigma$  (= perimeter) of the respective parameter and show intersections between neighbouring room types. Fig. 2 shows all data sets with spatial areas around the centroids, see [3].



**Figure 2:** Detailed view of all data records in the T-V- $G_{theo}$  plot. Marker colours: Room type (**concert halls**, **chamber music halls**, **opera houses**, **churches**, **theatres**, **lecture halls**, **rehearsal halls**). Coloured Areas: room type- Centroids  $\pm\sigma$ .

The characteristic area for concert halls (blue) with the surrounding standard deviation area around it is shown in blue, that for opera houses in pink, and they show an intersection.

This approach in a Cartesian coordinate system for the three main variables assumes, without limiting the generality of the statement, that the variables spanning the parameter space should be independent of each other, which is not necessarily the case for room acoustic parameters.

Although the variables volume, reverberation time and strength are not independent, they provide a first base for further investigations, especially as a theoretical strength  $G_{theo}$  can be calculated for all data sets from the information on room volume and reverberation time – assuming a diffuse sound field – according to the revised diffuse field theory of Kuttruff and Barron [9].

## Application in concert and opera hall acoustics

How can concert and opera halls with particularly good acoustics be identified? This question was the starting point for these studies. But, how does the height dimension fit in? The role of the room height  $H$  remains to be investigated.

To answer the question of the audibility of concert and opera houses, it is not sufficient to analyse only the objective data of the rooms in question; as with all similar questions concerning the assessment of the acoustic quality and suitability of a room, a correlation with subjective assessments has to be made.

For concert halls and opera houses, the aforementioned studies by Hidaka and Beranek are particularly useful [7],[8].

Fig. 3 shows an excerpt from the current data basis of the concert halls and opera houses with a quality ranking available and room height values added.

ID	City	Year	Room Type	Volume V (m³)	RT (s)	$G_{theo}$ (dB)	Quality Rating	Room Height H (m)
concert001	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert002	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert003	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert004	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert005	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert006	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert007	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert008	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert009	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert010	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert011	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert012	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert013	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert014	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert015	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert016	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert017	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert018	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert019	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert020	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert021	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert022	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert023	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert024	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert025	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert026	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert027	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert028	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert029	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert030	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert031	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert032	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert033	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert034	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert035	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert036	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert037	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert038	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert039	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert040	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert041	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert042	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert043	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert044	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert045	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert046	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert047	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert048	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert049	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert050	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert051	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert052	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert053	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert054	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert055	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert056	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert057	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert058	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert059	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert060	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert061	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert062	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert063	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert064	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert065	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert066	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert067	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert068	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert069	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert070	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert071	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert072	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert073	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert074	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert075	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert076	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert077	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert078	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert079	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert080	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert081	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert082	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert083	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert084	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert085	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert086	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert087	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert088	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert089	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert090	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert091	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert092	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert093	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert094	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert095	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert096	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert097	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert098	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert099	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0
concert100	Vienna	1869	Concert Hall	10700	2.0	10.5	1.0	12.0

**Figure 3:** View of the available data sets of concert halls and opera houses with quality ranking and room height. Font colours indicate: room type (**concert halls** and **opera houses**).

## Correlation analysis

With the aforementioned approach in a Cartesian coordinate system, the focus is on the visualisation and rapid visual identification of previously unknown relationships.

In the context of the complex phenomenon of audibility, it is not trivial to find independent variables to form a three- or multi-dimensional representation to span a parameter space, especially since not all data are consistently available for all data sets. It would be very interesting to extend the investigation to the lateral sound level, but unfortunately there is even less data for this particular parameter.

In order to gain further information from the available data of the concert halls and opera houses, the data sets, including the ranking and now the added room height  $H$  values, were subjected to a correlation analysis by using JASP.

With all due caution about spurious and hidden linear correlations (Pearson), the following conclusions can be drawn:

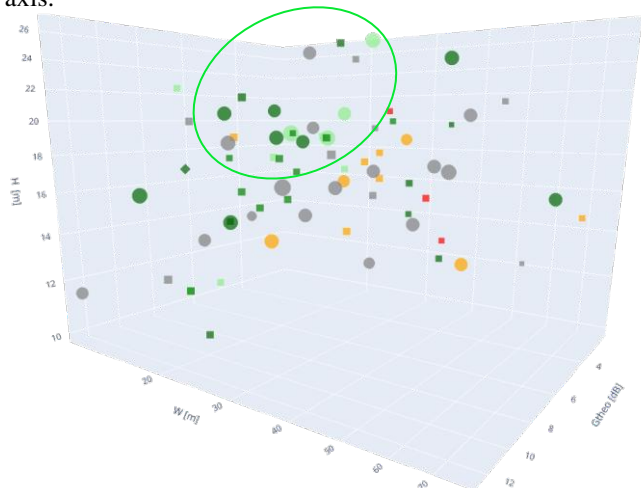
- In the concert hall publication by Leo Beranek [5], the quality rating of a concert hall correlates with higher values of measured strength  $G_{meas}$  ( $r$  0.648;  $p < 0.001$ ) and Binaural Quality Index BQI ( $r$  0.708;  $p < 0.001$ ), as well as with smaller room widths ( $r$  - 0.438;  $p$  0.001), lower clarity values ( $r$  - 0.4533;  $p < 0.001$ ) and a lower number of seats ( $r$  - 0.455;  $p < 0.001$ ).

- In the opera publication by Hidaka and Beranek [8], the quality rating of an opera hall correlates with the specific room volume  $V/N$  ( $r$  0.513;  $p$  0.025).

### Evaluation taking into account room height H

Again – how does the room height H fit into this picture?

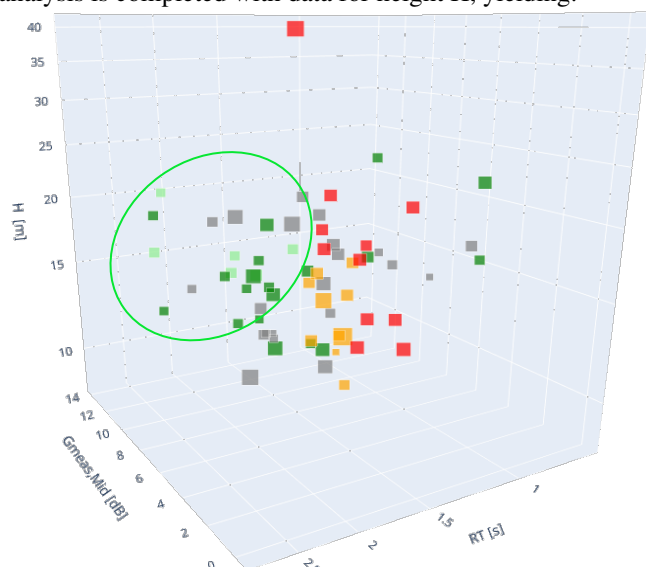
Figure 4 shows the available data sets of concert halls and opera houses with BQI, ranking and the room height on z-axis:



**Figure 4:**  $G_{theo}$ -W-H plot of concert halls and opera houses. Marker colours indicate quality rating (light green: excellent; dark green: good; orange: fair; red: less favourable; grey: not rated), marker: room type (concert halls: squares; opera houses: circles), marker size: ~ BQI values.

From fig. 4, it is obvious that the rooms with the highest quality rating pile up and can be found in the left upper corner in this view. For both room types, the better rated rooms are less than 30 m wide, all higher than 15 m, but less than 27 m.

In order to obtain more detailed results for both room types, they can be examined separately. For concert halls alone, the analysis is completed with data for height H, yielding:

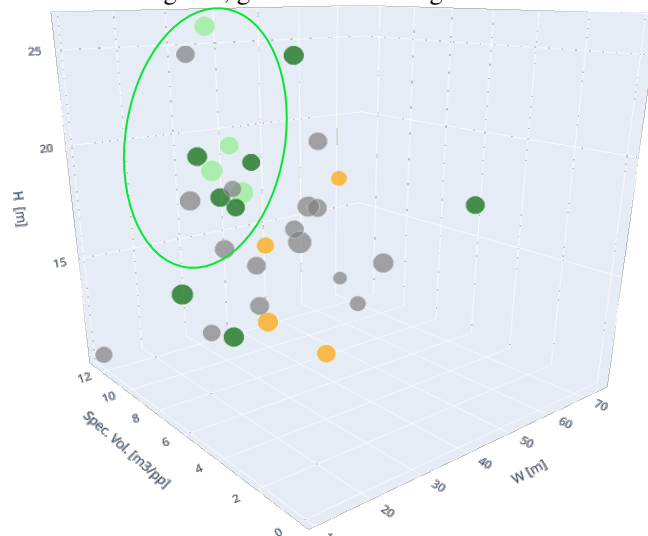


**Figure 5:** View of concert halls in a  $G_{meas}$ -RT<sub>occ3</sub>-H plot. Marker colours indicate the quality rating as in figure 4, marker size indicates the room width W.

In fig. 5 it can be seen, that the concert halls with the highest quality rating are located in the middle left part of the plot:

The best rated concert halls are less wide than 26 m and all higher than 15 m, but less than 27 m.

For the opera houses separately, this analysis, completed with data for the height H, gives the following results:



**Figure 6:** View of the opera halls with room height H in a W-Spec.Vol.-H Plot. Marker colours indicate rating as in fig. 4, marker size indicates the BQI value.

From fig. 6 it can be seen, that the opera houses with highest quality rating are located in the upper left part of the plot: The best-rated opera houses are all higher than 19 m and less than 29 m wide.

### Results. When is the sky the limit?

The investigation of room acoustic parameter values of 190 concert halls and 130 opera houses using the method of 3+ dimensional visual data analysis gives the following results regarding the question of suitable conditions for achieving a very good rated room acoustic quality in concert halls, taking into account the height (H):

#### Concert halls rated as particularly good

According to the investigations carried out so far on concert halls, the following ranges of average room values for the room acoustic parameters with distances from the source larger than two critical distances  $r_H$  are suitable target-oriented conditions for audibility rated as very good to excellent, at least according to Leo Beranek's quality ranking [7]:

Reverberation time T:	1.8 s	< T <sub>occ,3</sub> < 2.4 s	^
Strength G:	+5 dB	< G <sub>meas</sub> < +8 dB	^
BQI:	0.55	< BQI < 0.65	^
Clarity C <sub>80</sub> :	-3 dB	< C <sub>80,3</sub> < -1 dB.	

Note the "AND" links ("^").

For the concert halls considered here and rated as good to excellent, the following data for the concert halls in question can be used as a guide:

Room width W:	20 m	< W < 26 m	^
Number of seats:	800	< N < 2.000	^
Room volume V:	10 000 m <sup>3</sup>	< V < 20000 m <sup>3</sup>	^
Specific Volume:	11 m <sup>3</sup> /pP	< V/N < 12 m <sup>3</sup> /pP	^
<b>Room Height H:</b>	<b>16 m</b>	<b>&lt; H &lt; 27 m.</b>	

From these considerations it becomes obvious, that a room height of at least 16 m, and preferably towards ca. 27 m, combined with a limited width less than 26 m and a limited (and the same sufficient time large) room volume, is a suitable prerequisite for a concert hall to be considered acoustically very good.

### Opera Houses rated as particularly good

Based on the studies carried out so far, the following ranges of mean values for the room acoustic parameters with distances from the source larger than two critical distances  $r_H$  are therefore considered to be target-oriented prerequisites for very good audibility in opera houses:

Reverberation time T:	1.05 s	<	$T_{occ,3}$	<	1.65 s	^
Strength G:	+4 dB	<	$G_{theo,3}$	<	+6 dB	^
BQI:	0.55	<	BQI	<	0.8	^
$AL_{cons,theo}$ :	12 %	<	$AL_{cons}$	<	14 %	^

This goes hand in hand with the following parameters of the opera houses concerned, which can in principle be influenced by consultants and which can serve as a guide:

Room width W:	20 m	<	W	<	28 m	^
Number of seats N:	800	<	N	<	1800	^
Room volume V:	9 000 m <sup>3</sup>	<	V	<	15 000 m <sup>3</sup>	^
Specific volume:	8 m <sup>3</sup> /pP	<	V/N	<	10 m <sup>3</sup> /pP	^
<b>Room Height H:</b>	<b>19 m</b>	<	<b>H</b>	<	<b>27 m</b>	

It clearly can be stated that a room height of at least 19 m, preferably towards ca. 27 m, in combination with a limited width less than 28 m and a limited, but sufficiently large room volume, is a suitable prerequisite for an opera house to be classified as acoustically very good.

Thus, an appropriate room height H turns out to be an equally crucial quality factor as W. The optimal room height H should be larger than W/2 but has its limit of approx. less than 27 m for both room types.

While limited widths provide favourable early lateral reflections, a large room height ensures that a sufficient room volume can be achieved even if the distance of the seats furthest from the stage is limited, resulting in a room enveloping the listener with diffuse sound.

### Methodological criticism

The method of 3+ dimensional cluster analysis with addition of spatial centroids is a suitable tool for the objectives and implementation of this study.

1. The rankings according to Hidaka and Beranek used in these studies to characterise the subjectively assessed room acoustic quality, are not the only possible ones, but they are the ones with the largest current bases of auditoriums assessed in terms of audibility (58 concert halls, 20 opera houses). New studies with a larger data and test subject base may provide different subjective assessments of audibility in detail. However, the benefits of this analysis method are not limited by or dependent on the particular rating or ranking used.

2. This form of ranking is based on the evaluation of the suitability of a room as a whole, without taking into account local variations in audibility. For example, it would be possible to

make the extent of the markers dependent on the variation of the parameter values in a room.

3. Ranking and considerations were made without further differentiation of suitability for different instrumentations, repertoire or styles of performance.

4. Different sound preferences of different groups of listeners were not taken into account.

5. Lateral energy fraction LEF would be a worthwhile addition of the database and supplement to these investigations, but alas, LEF values are available only for relatively few halls.

### Outlook

The database is constantly being expanded to include more halls and rooms – data from other rooms is always welcome. It would also be very interesting to include parameters as Bass Ratio BR and/or LEF into account as further dimensions.

### Acknowledgements

Thanks must be given to the (technical) managements of the halls for facilitating and supporting the room acoustic measurements since 1996, and to all those involved in entering and analysing the data.

### Literature

- [1] Lorenz-Kierakiewitz, K.-H., et al, Mehrdimensionale visuelle Clusteranalyse raumakustischer Parameterwerte von Konzertsälen, Opernhäusern und anderen Raumtypen, DAGA - Fortschritte der Akustik, Kiel, 2017, p. 260-263.
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