Reduction Between Incommensurable Theories in a Rigid Structuralist Framework*

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Overview

1. What‘s wrong with the structuralist account of reduction?
2. An improved account of reduction
3. The philosophical merits of this improvement
4. Some final technical remarks

1. What‘s wrong with the structuralist account of reduction?

- There is hardly any other part of the structuralist framework that was criticized so often than the structuralist account of reduction
- Recall, in particular, the most prominent (and probably most relevant) critique by Thomas Kuhn
- We will point out here that there are serious problems with the structuralist account of reduction indeed.
- However, facing these problems we will immediately obtain a general strategy how to solve them.

„structurally reducible“

- We use a simplified version of both the structuralist framework and the structuralist account of reduction here (cf. *Architectonic*, ch. II+VI).
- A *theory* $\mathbf{T}$ shall be given in the form of a triple $(\mathbf{M_p}, \mathbf{M}, \mathbf{I})$. $\mathbf{T}$ is called *structurally reducible* to a theory $\mathbf{T}'$, iff there exists a reduction relation $\rho \subseteq \mathbf{M_p}' \times \mathbf{M_p}$ such that it holds
  1. $\mathrm{Rge}(\rho) = \mathbf{M_p}$
  2. for all x', x: if $x' \in \mathbf{M}'$ and $(x', x) \in \rho$ then $x \in \mathbf{M}$
  3. for all $x \in \mathbf{I}$ there exists a $x' \in \mathbf{I}'$ such that $(x', x) \in \rho$

First problem: *too wide (absurd cases)*

- There are obvious cases of „absurd“ reductions:
- Each theory $\mathbf{T}$ may be reduced to any theory $\mathbf{T}'$, which contains at least one structure x such that it holds $\mathbf{M_p} = \mathbf{M} = \mathbf{I} = \{x\}$. We just have to define $\rho \coloneqq \mathbf{M_p}' \times \mathbf{M_p}$.
- Such “absurd” reductions are possible, even in much stronger varieties of a structuralist account of reduction, e.g., if we require that ρ has to be a partially bijective function etc.

Example

- Take a theory $\mathbf{T}$, which only claims that a certain object c has a certain property P, i.e., the only "law" of the theory is $P(c)$. Say, $P(c)$ may express the fact that snow is white.
- Another theory $\mathbf{T}'$ may express the same fact, but in a different language, say, by means of the law $Q(d)$. And a third theory $\mathbf{T}''$ may express a completely different fact, say, that grass is green, by means of a different law $R(e)$.
- These three theories are structurally equivalent, of course. Thus, a purely structural reduction device of any form must allow for reductions between all of them.
- However, we are only willing to accept reductions between the first two theories, *because only reductions of that kind are considered to be true.*

Solution: ρ has to be *true*

- The solution to this first problem is quite obvious. We just have to reject the idea that ρ may be seen *as a purely structural device*.
- By contrast, the relation ρ characterizes a certain connection between two theories that not just depends from the presence of structural similarities.
- Rather, ρ has to fulfill *both* certain structural requirements *and* has to be (empirically) *true*.
- In other words, it is not sufficient to tie ρ to a mere *existence claim*, but we have to tie it to a certain *truth claim*.

Second problem: *too wide (incomplete)*

- In the case of more complex theories a mere mapping between models is generally not sufficient, in order to express the very relation between them.
- Take the case of phlogiston theory. Here the initial theory talks about ‘substances’ such as ‘air’ or ‘phlogiston’, which simply do not exist, according to the theory of modern chemistry.
- However, there are statements about ‘air’ and ‘phlogiston’ that are obviously empirically adequate and obviously have *counterparts* in the theory of modern chemistry.

- Let x be a model of phlogiston theory and x' a ‘corresponding’ model, in the context of modern chemistry (i.e., we have $(x', x) \in \rho$).
- Then x obviously has *parts* that do not have counterparts in x' at all (i.e., everything that is not covered by the theory of modern chemistry).
- But x also has parts that do have counterparts in x', namely, (at least) each empirical fact that is covered by both x and x'.
- In order to grasp the latter we have to consider a (true) relation *between the content of* x and x'.

Solution: ρ has to cover *the content* of the models of the respective theories

- In the case of incommensurable theories it is not sufficient to identify corresponding models. We also have to identify their corresponding (and not-corresponding) parts.
- This is a special case of Moulines‘ proposal to refine the reduction device, by means of incorporating the domains of models. (Cf. Carlos Ulises Moulines, “Ontological Reduction in the Natural Sciences”, in: Balzer et al. (eds.), *Reduction in Science*, 1984, 51-70)
- However, in our proposal we not just consider relations *between first-order objects*, but between *objects of any order*.

- In Maria Caamaño (2009), “A Structural Analysis of the Phlogiston Case”, *Erkenntnis* 70, 331-364 ρ was (1) restricted to the partial potential models and it was (2) accompanied by a relation between the respective first-order domains.
- According to our conception this is not sufficient, in order to being able to express the actual relation between these two theories.
- What we also need is a relation between the respective elements of higher-order (i.e., second-order) objects.
- Only the latter allows us to express what kind of fact exactly may correspond, in the context of the theory of modern chemistry, to each single appearance of ‘phlogiston’ and other substances of phlogiston theory.

Third Problem: *too restrictive*

- The structuralist account implies that a reducing theory has to provide counterparts to each model (i.e., each potential model, each actual model and each intended application) of the theory it reduces. (Conditions 1. to 3. of our definition)
- The intuition behind this is obviously that “a later theory [resolves] all problems solved by its predecessor and more besides” (Kuhn).
- However, as Kuhn demonstrated, the latter is generally *not* the case, i.e., after revolutionary changes, scientific theories are usually *weaker* than their predecessors.

Examples

- Theories that cover the whole universe versus later theories that just cover the solar system.
- Theories about infinitely small objects versus later theories that just cover extended particles.
- Theories that explain the distances between planetary orbits versus later theories that fail to provide such an explanation.
- Theories that also provide theological explanations of some sort versus later theories that fail to provide such explanations.
- Etc.

Solution: remove conditions 1. to 3.

- In order to fix this problem we simply have to remove our conditions 1. to 3. (and consider them only in the context of special cases)
- Kuhn‘s claim that it is generally *not* the case that “a later theory [resolves] all problems solved by its predecessor and more besides” involves that there are counterexamples to all of these three conditions.
- In other words, a proof that there is scientific progress is not so easy obtained, at least in the case of revolutionary changes. (But also compare our below remarks on ‘rationality gaps’.)

2. An improved account of reduction

An improved account of reduction has to take care to the following conditions:

1. *No general restrictions* to the reduction relation, in the sense of conditions 1. to 3. of the original structuralist account
2. A reduction device has to be formulated as a concrete theory about the relation between two theories, i.e., by means of a certain (empirical) *truth claim*.
3. Beside of a relation between the potential models of the old and the new theory, the reduction device also shall contain *relations between the content of the respective corresponding models*.

- Point 1 is just a simplication of our formalism, of course
- Point 2 is a matter of convention, which leads to a significant change of the overall framework, because theory-elements have to be accompanied here with (meta-)theories that express their relations
- Point 3 involves a significant change of our core formalism, because we also have to take into account the content of models here.

- In order to cover the content of a model we define, for each structure x the following set $\bar{x}$:

$$\bar{x} \coloneqq \bigcup_i x(D_i) \cup \bigcup_j (\{R_j\} \times x(R_j))$$

The domain D_i, as specified by x

The "label" of relation R_j

The relation R_j, as specified by x

- On this basis we define a relation $\mathbf{R}$ between two theories $\mathbf{T}$ and $\mathbf{T}'$ as an ordered pair

$$\left(\rho, \left\{\rho_{(x',x)} \middle| (x', x) \in \rho\right\}\right)$$

that consists of a relation $\rho \subseteq \mathbf{M_p}' \times \mathbf{M_p}$ and a set of relations $\left\{\rho_{(x',x)} \middle| (x', x) \in \rho\right\}$, such that each instance $\rho_{(x',x)}$ is defined as a relation between $\overline{x'}$ and $\bar{x}$.

- The only further condition that such a relation $\mathbf{R}$ has to fulfill is that it shall preserve 'homogeneous' elements:
  1. $\forall x \colon x \in \mathbf{M_p}' \wedge x \in \mathbf{M_p} \rightarrow \rho(x, x)$
  2. $\forall x', x \colon (x', x) \in \rho \rightarrow$
     $(\forall p \colon p \in x' \wedge p \in x \rightarrow \rho_{(x', x)}(p, p))$
- Cf. Moulines' distinction between heterogeneous and homogeneous reductions in Moulines, op. cit., p. 60.

3. The philosophical merits of this improvement

We consider the following cases:

A. ρ relates just identical models to each other (=specialization)

B. ρ is heterogeneous at the level of models, but homogeneous at the level of their content

C. ρ is heterogeneous, both at the level of models and at the level of their content

- CI. Approximate identity
- CII. Fully-fledged incommensurability
- CIIa. The closure of ‘rationality gaps’ in Kuhn
- CIIb. The problem of theory-ladenness

CI. Approximate identity

- This is the case of a relation **R**, which relates non-identical elements of models, only if they are *approximately identical*.
- The standard example for that case is the *shift from Newtonian mechanics to special relativity*.
- A model of Newtonian physics characterizes the movements of a number of particles. Here, we may pick out these movements whose velocities are relatively small and identify some (one or more) suitable models of the theory of special relativity and associate only these approximately true parts of the former model with some (one or more) corresponding elements of the latter model.
- For that task we may use the usual structuralist technics of approximation, of course. (See *Architectonic*, ch. VII)

CII. Fully-fledged incommensurability

- Our framework is restricted to just one aspect of incommensurability, namely *referential incommensurability*.
- This is the case of two theories being incommensurable, in that both theories contain terms that do not have any direct counterpart in the respective other theory, i.e., are by no means *literally translatable*.
- *Examples*: terms such as ‘air’ or ‘phlogiston’ or ‘(Newtonian) space’

CIIa. The closure of ‘rationality gaps’ in Kuhn’s theory of incommensurability

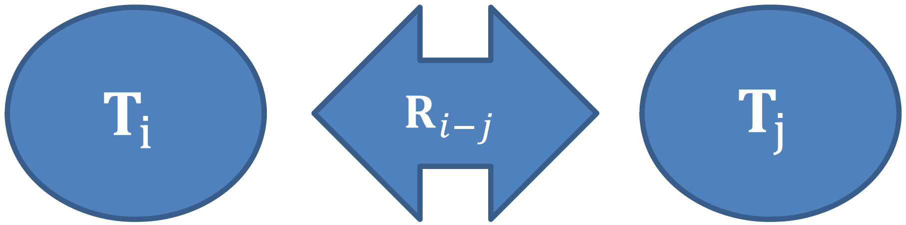
- Stegmüller’s attempt to show that „a later theory [resolves] all problems solved by its predecessors and more besides“ had failed.
- On the other hand, our improved version of reduction provides a formal tool that allows us to identify all these parts of two theories that *correspond*, insofar as they describe the same (empirical or theoretical) matter of fact
- Insofar as Kuhn‘s theory does not provide such an account and our improved version of structuralism does, we indeed may be able to close a certain rationality gap here.
- However, this solution is considerably *weaker* than Stegmüller’s failed proposal. It only implies that there is *continuity* in the sciences. The question of whether there may be real *success* has to be left open here!

CIIb. The problem of theory-ladenness

- The latter observation also leads to a solution to another, even more classical philosophical problem, namely, the problem of theory-ladenness.
- If all our observation-terms are theory-infected, is it possible at all to identify the objective content of any observation?
- The answer is *yes*, as soon as we are able to provide a suitable reduction device that associates the corresponding empirical content of different theories, even if these theories appear to be heavily incommensurable.
- There can be no doubt that our improved account of reduction must be capable for that job.

Theories $\mathbf{T}_1, \dots, \mathbf{T}_n$
Relations $\mathbf{R}_{i-j}$ for i, j with $1 \leq i \leq n$ and $1 \leq j \leq n$

- phlogiston, etc.
- fiery swords



- oxygen, etc.
- sunset

- Suppose that $\mathbf{T}_\mathrm{j}$ is the scientific account we actually adopt.
- Then, we certainly have to *know*, which parts of a model of $\mathbf{T}_\mathrm{j}$ represent *observable facts*.
- If $\mathbf{T}_\mathrm{i}$ is any other theory (about the same subject than $\mathbf{T}_\mathrm{j}$), then, in order to *understand* $\mathbf{T}_\mathrm{i}$, we need to understand which parts of a model of $\mathbf{T}_\mathrm{i}$ correspond to which parts of a model of $\mathbf{T}_\mathrm{j}$.
- In other words, we need to have an accurate picture of $\mathbf{R}_{i-j}$.
- On this basis it does not matter at all, in what sense any empirical term of $\mathbf{T}_\mathrm{i}$ may be theory infected, because the empirical basis of $\mathbf{T}_\mathrm{i}$ is given, by means of a suitable formal picture of $\mathbf{T}_\mathrm{j}$, $\mathbf{T}_\mathrm{i}$, and $\mathbf{R}_{i-j}$, plus our knowledge about the observable parts of $\mathbf{T}_\mathrm{j}$.

4. Some final technical remarks

- In the empirical sciences we never talk about pure structures, but always about concrete objects having concrete properties.
- Thus, a formal language, in the case of the empirical sciences, always has to be *fully interpreted*.
- In the case of structuralism, this full interpretation is realized, by means of the stipulation of intended applications.

- However, the structuralist conception fails to provide a full interpretation, for relations between theories.
- My claim is that this is the main (and possibly the only serious) failure of structuralism.
- In order to improve the original account of structuralism we simply have to establish a full interpretation, even at the level of relations between theories.
- Roughly, we have to stipulate *the intended* ρ, which allows us to express the actual relation between two theories. (In much cases this intended ρ will appear to be empty, of course.)

- If one accepts our improvement to the notion of reduction, then it turns out that structuralism is hardly a “structural” account at all.
- Structuralism by no means talks about mere structures, but about concrete objects having concrete properties.
- Thus, it seems to be much more straightforward to use the framework of „rigid logic“ here, rather than the framework of model theory.
- See Christian Damböck, „Theory Structuralism in a Rigid Framework“, *Synthese* (online first)