Can Auctions Maximize Welfare in Markets After the Auction?

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Abstract

Consider a multi-unit auction followed by a market. Bidders are firms and the auctioned licenses reduce the production costs for a consumption good. The objective is to design an auction that maximizes social welfare, so the sum of industry profits and consumer surplus. It is shown that no socially efficient auction exists. Auction formats are presented that can balance maximizing industry profits and consumer surplus. These auctions require bids on the entire license allocation, locally maximize bidders' welfare, and feature caps and set-asides.

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1 Introduction

Many auctions allocate production inputs. The bidders are firms who do not only compete in the auction, but also in a market for a downstream consumption good. For example, governments around the world use auctions to allocate spectrum licenses to telecommunication companies. These companies need the licenses for the provision of their services, as mobile phone calls and mobile data can only be transmitted via electromagnetic spectrum. Online position auctions are another example. An online shop's production process includes all the activities that influence the sales. The ad positions are an input in this process, because they determine the likelihood of attracting potential customers to the online shop. Companies like Facebook or Google set up auctions for ad positions on their websites and competing online shops are among the bidders. Other auctions of production inputs allocate oil and gas leases, pollution permit rights, rough diamonds and fishing quota.

Efficiency is frequently the objective of multi-unit auctions. In the context of auctions, efficiency is usually understood as maximizing bidders' welfare. Spectrum auctions, for example, are supposed to "put spectrum into the hands of those who value it the most".[1] Online ad auctions are also designed to be efficient. I refer to this notion of efficiency as *auction-efficiency* to distinguish it from *social efficiency*. In the context of a market, social efficiency is usually understood as maximizing the sum of producer and consumer surplus. Auction-efficiency corresponds to maximizing producer surplus as the bidders are the downstream producers. This can be a good objective for a competing platform like Facebook as this will attract bidders. In contrast, a regulator is legally required to maximize social welfare and not just the profits of the firms (EU Directive, 2002).

This paper studies the design of auction-efficient and socially efficient auctions when there is a market after the auction. I investigate the design in a simple and flexible model where an auction is followed by Cournot competition. The auction allocates multiple licenses that reduce downstream production costs. There are two design objectives in addition to the respective notion of efficiency. On top of efficiency the details of the auction should not depend upon the specific demand

[1] Milgrom (2004) ascribes the quote to the former US Vice President Al Gore. In the design of the British 3G auction that ended in April 2000, Binmore and Klemperer (2002) state that "Efficiency was understood as putting the licenses into the hands of the bidders with the best business plans. Since a bidder with a better business plan will generally value a license more, this aim roughly reduces to seeking to maximize the sum of the valuations of the bidders who are awarded licenses."

and production cost functions. If such an auction exists, then the auctioneer does not need to learn the details of the market in order to maximize social welfare. This is important, as the auctioneer typically does not have access to the same information as the bidders. Another design objective is that optimal bidding is simple in terms of bidders having a dominant strategy. Simplicity is a frequently formulated objective in auction and market design (e.g. Cramton, 2013; Vulkan et al., eds, 2013), where it is often strengthened to requiring that "truthful" behavior is optimal.

The central result of this paper is that there is no auction that implements the socially efficient license allocation in dominant strategies and that does not depend upon the specific demand and cost functions. The reason for this negative result is the fact that consumers have preferences over auction outcomes that are often not aligned with firms' preferences. Consumers cannot express their preferences in the auction and this leads to the social inefficiency. A consequence of the result is that the auctioneer cannot use the same auction format for all possible demand and cost functions. When detailed information about demand and costs is not available, or when the design cannot be conditioned on the information, then no auction is guaranteed to achieve the first best outcome.

A second result is that an auction-efficient format exists, but that it needs to allow bids on full license allocations. In auction theory it is typically assumed that bidders only care about their own quantity. In the market context, however, it is plausible that they also care about who wins what. The auction outcome determines the production costs of all firms and firms naturally care about their own and their competitors' costs. Thus, they have preferences over full license allocations. A bidder with preferences over full license allocations has multiple values for winning the same quantity, as given the own quantity there are many possible ways to allocate the remaining licenses among the other bidders. An auction with bids only on quantities does not allow to fully express these richer preferences. In particular, it cannot have a dominant strategy, as the bids of the other bidders determine which value is realized. The Vickrey–Clarke–Groves (VCG) auction implements the auction-efficient license allocation in dominant strategies and for all possible demand and cost functions, but only if it allows bids on full license allocations. In practice, bidders have only been able to bid on their own quantity and not on the full allocation. It is difficult to get empirical evidence on the preferences of firms, but in a review of the German 2015 spectrum auction, Bichler et al. (2017) argue that the bidding process can best be explained by firms caring about the full allocation and not only about the quantity they win.

In addition, I show that the non-existence of socially efficient auctions can be mitigated by certain design ideas. Caps are upper bounds on how much a bidder can win and are used in practice to bias the outcome of an auction in favor of consumers (e.g. Binmore and Klemperer, 2002).[2] In particular, caps insure that firms remain relatively similar. My model allows evaluating when caps are a good idea. Consider the VCG auction with bids on full license allocations and caps. Suppose all firms have access to the same cost reduction technology that becomes less effective the more licenses are won. It turns out that industry profits might be maximized by very unequal license allocations, whereas consumers prefer very equal license allocations. In this case, caps rule out the unequal global maximizer of industry profits and unambiguously raise consumer surplus. Industry profits are still maximized, but only locally, by the VCG auction with bids on license allocations.[3] The choice of the caps requires some information about the demand and cost functions, however. When firms have very asymmetric cost reductions or when the firms' cost reduction technologies are increasingly effective, then consumers prefer more unequal license allocations. Caps might then have adverse effects.

In an extension, it is shown that set-asides can improve consumer surplus when there is potential entry. Set-asides are licenses that are allocated non-competitively to potential entrants or small firms.[4] Suppose there is a firm that needs to win a certain amount of licenses in order to be active in the market after the auction. This firm can be a potential entrant, or an incumbent that has to exit the market without additional licenses. Without set-asides, there would be no successful entry after an auction-efficient auction, as industry profits are higher with only $n-1$ active firms. For consumers there is a potential trade-off. On one hand, consumers prefer more active firms over less. On the other hand, an entrant wins licenses that could also further decrease the incumbents' costs. It turns out that consumer surplus is higher under successful entry when consumers' demand for the downstream good is high. Sufficiently high set-asides enable entry, so that the VCG auction with bids on full license allocations together with caps finds a local

[2] For instance, Ofcom (2017) will use caps in the upcoming spectrum auction in the UK in order to ensure a competitive market among four firms. In position auctions a bidder can usually win only one position. This can be seen as a very strict cap.

[3] In position auctions, consumers might have higher search costs if all ad positions were won by a single firm. This suggests that consumer surplus might be lower without caps, whereas industry profits might be higher. Thus, caps can bias the auction outcome in favor of consumers.

[4] For example, the Austrian regulator RTR (2013) reserved two out of six available 800 MHz blocks for a potential entrant. If there had been potential entrants, there would have been an auction before the main auction for the two (bundled) blocks. With a single potential entrant, the pre-auction would have concluded at the reserve price.

maximum of industry profits in a neighborhood around the allocation optimal for consumers. The auctioneer's assessment about the demand and cost functions is important for the choice of the set-asides and the caps.

Related Literature

The paper is related to the discussion of auction objectives. For some authors auction-efficiency is the primary objective in spectrum auctions (e.g. Cramton, 2002). It has also been pointed out, however, that auction-efficiency is not the adequate objective when there is a market after the auction (e.g. Klemperer, 2002; Jehiel and Moldovanu, 2003; Cramton, 2013). According to Klemperer (2002), good auction design also considers the market after the auction. Cramton et al. (2011) informally discuss the positive role of caps and set-asides on after-market competition. Bichler and Goeree (2017) ask for robust auction formats that take care of bidders' preferences and that help regulators achieve their goals. I add to this literature by providing a formal model that derives rather than assumes bidders' and consumers' preferences. This allows me to formally assess the implications of design instruments and auction objectives on the market structure.

This is the first paper that shows that there is no socially efficient auction in a setting in which the auction does not select the active firms. Dana and Spier (1994) and Rey and Salant (2017) consider the case in which the auction determines which firms will operate in the market after the auction. I investigate the sale of cost reductions and not of entry licenses.[5] The firms in my model are established incumbents. There are no potential entrants and the cost reduction technology is not "revolutionary" in the sense that a firm that does not win enough licenses has to exit the market. This seems to be a reasonable approximation of some auctions. For example, in the US Incentive Auction (600 MHz) Verizon did not buy any new spectrum. Nevertheless, it does not seem to exit the market.

The most closely related model is by Mayo and Sappington (2016) who have two firms with complete information in differentiated Bertrand competition. They analyze the allocation of a marginal change in marginal costs to the firm with the larger increase in profits, which is the firm with the larger pre-auction market share. It is, however, not always socially efficient to assign the cost reduction to

[5]Hoppe et al. (2006) study the impact of the number of entry licenses on the market structure. Gebhardt and Wambach (2008) consider the optimal number of active firms when they have privately known fixed costs. Janssen and Karamychev (2010) show that auctions do not necessarily select the firms with the lowest costs in a unit-demand setting. This is in contrast to the case when there is a single monopoly license for sale (Demsetz, 1968; Laffont and Tirole, 1999).

this firm. In contrast, I consider auction design and the allocation of a perfectly divisible good among n firms in Cournot competition. I show that it matters for auction design if there are two firms or more. In many industries, firms first produce a capacity and then compete in prices. The capacity can be the antennas in a network, the range of goods in an online shop, the number and intensity of developed oil fields, or the fishing fleet. Under certain conditions the outcome in the capacity-then-price game is equal to the outcome in the standard Cournot game (Kreps and Scheinkman, 1983). For simplicity, I abstract away from the pricing stage and only consider the capacity choice. Eső et al. (2010) consider an auction-efficient auction that allocates capacity constraints to ex-ante symmetric firms in capacity-constrained Cournot competition with complete information. Unlike in my model, they do not consider the effect of cost reduction, which is an important aspect of many auctions of production inputs. The more spectrum a firm has, the fewer antennas are needed and the lower the maintenance costs. When an online shop wins a good ad positions, it has lower costs of selling an additional unit. Oil companies can have economies of scale when they develop more or adjacent oil fields. Winning more quotas for different fish species can reduce the fines that have to be paid due to by-catch (Marszalec, 2017).

Other papers assume that agents have allocative and identity-dependent externalities (preferences over auction allocations) and consider bidding in standard auctions and mechanism design (e.g. Das Varma, 2002; Jehiel and Moldovanu, 2001, 2006). They do not, however, consider the impact of the mechanism on consumer surplus. The literature on process innovations and R&D consider related models related to mine (e.g. Dasgupta and Stiglitz, 1980; Tandon, 1984). A major difference, however, is that innovations are typically unlimited, whereas the supply of licenses is scarce.

The next section presents the model and some preliminary results about the market after the auction. Section 3 analyzes firms' and consumers' preferences over license allocations. Section 4 presents the main results on efficient auction design. Sections 5 and 6 discuss the positive role of caps and set-asides for consumer surplus, respectively. I conclude and discuss implications for auction design in Section 7. Proofs are in Appendix A.

2 The Model and Preliminary Results

There is an auction for multiple licenses that is followed by a downstream market for a consumption good. Firms can buy the licenses and compete against each after the auction, where the licenses reduce the buyers' production costs. The model has the following timing. Firms participate in the auction by submitting bids. The auction allocates the licenses according to the auction rules and the submitted bids. Firms learn the auction outcome and then choose their actions in the market after the auction. Finally, profits and consumer surplus are realized.

The auction allocates production cost reducing licenses to the bidders. Licenses are perfectly divisible and available in supply with unit measure. The auction determines a feasible allocation of the licenses, that is, it implements a vector $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n_+$ such that $x \geq 0$ and $x_1 + x_2 + \cdots + x_n \leq 1$. A *no-undersell allocation* of licenses is a feasible license allocation in which the entire supply is allocated, i.e. $x_1 + \cdots + x_n = 1$. The set X denotes the set of all feasible allocations, whereas the set $\bar{X}$ denotes the set of all no-undersell allocations and coincides with the $n-1$ dimensional simplex. The licenses cannot be resold after the auction.

The Market After the Auction

There is a market for a homogeneous consumption good that takes place after the auction. In this market $n \geq 2$ firms are in Cournot competition, that is, firm $i \in N = \{1, \ldots, n\}$ chooses a quantity $q_i \geq 0$ in order to maximize profits. Industry output is denoted by $Q = q_1 + \cdots + q_n$. It determines the market price through the inverse demand function $P(Q) \geq 0$ with $P' < 0$ for the interval for which $P > 0$. The production costs $C_i(q_i|x_i)$ depend on the level of output and the share of licenses obtained in the auction. In particular, the marginal costs are decreasing in x_i, i.e. $\partial^2 C_i(q_i|x_i)/\partial q_i \partial x_i < 0$. Let marginal costs be denoted by $C_i'(q_i|x_i) = \partial C_i(q_i|x_i)/\partial q_i$ and similarly, let $C_i''(q_i|x_i) = \partial^2 C_i(q_i|x_i)/\partial q_i^2$. The inverse demand function and the cost functions are twice continuously differentiable in all the arguments. Auction expenditure is sunk. Firm i's profits conditional on auction allocation x and production vector q are

$$\pi(q|x) = P(Q) \cdot q_i - C_i(q_i|x_i). \tag{1}$$

All after-markets considered in this paper are contained in the collection of markets $\mathcal{M}$. Formally, *market* $M = (P, C)$ consists of an inverse demand function $P : \mathbb{R}_+ \to \mathbb{R}_+$ and a profile of cost functions $C = (C_1, \ldots, C_n)$, where $C_i : \mathbb{R}_+ \times [0, 1] \to R_+$. Proposition 1 will use a result of Kolstad and Mathiesen (1987) to show that the following conditions on markets in $\mathcal{M}$ are necessary and sufficient so that there is a unique equilibrium in every Cournot sub-game in which n firms are active.[6] Market is in $\mathcal{M}$ if and only if for all $x \in X$ and for all $q_1, \dots, q_n$ with $Q = q_1 + \cdots + q_n$ that satisfy the n first-order conditions of profit maximization, and for all $i \in N$ *(i)* $q_i > 0$, *(ii)* $C_i''(q_i|x_i) - P'(Q) > 0$, and *(iii)* $\frac{P'(Q)+q_i P''(Q)}{C_i''(q_i|x_i)-P'(Q)} < \frac{1}{n}$.

Costs $C_i(q_i|x_i) = c_i(x_i) \cdot q_i$ can be linear in the produced quantity. Marginal costs $c_i(x_i) = \theta_i + \rho_i(x_i)$ then only depend on the share of licenses won in the auction and are the sum of initial marginal costs θ_i and the effect of the continuously differentiable *cost reduction technology* ρ_i, where $\rho_i(0) = 0$. All licenses are effective in reducing marginal costs, so $\rho_i' < 0$. For the case of production cost linear in q_i, two special cases of the cost function profiles are worth mentioning. First, firms can have the *same* cost reduction technology, that is, $\rho_i(\tilde{x}) = \rho_j(\tilde{x})$ for all firms i and j and $\tilde{x} \in [0, 1]$. Note that the initial marginal costs do not need to be the same. The second special case are *linear* cost reduction technologies, so $\rho_i(x) = r_i \cdot x$, with $-\theta_i \leq r_i < 0$.

Firms have perfect and complete information. The auction outcome is publicly announced after the auction, as is commonly done in real-world auctions. Moreover, firms know the market and all other ingredients of the model. One can argue that this assumption is somewhat justified for well-established firms that know each other, and that have access to similar technologies, consultants, and business cases. There might be uncertainty about future demand and costs, but all firms have the same expectations. Incomplete information that impacts the market tends to lead to interdependent values (informational externalities) and introduces several complications that seem to be of second-order interest for the market structure. For example, an important aspect of auction design becomes information design, as different information provides different signaling opportunities (e.g. Goeree, 2003). Moreover, when the private information is multi-dimensional, not even auction-efficient mechanisms need to exist (e.g. Dasgupta and Maskin, 2000; Jehiel and Moldovanu, 2001).

The solution concept is sub-game perfect Nash equilibrium. The firms use backward induction to derive the values in the auction. The auction is part of the game and induces potentially many sub-games that have the same auction allocation $x \in X$. As the auction expenditure is sunk, however, it is convenient to summarize all sub-games that follow the auction allocation $x \in X$ to one Cournot continuation game. Kolstad and Mathiesen (1987) show that the assumptions on markets in $\mathcal{M}$ are necessary and sufficient for the existence of a unique equilibrium for every $x \in X$.

[6] The case in which not all firms are active in all sub-games is analyzed in Section 6.

Proposition 1. *There exists a unique equilibrium in the Cournot continuation game that follows auction allocation $x \in X$ for all $M \in \mathcal{M}$.*

Let $\pi_i(x)$ denote the indirect profit function derived from the unique equilibrium in the Cournot continuation game induced by $x \in X$. It can also be written as $\pi_i(x_i, x_{-i})$, where I follow the usual convention that (x_i, x_{-i}) denotes x. Firm i's Cournot continuation equilibrium production is denoted by $q_i(x)$. Note that $q_i(x) > 0$ due to the requirement *(i)* above. The indirect industry profit function is $\pi(x) = \pi_1(x) + \cdots + \pi_n(x)$. Indirect consumer surplus is given by $CS(x) = \int_0^{Q(x)} P(y)\,dy$ and strictly increasing in aggregate output $Q(x) = q_1(x) + \cdots + q_n(x)$. Total welfare $W(x) = \pi(x) + CS(x)$ is the sum of industry profits and consumer surplus.

The linear Cournot model is a special case that will play a prominent role. In the linear Cournot model, inverse demand $P(Q) = a - b \cdot Q$ is linear in Q, with $a > 0$ and $b > 0$, and production costs C_i are linear in q_i. The linear inverse demand satisfies $P' < 0$ for $P > 0$ and is log-concave. Markets $M \in \mathcal{M}$ with linear inverse demand must satisfy

$$\min_{x \in X} a - nc_i(x_i) + \sum_{j \neq i} c_j(x_j) > 0 \tag{2}$$

for all $i \in N$, as this condition guarantees that all firms are active independent of the allocation. The size of the market is captured by the parameter a. Standard techniques allow to characterize the Cournot continuation equilibrium (e.g. Belleflamme and Peitz, 2010). Firm i's Cournot continuation equilibrium production equals

$$q_i(x) = \frac{a - nc_i(x_i) + \sum_{j \neq i} c_j(x_j)}{b(n+1)}. \tag{3}$$

The condition stated in Equation (2) implies that $q_i(x) > 0$ for all feasible license allocations $x \in X$, so all firms are active independent of the auction outcome. Cournot continuation equilibrium profits are given by

$$\pi_i(x) = \frac{\Big(a - nc_i(x_i) + \sum_{j \neq i} c_j(x_j)\Big)^2}{b(n+1)^2}. \tag{4}$$

Consumer surplus is given by $CS(x) = bQ(x)^2/2$. Industry production can be written as

$$Q(x) = \frac{an - \sum_{i=1}^{n} c_i(x_i)}{b(n+1)}. \tag{5}$$

Auction Design

The objective is to design a standard auction that implements the auction-efficient or socially efficient allocation in weakly dominant strategies for all possible markets. A standard auction *(i)* elicits a willingness-to-pay, i.e. participants must submit bids, and *(ii)* selects the allocation that maximizes the elicited willingness-to-pay (Krishna, 2010). The restriction on standard auctions can come from an institutional or a legislative constraint. Simplicity is an additional design objective, where an auction is simple if it has a dominant strategy.[7] A bidding function is dominant strategy when it is a best response against all profiles of other bidders' bidding functions (Krishna, 2010).

The bid elicitation method (bidding language) and the payment rule are subject to design. Bidder i expresses the willingness-to-pay through the (upper semi-continuous) bidding function $B_i : \mathcal{Y}_i \to \mathbb{R}_+$, where $\mathcal{Y}_i$ is the bidding language. The designer allows bidder i to bid on license allocations in the set $Y_i \subseteq X$. The set $\mathcal{Y}_i$ is a partition of Y_i. The auctioneer treats allocation $x \in X \setminus Y_i$ as if bidder i bid zero on it, i.e. B_i is extended to $B_i(x) = 0$ for all $x \in X \setminus Y_i$. I slightly abuse notation by writing $B_i(x)$ instead of $B(y)$ for $x \in y \in \mathcal{Y}_i$. Two important classes of bidding languages are *bids on licenses* and *bids on the full license allocation.* Suppose bids can be made on all license allocations, so $Y_i = X$. When there are only bids on licenses, then the cells of the partition are indexed by $\gamma \in [0, 1]$ and defined as $y(\gamma) = \{x \in X | x_i = \gamma\}$. When there are bids on full license allocations, then every license allocation is a cell of the partition, i.e. $\mathcal{Y}_i = \cup_{x \in X}\{\{x\}\}$. In any case, a standard auction implements the license allocation that maximizes the elicited willingness-to-pay, i.e.

$$x \in \arg\max_{\tilde{x} \in X} \sum_{j=1}^{n} B_j(\tilde{x}). \tag{6}$$

The auction design also includes a payment rule, which is a mapping from the bidding functions and the implemented license allocation to the positive reals for every bidder.

Regulators arguably use auctions to allocate licenses, because they do not know the socially efficient allocation. Hence, the objective is to find an auction that is socially efficient for all markets $M \in \mathcal{M}$, where the formal definition of a socially

[7] Multi-unit auctions tend to have many equilibria. An alternative objective could be to design an auction that has one socially efficient equilibrium. Due to the multiplicity of equilibria, the auction might also have other inefficient equilibria. The focus on dominant strategies avoids the problem of equilibrium selection.

efficient auction is the following.

Definition 1. An auction is *socially efficient* for market $M \in \mathcal{M}$ if it implements a socially efficient license allocation $x^* \in \arg\max_x W(x)$ in dominant strategies.

If it is possible to design such an auction, then this auction can be used irrespective of the information available to the auctioneer. In addition, the auction format can be used without justification that might be necessary when the design is conditional on the specific market. Suppose the regulator has the non-verifiable information that the market is such that allocation x is socially efficient. Any standard auction that only allows bids on allocation x is socially efficient. However, it might be difficult to implement or justify such a restricted bidding language. Firms might object, because it might be that industry profits are low at x. Likewise, the government might object due very low expected auction proceeds.

Another important design objective is auction-efficiency, where the notion of efficiency is restricted to the bidders in the auction. In the present model, if all n firms participate in the auction, then auction-efficiency means that a license allocation $x \in \arg\max_{\tilde{x}} \pi(\tilde{x})$ is implemented that maximizes industry profits.

Definition 2. An auction is *auction-efficient* for market $M \in \mathcal{M}$ if it implements a license allocation that maximizes bidders' welfare in dominant strategies.

An auction that can be auction-efficient when bidders have private values is the Vickrey–Clarke–Groves (VCG) auction. The VCG auction can be run with different bidding languages. In the paper, I use two different variants. In both cases the implemented allocation is chosen according to Equation (6). Moreover, every bidder i pays the VCG price

$$p_i(x_i) = \max_{\tilde{x} \in X} \sum_{j \neq i} B_j(\tilde{x}) - \sum_{j \neq i} B_j(x),$$

which is the externality she imposes on the other bidders. The difference is the bidding language. The first format is the "standard" *VCG auction with bids on licenses*. Bidders bid on all possible $x_i \in [0, 1]$. Truthful bidding is a weakly dominant strategy when a bidder has private values for licenses. The second variant is the *VCG auction with bids on full license allocations*, so bidders express values for all possible allocations $x \in X$. It is auction-efficient, when bidders have private values over the allocations. In general, the VCG auction has a dominant strategy when the bidding language is rich enough so that bidders can report their preferences. The next section considers the preferences of the firms and the consumers.

3 Firms' and Consumers' Preferences

Auction design begins by studying the bidders' and the consumers' preferences over auction outcomes. The auction outcome is a license allocation that determines the cost structure of the industry. Knowing the preferences over the cost structures is essential for designing an efficient auction. A firm might care about the marginal costs of its competitors, as their marginal costs influence the output choice and through the aggregate output the firm's profits. Similarly, consumers might care about the cost structure, as the cost structure might influence the degree of industry concentration and industry output. The section first characterizes the domain of firms' and consumers' preferences. In order to get a better understanding of the shape of the preferences, the section also presents some results for the linear Cournot model.

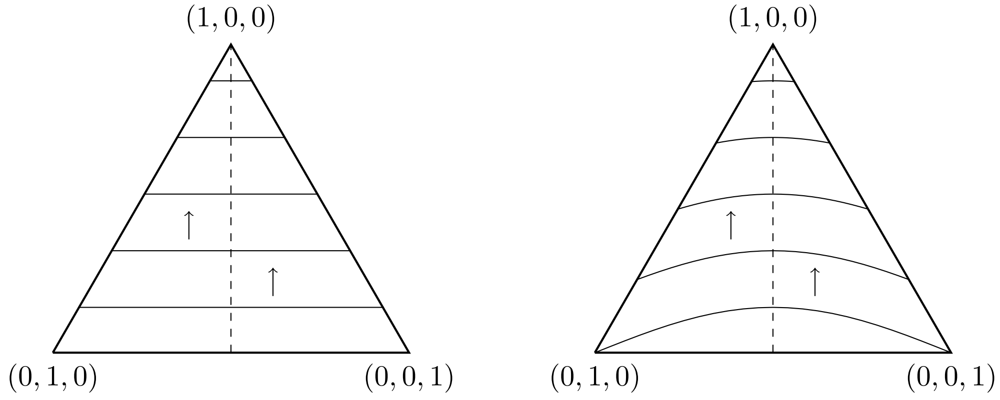
Firm i's indirect profit function $\pi_i(x)$ is a function of the entire license allocation x in general. On the one hand, the profits depend on how many licenses $x_1 + \cdots + x_n$ are allocated. For example, conditional on winning x_i, firm i's profits might be different when $1 - x_i$ is allocated among the other bidders, and when $1 - x_i$ is not allocated. In the latter case, no other firm reduces marginal costs, while in the former case some firms do end up with lower marginal costs. It is therefore interesting to study π_i when the amount of allocated licenses is constant. Conditional on allocating the full supply, firms might or might not care about *how* licenses that they do not win are allocated. The following two definitions formalize this distinction. In the first case, profits are a function of the entire license allocation.[8]

Definition 3. Firm i has *preferences over full license allocations* when there exist no-undersell allocations x, $x' \in \bar{X}$ with $x_i = x'_i$ and $\pi_i(x) \neq \pi_i(x')$.

The opposite of preferences over full license allocations is when bidder i only cares about the share that she wins in the auction. This assumption is made implicitly in the majority of auction models. Firms have preferences over own licenses if for any no-undersell allocation every firm assigns a unique value to every share.

Definition 4. Firm i has *preferences over own licenses* if $\pi_i(x) = \pi_i(x')$ for all no-undersell allocations x, $x' \in \bar{X}$ with $x_i = x'_i$.

[8] In the auctions and mechanism design literature this property has also been called allocative and identity-dependent externalities (e.g Jehiel and Moldovanu, 2006).



(a) Preferences over own licenses

(b) Preferences over full license allocations

Figure 1: Iso-profit lines of firm 1

The following proposition states necessary and sufficient conditions for firms having preferences over own licenses. With only two firms there is a unique no-undersell allocation in which firm i receives x_i. Hence, both firms have preferences over own licenses. Suppose there are at least three firms. Then aggregate output

Proposition 2. *Firms have preferences over own licenses if and only if there are two firms or when all firms* $i \in N$ *have costs* $C_i'(q_i|x_i) = \theta_i + r \cdot x_i$ *for* $q_i \in [\min_{x \in \bar{X}} q_i(x), \max_{x \in \bar{X}} q_i(x)]$ *and* $r < 0$.

Figure 1 illustrates possible iso-profit lines of firm 1 when there are three firms. The triangle depicts the set of no-undersell allocations. Figure 1a shows firm 1's iso-profit lines for the case when firm 1 has preferences over own licenses. The edge opposite of vertex $(1, 0, 0)$ is the set of no-undersell allocations in which firm 1 receives nothing. Iso-profit lines are parallel shifts of this edge. Firm 1 is indifferent between all no-undersell allocations in which she gets the same share. Profits are strictly increasing in the own share. Figure 1b displays possible iso-profit lines when firm 1 has preferences over full license allocations. The iso-profit lines are no longer parallel shifts of the edge opposite of vertex $(1, 0, 0)$. For the shown preferences and a given $x_1 < 1$, firm 1's profits are higher the more unequal the allocation among firms 2 and 3.

There are comparative statics results in the literature that can be applied to this model. Suppose P is log-concave, so all best response functions are downward sloping in equilibrium (Vives, 1999). Dixit (1986) shows a marginal decrease in firm i's marginal costs increases firm i's profits and decreases firm j's profits for $j \neq i$. The question is then whether industry profits are also maximized by no-undersell license allocations. In the linear Cournot model this is the case.[9] However, Seade (1987) shows in a symmetric model the somewhat counterintuitive result that industry profits can be higher when firms have higher marginal costs. The intuition is that for certain inverse demand functions higher marginal costs lead to a so much higher market price, so that profits are higher. Seade (1987) also considers the asymmetric case, but concludes that it is difficult to find conditions that unambiguously sign the effect. Hence, there are probably markets for which industry profits are not maximized by a no-undersell allocation.

Consumers do not participate in the auction, but can have preferences over auction allocations. Dixit (1986) shows that if firm i marginally reduces its costs, total output $Q(x)$ increases. Hence, consumer surplus is always maximized by no-undersell license allocations. The more licenses are allocated, the lower marginal costs, hence the larger aggregate output. As consumers prefer more mass of licenses to be allocated, it is interesting to consider their preferences over the set of no-undersell allocations.

Proposition 3. *Consumers are indifferent between all no-undersell allocations if and only if all firms have marginal costs constant in q_i and the same linear cost reduction technology, i.e. all firms $i \in N$ have costs $C_i'(q_i|x_i) = \theta_i + r \cdot x_i$ for $q_i \in [\min_{x\in\bar{X}} q_i(x), \max_{x\in\bar{X}} q_i(x)]$ and $r < 0$.*

As long as the cost reduction is not the same linear function, consumers have non-trivial preferences over the license allocations. Suppose all firms have a cost function affine in q_i and the same linear cost reduction technology. In this case, aggregate output depends on the sum, but not on the allocation of marginal costs when production costs are linear in the produced quantity (e.g. Bergstrom and Varian, 1985). When firms do not have the same linear cost reduction technology, then the sum of marginal costs is different for different license allocations, and therefore consumer surplus depends on the license allocation.

In order to obtain a better understanding of consumer surplus and industry profits, I consider the maximization of consumer surplus and industry profits for the linear Cournot model, because it allows an explicit characterization of consumer surplus and industry profits. The following proposition considers linear cost reduction. The first insight is that firm i's profit function is convex, that is, the increase in profits is increasing in the share of licenses won. Note that convex profits (or values) are usually associated with complementarities (e.g. Goeree and Lien, 2014). Industry profits are convex and can only be maximized by an allocation

[9] I show this in Footnote 15 in the Proof of Proposition 5.

in which one firm wins the full supply. When firms have the same cost reduction, industry profits are highest when the firm with the lowest initial marginal costs wins the full supply. The intuition is the following. Suppose firm 1 has the lowest initial marginal costs, so it has the largest price margin and the largest market share without any additional licenses. A large market share leverages a decrease in marginal costs. This increase in profits is larger than the decrease in profits of the firms who do not receive anything, because they have a lower original margin due to higher initial marginal costs. Consumer surplus is maximized by any no-undersell allocation, so there is no trade-off between industry profits and consumer surplus.

When the cost reduction is not the same, consumer surplus is highest when the firm with the most effective cost reduction technology (the smallest r_i) wins the full supply. Characterizing the industry profit maximizing allocation is difficult, because there is a non-trivial interaction of initial marginal costs θ_i and cost reduction r_i. Instead of a complete characterization, the proposition provides two sufficient conditions for when there is no trade-off between maximizing consumer surplus and industry profits. The first condition is when the market size, measured by the intercept of the linear inverse demand, is large enough. The second condition is when initial marginal costs are the same.

Proposition 4. *Consider the linear Cournot model with $P(Q) = a - b \cdot Q$ and bidders with a linear cost reduction technology, so $c_i(x_i) = \theta_i + r_i \cdot x_i$ for all $i \in N$. Firm i's profits $\pi_i(x)$ and industry profits $\pi(x)$ are convex on X. Suppose firms have the same cost reduction technology, i.e. $r_1 = r_2 = \cdots = r_n = r$. Industry profits are maximized by the license allocation in which the firm with the lowest initial marginal costs wins the full supply. Suppose the cost reduction has $r_1 \leq r_2 \leq \cdots \leq r_n$. Consumer surplus is maximized by the license allocation $(1, 0, \ldots, 0)$. This license allocation maximizes industry profits when a is large enough or when $\theta_1 = \theta_2 = \cdots = \theta_n$.*

The next proposition looks at firms that have the same strictly convex reduction technologies.[10] A convex cost reduction means that additional licenses are less effective in reducing marginal costs. It turns out that firm i's indirect profit function π_i is never concave. Note that concave profits (over own shares) are assumed in many auction models in order to get interior solutions (e.g. Ausubel, 2004; Levin and Skrzypacz, 2016). Industry profits can be a concave, but also a convex

[10] When the cost reduction technology is concave, then consumer surplus is maximized by the license allocation in which firm i wins the full supply, where i is such that $\rho_i(1) \leq \rho_j(1)$ for all $j \in N$.

function. Consumer surplus is always maximized by the allocation $(1/n, \dots, 1/n)$, because it minimizes the sum of marginal costs. The cost reduction becomes less effective the more licenses a firm obtains, so the sum of marginal costs is minimized so that the derivative of the cost reduction technologies is the same. A necessary condition for this license allocation to maximize industry profits is that all firms have the same initial marginal costs.

Proposition 5. *Consider the linear Cournot model with inverse demand* $P(Q) = a - b \cdot Q$ *and bidders with the same strictly convex cost reduction technology, so* $c_i(x_i) = \theta_i + \rho(x_i)$ *for all* $i \in N$ *and* $\rho'' > 0$*. Firm* i*'s profits* $\pi_i(x)$ *are never concave on* X*. Industry profits can be convex, but are concave when* a *is sufficiently large. In any case, consumer surplus is maximized by the license allocation* $(1/n, \dots, 1/n)$*. This license allocation is a critical point of industry profits only if* $\theta_1 = \theta_2 = \dots = \theta_n$*.*

The proposition shows that the license allocation $(1/n, \dots, 1/n)$ maximizes consumer surplus when the firms have the same strictly convex cost reduction technology. Moreover, it shows that the consumer surplus maximizing allocation can maximize industry profits when the initial marginal costs are the same. In the next example, however, the industry profit maximizing license allocation minimizes consumer surplus on the set of no-undersell license allocations. The example and the proposition together illustrate that the industry profit maximizing allocation can minimize, but also maximize consumer surplus.

Example 1. Let $n = 3$, $P(Q) = 10 - Q$, and $c_1(x) = c_2(x) = c_3(x) = 3 - \sqrt{3x}$. As all firms have the same strictly convex cost reduction technology, consumer surplus is maximized by the allocation $(1/3, 1/3, 1/3)$ and minimized on $\bar{X}$ by any allocation in which one firm wins the whole supply. Industry profits are convex and maximized by an allocation in which one firms wins the whole supply.

The next section uses the results on the firms' and consumers' preferences for auction design.

4 Efficient Auction Design

This section studies the design of efficient auctions. In particular, I consider the objective of maximizing the bidders' welfare (auction-efficiency) and the objective of maximizing the welfare of all agents (social efficiency). As a warm-up, I first consider an auction format that is typically deemed auction-efficient. This auction format is the "standard" VCG auction with bids on licenses. It will turn out that this auction design is not auction-efficient for all markets. This negative result is due to the restrictive bidding language, as only bids on licenses are allowed. An auction with a sufficiently rich bidding language will be auction-efficient. The main objective of this section is to the design an auction that is socially efficient for all possible markets. Social efficiency will not be implementable for all markets.

The VCG auction with bids on licenses is commonly believed to be auction-efficient. This belief carries over to practical auction design, as this auction format has been used, for example, in recent spectrum auctions and in auctions for online ad positions. In the context of spectrum auctions, the Combinatorial Clock Auction (CCA) is a dynamic auction that has a VCG auction in its core. Cramton (2013) reports that the VCG pricing was chosen in the CCA in order to make truthful bidding optimal. Truthful bidding is simple and leads to auction-efficient outcomes.[11] The CCA has allowed bids on auctioned quantities or packages, but not on full allocations. The VCG auction is also used with bids on licenses in auctions for online ad positions by Facebook or Google. Google uses the related generalized second-price auction for auctions associated with its search engine, and the VCG auction with bids on licenses to sell ads outside the search engine (Varian and Harris, 2014). Truthful bidding is again the argument in favor of the VCG auction with bids on license. The following proposition shows, however, that the VCG auction with bids on licenses is not auction-efficient for all markets. The problem is that there are many markets for which it is not clear what truthful bidding means, so that there is no dominant strategy.

Proposition 6. *The VCG auction with bids on licenses is not auction-efficient for all markets $M \in \mathcal{M}$. Let P be log-concave and suppose industry profits are maximized by a no-undersell allocation. The auction is auction-efficient if and only if all firms have a cost function linear in q_i and the same linear cost reduction technology, i.e. all firms $i \in N$ have costs $C_i'(q_i|x_i) = \theta_i + r \cdot x_i$ for $q_i \in [\min_{x \in \bar{X}} q_i(x), \max_{x \in \bar{X}} q_i(x)]$ and $r < 0$.*

Truthful bidding is not a dominant strategy for all markets in the VCG auction with bids on licenses. Proposition 2 shows that there are many markets for which bidders have preferences over full license allocations. For such a market, bidders have multiple values for winning the same license, but the auction with bids on licenses allows them to express only one of them. The final license allocation depends on the bids of the other bidders and therefore the bids of the other bidders

[11] Levin and Skrzypacz (2016) and Janssen and Kasberger (2017) show, however, that the CCA has many equilibria that are not auction-efficient. This exemplifies the equilibrium selection problems one can have when there is no dominant strategy.

determine the value for the license. There cannot be a dominant strategy. The condition that P is log-concave is needed to insure that the bidding function is increasing. Suppose industry profits are maximized by not assigning any licenses. The example of Seade (1987) suggests that this can be the case. It cannot be a dominant strategy for a bidder not to bid in the VCG auction.

Bidders with preferences over full allocations face the *allocative exposure problem* when bids can only be placed on shares. This problem can best be illustrated in a simple example. Suppose there are three bidders, three indivisible goods and bidder 1 can win at most one unit. Bidder 1 has preferences over full allocations, so it matters, conditional on winning one unit, whether the final allocation is (1,2,0) or (1,1,1). Suppose profit is higher when the allocation is more unequal, i.e. $\pi_1(1,2,0) > \pi_1(1,1,1)$. Others might bid in such a way that bid $B_1(1) > \pi_1(1,1,1)$ implements the allocation (1,2,0). Bid $B_1(1)$ might be optimal when indeed (1,2,0) is implemented. Other bidders, however, might bid differently so that bid $B_1(1)$ leads to the allocation (1,1,1). The price for the license can be as high as $B_1(1)$, so bidder 1 might make a loss as $\pi_1(1,1,1) - B_1(1) < 0$. Thus, there are other bidders' bidding functions such that the bid $B_1(1) > \pi_1(1,1,1)$ is optimal. However, such a high bid also exposes the bidder to the risk of a potential loss.

The next proposition shows that the VCG auction is auction-efficient for all markets if it has a rich enough bidding language, so if it allows bids on full license allocations. For this auction the allocative exposure problem does not arise. Truthful bidding is possible and optimal, so bidder 1 can choose $B_1(1,1,1) = \pi_1(1,1,1)$ and $B_1(1,2,0) = \pi_1(1,2,0)$ in the example of the previous paragraph. Note that to the best of my knowledge, no auction has allowed bids on allocations in practice. Bids on allocations have certain disadvantages. One negative aspect is that the rich bidding language increases the complexity of the auction, so it might be computationally infeasible to compute or report values for all allocations. Another property that might be difficult to implement is that losers of the auction might have to pay. Since bidders can submit positive bids on allocations in which they receive nothing, they might also have to pay for their influence on the final allocation. This makes sense from an economic perspective, but might face objections in practice.

Proposition 7. *The VCG auction with bids on full license allocations and $Y_i = X$ for all $i \in N$ is auction-efficient for all $M \in \mathcal{M}$.*

I now come to the design of socially efficient auctions. A license allocation is socially efficient if it maximizes the sum of industry profits and consumer surplus.

Industry profits and consumer surplus can, in principle, be maximized by the same license allocation, but they can also be maximized by two different allocations. In the first case, maximizing social welfare is relatively simple, because both terms are maximized by the same license allocation. If the two maximizing allocations are different, however, then social welfare might be maximized by a third allocation.

Consumers' preferences over auction outcomes matter for the social efficiency of auctions. Suppose that consumers are indifferent between the no-undersell allocations, and that industry profits are maximized by no-undersell allocations. Then auction-efficiency and social efficiency would coincide and the VCG auction with bids on license allocations would be socially efficient. However, Proposition 3 shows that consumers are indifferent between the no-undersell license allocations if and only if marginal costs are constant in q_i and all firms have the same linear cost reduction technology. In this special case firms have preferences over their own licenses and the VCG auction with bids on licenses is auction-efficient. Hence, under certain conditions the VCG auction with bids on licenses is socially efficient if and only if it is auction-efficient. In the general case, bids on licenses are required so that the auction has a dominant strategy.

The following proposition shows that there is no socially efficient auction. The proof basically argues that an auction that is socially efficient for all markets $M \in \mathcal{M}$ must also be auction-efficient, because Propositions 4 and 5 show that there are markets in which the auction-efficient license allocation is socially efficient. In these cases, consumer surplus and industry profits are maximized by the same license allocation. However, the propositions also imply that there are markets for which the auction-efficient allocation is not socially efficient, as consumer surplus and industry profits are maximized by distinct license allocations.

Proposition 8. *There is no auction that is socially efficient for all markets $M \in \mathcal{M}$.*

The fundamental reason for this impossibility result is that only n out of $n+1$ agents participate in the mechanism. The participating agents are the firms and the non-participating agent is the representative consumer. If the consumer would also participate in the mechanism, then the VCG mechanism with bids on full license allocations would be socially efficient. Note that the representative consumer does not want to win any licenses. She would only submit positive bids for license allocations in which she receives zero licenses. The consumer would have to pay for her influence on the license allocation through the VCG price, and for the services consumed in the market after the auction. The feasibility of consumers participating in a license allocation seems questionable.

Another remark concerns the generality of the result. In the model firms produce a homogeneous consumption good in Cournot competition, and the licenses are homogeneous. The impossibility result continues to hold when an auction is to be designed that is socially efficient for a set of models that includes the current model.

To summarize, it is impossible to design an auction that is socially efficient for all markets $M \in \mathcal{M}$. There are two important implicit objectives in the last sentence. The first requires that the auction is socially efficient. The second objective is that the auction is socially efficient for all markets. Both objectives will be weakened in the remainder of the paper. The auctioneer might have some information about the environment and this information might turn out to be useful for auction design. One can also give up the objective of finding the first best license allocation and rather look for an allocation that finds a balance between maximizing industry profits and maximizing consumer surplus. The next section looks for auction designs that do this for a certain class of markets.

5 Caps as a Design Instrument

In this section, I discuss cases in the linear Cournot model in which the VCG auction with bids on license allocations and caps can perform well in terms of industry profits and consumer surplus. From above it is known that this auction format certainty is not socially efficient for all markets. Many spectrum auctions are supposed to be auction-efficiency, but feature caps (upper bounds) on the amount a bidder can win. These caps certainly do not improve auction-efficiency, because it might be auction-efficient that one bidder wins the full supply, which is ruled out by caps. The informal argument in favor of the caps is that they are remedies to insure competition in the market after the auction. In particular, they are supposed to insure that all firms are relatively similar after the auction. Caps allow firm i to win at most $\bar{x}_i \leq 1$. Formally, the bidding language is such that $Y_i \subseteq \{x \in X | x_i \leq \bar{x}_i\}$. The caps do not imply undersell, i.e. $\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_n > 1$. Firms can face different, but also identical caps. For simplicity, suppose all firms face the same cap $\bar{x} \geq 1/n$.

Caps can have ambiguous effects on consumer surplus and social welfare. On the one hand, caps unambiguously decrease social welfare when it is socially efficient that one firm wins the full supply. This can be the case when the cost reduction is linear (Prop. 4). On the other hand, caps unambiguously raise consumer surplus when all firms have the same strictly convex cost reduction technology. Proposition 5 shows that aggregate output is a symmetric and concave function on $\bar{X}$, and that consumer surplus is maximized by the allocation $(1/n, \ldots, 1/n)$. Caps decrease the set of license allocations that can be implemented. This decrease is in favor of the consumers, as the final allocation will be closer to the consumers' bliss point. Setting caps in favor of consumers does not require detailed knowledge about the market, as consumer surplus is non-decreasing in $\bar{x}$ when the cost reduction is the same strictly convex function for all firms. This is the first part of the following proposition. Industry profits are certainly non-increasing in caps when the VCG auction with bids on full allocations and caps is used. Thus, social welfare does not need to be non-decreasing in the caps.

The next proposition shows that the VCG auction with bids on full allocations together with caps can strike a balance between maximizing industry profits and maximizing consumer surplus. Suppose all firms are identical and have the same strictly convex cost reduction technology. Example 1 illustrates that the VCG auction with bids on full allocations and without caps can minimize consumer surplus on the set of no-undersell allocations. Proposition 5, however, shows that industry profits and consumer surplus are maximized by the same license allocation when the market is sufficiently large. In the latter case, industry profits are concave on $\bar{X}$. Global concavity is not needed when there are caps. In the proof, I show that industry profits have a local maximum at the consumer surplus maximizing allocation $(1/n, \ldots, 1/n)$ when the market size a is sufficiently large relative to the cost function and its derivatives.[12] Sufficiently large caps transform the local maximum into the global maximum on the restricted domain. Hence, caps rule out the possible global maximum of industry profits that could lead to a very small consumer surplus, whereas the VCG auction locally maximizes industry profits.

Proposition 9. *Consider the linear Cournot model with inverse demand $P(Q) = a - b \cdot Q$, and let all firms have the same strictly convex cost reduction technology. Consider the VCG auction with bids on license allocations. Consumer surplus is non-increasing in the cap $\bar{x} \in (1/n, 1)$. Moreover, with identical initial marginal costs, and the market size a sufficiently large, there exists caps $\bar{x}$ such that the auction outcome maximizes consumer surplus.*

[12]The market size must be large in the second part of the proposition. The proof in the appendix shows that the proposition can also be formulated to require that the curvature of the cost reduction technology $\rho''(1/n)$ is sufficiently large at $1/n$, or that the absolute value of $\rho'(1/n)$ is sufficiently small. The parameters can be such that the local maximum is also the global maximum of industry profits.

Depending on the parameters of the model, the consumer surplus maximizing allocation can be socially efficient but not auction-efficient, so caps can restore social efficiency. This is the case in the example below.

Example 2. Let $n = 3$, $a = 10$, $b = 1$, and $c_1(x) = c_2(x) = c_3(x) = 3 - \sqrt[4]{3x}$. As all firms have the same strictly convex cost reduction technology, consumer surplus is maximized by the allocation $(1/3, 1/3, 1/3)$. Industry profits are globally maximized by allocations in which two firms win 1/2 each, as $\pi(1/2, 1/2, 0) = 12.04$, $\pi(1/3, 1/3, 1/3) = 12$, and $\pi(1, 0, 0) = 11.53$. Social welfare is maximized by the consumer surplus maximizing allocation. The VCG auction with bids on allocations and without caps would implement the allocation $(1/2, 1/2, 0)$. With caps $\bar{x} = 0.4$, however, the local maximum $(1/3, 1/3, 1/3)$ of π is the global maximum on the restricted domain.

The second part of the proposition holds for a purely symmetric setting in which all firms have the same cost function. Suppose there are small asymmetries in the initial marginal costs. The consumer surplus maximizing allocation does not change, but the local maximum of industry profits is different. Hence, the VCG auction with caps does no longer lead to the maximization of consumer surplus, but it nevertheless still leads to a balance between industry profits and consumer surplus. When the asymmetries are not too large, then by continuity, a license allocation is implemented that locally maximizes industry profits and that is close to the consumer optimal allocation. Asymmetries in the cost reduction technologies also change the consumer optimal allocation, but the basic notion of a balance remains. The auction-efficient allocation might lie on the boundary of $\bar{X}$ and might minimize consumer surplus. This allocation is ruled out by the caps. Industry profits are then maximized locally. To summarize, the VCG auction with bids on license allocations together with appropriate caps can balance maximizing industry profits and consumer surplus. It locally maximizes industry profits in a neighborhood of the consumer surplus maximizing allocation.

Setting optimal caps certainly requires information about the market that might not always be available. For consumers, caps cannot be too high when the cost reduction technologies are identical, because then consumer surplus is non-increasing in the size of the caps. When the cost reduction technologies are not the same, then caps close to $1/n$ might be too high, as these high caps might exclude the consumer surplus maximizing allocation. Industry profits might not have a local maximum in the interior of the set of no-undersell allocations. In this case the caps will be binding for some bidders in the final allocation. Caps are also binding if industry profits have a local maximum in the interior of $\bar{X}$, but the caps were chosen too small. Suppose firms have already existing license holdings and the same convex technology. Consumer surplus is maximized by the license allocation that equalizes the license holdings. When there are reasons to assume that firms use the same convex technology, global caps might be beneficial for consumer surplus.

6 Extension: Entry and Exit and Set-Asides

So far it the case has been considered in which all firms are active in the market after the auction for all auction outcomes. In this section, I analyze the case in which one firm is not always active in the linear Cournot model. This allows formally studying the effects of set-asides on consumer surplus and industry profits. Set-asides are licenses reserved for a specific group of bidders and a design instrument that is deemed to improve the competition after the auction (Cramton et al., 2011). The section first introduces the formal changes in the model. Subsequently, I review the above findings in the light of the changes. Finally, I discuss the trade-off between auction-efficiency and consumer surplus.

The model is changed in the following way. The first $n-1$ firms have marginal cost functions such that they are active independent of the auction allocation. Firm n, however, has a handicap as it needs to win at least $\epsilon \in (0,1)$ such that it is active.[13] The firm can be considered as a potential entrant who needs a certain mass of licenses in order to profitably operate its business. Alternatively, it can be seen as a firm that has been active in the market, but has to exit if it does not win sufficiently many licenses. These asymmetries can arise, for instance, endogenously through sequential auctions. New notation is required. Let $\pi(x|k)$ be the equilibrium industry profits in the Cournot continuation game, where the license allocation x is such that $k \in \{n-1, n\}$ firms are active. Other functions are defined analogously.

The characterization of the preferences changes in this model. For firm $i < n$ it naturally makes a difference whether $x_n \geq \epsilon$ or not. Conditional on firm i winning x_i, it wants firm n to win as much as possible as long as $x_n < \epsilon$. This holds true even when all firms have the same linear cost reduction. As a result, it always has preferences over full license allocations. Firm n has preferences over own licenses if and only if other firms have the same linear cost reduction technology.[14] As

[13]The cost function $c_n(x)$ does not need to be differentiable at ϵ and does not need to be strictly decreasing on $[0, \epsilon)$.

[14]Profits are zero for $x_n < \epsilon$ and might make a jump at ϵ. This provides a micro foundation for complementarities in utility functions.

consumer surplus depends on the number of active firms, consumers always have non trivial preferences over no-undersell license allocations.

A set-aside is a certain amount of licenses reserved for a particular bidder or a group of bidders. In practice set-asides are used to support small businesses in auctions. It is modeled as an amount $x^{sa} \in [0, 1]$ reserved for firm n. The timing is the following. The auctioneer announces an auction format including the set-aside. Firm n decides on how much $y_n \leq x^{sa}$ to buy before the auction. The difference $1 - y_n$ is allocated through the conventional auction. Firm n can participate in the regular auction after receiving y_n. The price for y_n can be some reservation price, for example zero. Firm n buys the set-aside if it knows that it will be active in the market after the auction.

I now consider a situation in which the set-aside improves consumer surplus. Suppose the first $n - 1$ firms are symmetric, that is $c = c_1 = c_2 = \cdots = c_{n-1}$, and have strictly convex cost reduction technologies. The handicap is not too large, $\epsilon < 1/n$, and as long as firm n wins more than ϵ, it is identical to the other firms, so $c_n(x) = c(x)$ for $x \in [\epsilon, 1]$. Consumer surplus is potentially maximized by two allocations. The first candidate maximizes consumer surplus when all firms are active, i.e. $(1/n, \ldots, 1/n)$. The second candidate maximizes consumer surplus conditional on firm n not being active, that is, $(1/(n-1), \ldots, 1/(n-1), 0)$. Consumers prefer more active firms over less, but they also prefer lower sums of marginal costs. Suppose the cost reduction technology is very effective. Then it might be that aggregate output is higher with $n - 1$ active firms, because the cost reduction outweighs the competitive effect. On the other hand, when the cost reduction hardly matters, consumers prefer an allocation in which n firms are active in the market after the auction. This effect dominates when, for a fixed profile of cost functions, the market size is large enough.

An auction-efficient auction will always lead to a license allocation in which no entry occurs ($x_n = 0$), because industry profits are higher when fewer firms are active. Depending on the model, this can be socially efficient, but also inefficient. If the auctioneer knows the market sufficiently well, she might set the correct set-aside. When the market is large enough, then a set-aside $x^{sa} \in [\epsilon, 1/n]$ corrects the unambiguous trade-off between industry profits and consumer surplus in favor of consumers. Moreover, like in Proposition 9, the VCG auction with bids on license allocations and caps locally maximizes industry profits at the allocation at which consumer surplus is maximized.

Proposition 10. *Consider the linear Cournot model with inverse demand* $P(Q) = a - b \cdot Q$ *and all firms with identical marginal cost functions. Firm* n *needs to win*

ϵ in order to be active. When the market size a is sufficiently large, then the VCG auction with bids on license allocations and appropriate caps and set-asides maximizes consumer surplus.

In the next example I show that without caps and set-asides the VCG auction with bids on allocations implements the allocation in which only two firms are active. Caps and set-asides change the auction outcome in favor of the consumers. Choosing the right set-aside obviously requires sufficient information about the market. If the set-aside is too small, then no entry will occur. For consumer surplus, a set-aside can only be too high when the handicap is too large or the market size too small, so when consumer surplus would be higher with no additional entry.

Example 3. Reconsider the setting of Example 2, but with firm 3 having costs $c_3(x) = 10$ for $x \in [0, 0.25)$ and $c_3(x) = 3 - \sqrt[4]{3x}$ for $x \in [0.25, 1]$. Industry profits are highest when only the first two firms are winners, i.e. at the allocation $(1/2, 1/2, 0)$. Consumer surplus is highest when $(1/3, 1/3, 1/3)$, as $Q(1/3, 1/3, 1/3|3) = 6$ and $Q(1/2, 1/2, 0|2) = 5.4$. A set-aside $x^{sa} \in [0.25, 1/3]$ guarantees that all three firms are active after the auction. From above it follows that the VCG auction with bids on license allocations and caps $\bar{x} = 0.4$ then implements the consumer surplus maximizing allocation.

7 Conclusion

This paper considers auctions that are followed by market interaction between the bidders. Two notions of efficiency are contrasted: auction-efficiency and social efficiency. An auction is auction-efficient when only bidders' welfare (profits) is maximized. An auction is socially efficient when the welfare of firms and consumers is maximized. For some auctioneers it can be optimal to design auctions in favor of the bidders. Platforms like Facebook or Google compete for the sale of ad positions and use the auction proceeds for financing their own services for the final customers. An auction-efficient auction might attract many bidders and might maximize the platform's profits in the long run. Other auctions involve public to business transactions. In these auctions the appropriate objective can be social efficiency.

The paper shows in a simple and flexible model that there is no auction that is socially efficient and that does not depend upon details of the market. Instead of maximizing social welfare, one can also look for a balance between maximizing industry profits and consumer surplus. Under certain conditions, the VCG auction with bids on full license allocations and caps implements a license allocation that is close to the consumer optimal license allocation and that locally maximizes industry profits. Without caps, the VCG auction with bids on full license allocations is optimal for firms, but might minimize consumer surplus. Set-asides are another design feature that can greatly improve consumer surplus. Again under certain conditions, there is no entry when the auction is auction-efficient, as industry profits are larger the fewer firms. Consumer surplus increases, however, in the number of active firms when demand is sufficiently large. To summarize, set-asides and caps can help an entrant and improve consumer surplus at the expense of auction-efficiency. The correct implementation of caps and set-asides requires some information about the market.

The paper provides theoretical insights that can be used for practical auction design. First, I show that bids on the entire allocation might be necessary for auctions to have a dominant strategy. Second, in my model auction-efficiency is not a good proxy for social efficiency. Third, I provide a theoretical foundation for nevertheless using an auction format that is auction-efficient. However, it has to be combined with caps and set-asides. This can be crucial for consumer surplus. For example, in the Austrian 2013 spectrum auction one of the three active firms did not win any licenses in the 800 MHz band. This band is important for the provision of fast mobile data services. It has been argued informally that the high revenue of two billion euro was reached because the other two companies tried to foreclose these licenses (Janssen, 2015). It might be that the third competitor will be a weak competitor or even exit the market if it does not win enough low band licenses in a future auction. My model suggests that an auction-efficient auction without caps and set-asides will make the endeavor of the two firms successful. Appropriate design, however, might rule out this outcome in favor of consumer surplus. My model also suggests that if firms have access to the same convex cost technology, the caps should be set so that all firms have similar license holdings. Telecommunication companies arguably do have the same or similar cost technology, as they do have access to the same technology and similar subcontractors who build the network.

Auctions have a dominant strategy only if it allows bids on full allocations. To the best of my knowledge, no auction has used bids on allocations in practice. Allowing bids on full allocations solves the allocative exposure problem, but it also creates some issues that do not arise when bids are only on quantities. One issue is the increased complexity, as the number of license allocations can be very large. When there are only few licenses and few bidders, then the number of required bids might not be higher than for existing package auctions. Another issue is that in auctions with bids on allocations bidders can submit positive bids on allocations in which they receive nothing. The auction outcome can then be such that losers have to pay positive amounts. This might be difficult to implement in reality. More research on auction design is needed to see whether it is worth to use bids on full allocations or not.

A Proofs

Proposition 2. *Firms have preferences over own licenses if and only if there are two firms or when all firms $i \in N$ have costs $C_i'(q_i|x_i) = \theta_i + r \cdot x_i$ for $q_i \in [\min_{x \in \bar{X}} q_i(x), \max_{x \in \bar{X}} q_i(x)]$ and $r < 0$.*

Proof. I first show that the stated conditions imply preferences over own licenses. Let $n = 2$. Conditional on firm i winning x_i, in a no-undersell allocation the other bidder necessarily wins $1 - x_i$. Therefore bidder i has a unique value for winning x_i units. Let $n > 2$ and let all $j \neq i$ have marginal costs given by $c_j(x) = \theta_j + r \cdot x_j$. I will show that $\pi_i(x) = \pi_i(x')$ for all $x, x' \in X$ with $x_i = x_i'$. Let x and x' be two such license allocations. The first-order condition of firm i's profit is

$$P(Q) + P'(Q)q_i - c_i(x_i) = 0. \tag{7}$$

The sum over the n FOCs gives

$$nP(Q) + P'(Q)Q = \sum_{i=1}^{n} c_i(x_i). \tag{8}$$

The right-hand side $\sum_i c_i(x_i) = r + \sum_i \theta_i$ is the same for both allocations. Proposition 1 guarantees a unique Q that solves Equation (8). The indirect profits $\pi_i(x)$ and $\pi_i(x')$ are therefore the same.

For the other direction, I consider bidder 1 without loss of generality. Suppose bidder 1 has preferences over own licenses, i.e. $\pi_1(x) = \pi_1(x')$ for $x_1 = x_1'$ and $x, x' \in \bar{X}$. For any $x \in \bar{X}$ with $x_1 < 1$, I will show that aggregate output does not change when one changes the allocation of licenses among two other firms. This is then used to show that for $i > 1$ the cost function must be linear in q_i and the marginal cost reduction technology must be the same linear function in the relevant range.

First, I will show that aggregate output does not change when licenses are reallocated among two other firms. Let $x \in \bar{X}$ with $x_1 \in [0, 1)$ and $x_2 > 0$. Consider license allocations in which bidder $k \neq 2, 3$ receives x_k, bidder 2 receives

$x_2 - t$ and bidder 3 receives $x_3 + t$, where t small enough. For these license allocations, firm 1's has to solve the parametric optimization problem

$$\max_{q_1 \geq 0} \pi_1(q_1, Q_{-1}(t)) = \max_{q_1 \geq 0} P(q_1 + Q_{-1}(t))q_1 - C_1(q_1|x_1),$$

where $Q_{-1}(t)$ is the aggregate Cournot continuation equilibrium production of the $n-1$ other firms. The value function $\pi_1(t)$ of this maximization problem is then just a function of the parameter t. As the bidder has preferences over own licenses, the equality $\pi_1(t+\epsilon) - \pi_1(t) = 0$ holds for all small t and $\epsilon \neq 0$. Dividing by ϵ and taking $\epsilon \to 0$ implies $\pi_1'(t) = 0$. Theorem 1 of Milgrom and Segal (2002) shows that $\pi_1'(t) = q_1^* P'(Q(t)) \sum_{i>1} \frac{\partial q_i(t)}{\partial t}$. This implies that $\sum_{i>1} \frac{\partial q_i(t)}{\partial t} = 0$, i.e. that $Q_{-1}(t)$ is independent of t. Hence, firm 1's best response is the same for all t and therefore Q is independent of t. From this and Selten's idea of the cumulative best reply it also follows that firm $k > 3$ produces the same quantity for all t (e.g. Vives, 1999, p. 97).

I will now use the information that Q does not depend on the specific allocation of a fixed mass of licenses among two bidders to show that the slope of the marginal cost function is the same for both bidders and therefore independent of the individual output and share. Consider license allocations parameterized by ϵ. Let bidder 1 receive $x_1 - \epsilon$, bidder 2 $x_2 + \epsilon$, and bidder 3 x_3. Slightly abuse notation and let x denote this license allocation. Let x' denote the license allocation in which bidder 1 receives $x_1 - \epsilon$, bidder 2 receives x_2, and bidder 3 gets $x_3 + \epsilon$. The previous paragraph showed that aggregate output Q is the same for both allocations for all ϵ. Let $q_i = q_i(x)$ and $q_i' = q_i(x')$. The sum of first-order conditions is 0 for both allocations. Thus, for any ϵ small enough (in absolute value), the sum of first-order conditions simplifies to

$$nP(Q) + P'(Q)Q - \sum_{i=1}^{n} \frac{\partial C_i(q_i|x_i)}{\partial q_i} = nP(Q) + P'(Q)Q - \sum_{i=1}^{n} \frac{\partial C_i(q_i'|x_i')}{\partial q_i} \tag{9}$$

$$\frac{\partial C_2(q_2|x_2+\epsilon)}{\partial q_2} + \frac{\partial C_3(q_3|x_3)}{\partial q_3} = \frac{\partial C_2(q_2'|x_2)}{\partial q_2} + \frac{\partial C_3(q_3'|x_3+\epsilon)}{\partial q_3}.$$

Rearranging, dividing by ϵ, and taking the limit gives

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon} \left(\frac{\partial C_2(q_2|x_2+\epsilon)}{\partial q_2} - \frac{\partial C_2(q_2'|x_2)}{\partial q_2} \right) = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left(\frac{\partial C_3(q_3'|x_3+\epsilon)}{\partial q_3} - \frac{\partial C_3(q_3|x_3)}{\partial q_3} \right)$$

$$\frac{\partial^2 C_2(q_2|x_2)}{\partial q_2 \partial x_2} = \frac{\partial^2 C_3(q_3|x_3)}{\partial q_3 \partial x_3}.$$

The last equality says that the derivative of marginal costs with respect to x_i is the same for both firms, independent of the levels of q_2 and q_3, and independent of the levels of x_2 and x_3. This can only be the case when the costs are affine in q_i (in the relevant range), and when both firms have the same linear cost reduction technology. $\square$

Proposition 3. *Consumers are indifferent between all no-undersell allocations if and only if all firms have marginal costs constant in q_i and the same linear cost reduction technology, i.e. all firms $i \in N$ have costs $C_i'(q_i|x_i) = \theta_i + r \cdot x_i$ for $q_i \in [\min_{x\in\bar{X}} q_i(x), \max_{x\in\bar{X}} q_i(x)]$ and $r < 0$.*

Proof. Let consumers be indifferent between all no-undersell allocations, i.e. $Q(x) = Q(x')$ for all x, $x' \in \bar{X}$. The proof is as in the proof of Proposition 2, as Equation (9) implies that the sum of marginal costs must be the same for all no-undersell allocations. This can only be the case when marginal costs are constant in q_i and firms reduce marginal costs in the same linear way.

The proof in the other direction is also as in Proposition 2. Plugging $c_i(x_i) = \theta_i + r \cdot x_i$ into Equation (8) shows that Q is the same for all allocations. Thus, consumer surplus is maximized by any no-undersell allocation. $\square$

Proposition 4. *Consider the linear Cournot model with $P(Q) = a - b \cdot Q$ and bidders with a linear cost reduction technology, so $c_i(x_i) = \theta_i + r_i \cdot x_i$ for all $i \in N$. Firm i's profits $\pi_i(x)$ and industry profits $\pi(x)$ are convex on X. Suppose firms have the same cost reduction technology, i.e. $r_1 = r_2 = \cdots = r_n = r$. Industry profits are maximized by the license allocation in which the firm with the lowest initial marginal costs wins the full supply. Suppose the cost reduction has $r_1 \leq r_2 \leq \cdots \leq r_n$. Consumer surplus is maximized by the license allocation $(1, 0, \ldots, 0)$. This license allocation maximizes industry profits when a is large enough or when $\theta_1 = \theta_2 = \cdots = \theta_n$.*

Proof. The proof first shows that firm i's indirect profit function is convex and that therefore also industry profits are convex. Second, industry profits are maximized for identical cost reduction. Finally, I consider consumer surplus and sufficient conditions for the maximization of industry profits in the asymmetric case.

First, without loss of generality, I will show that $\pi_1(x)$ is convex. The function $\pi_1(x)$ is convex if the Hessian $H = H_x\pi_1(x)$ is positive semi-definite, i.e. if $z^T Hz \geq$

0 for all $z \in \mathbb{R}^n \setminus \{0\}$. The Hessian is given by

$$H_x\pi_1(x) = 2\begin{pmatrix} \frac{n^2c_1'(x_1)^2}{b(n+1)^2} - \frac{nc_1''(x_1)q_1(x)}{n+1} & \frac{-nc_1'(x_1)c_2'(x_2)}{b(n+1)^2} & \cdots & \frac{-nc_1'(x_1)c_n'(x_n)}{b(n+1)^2} \\ \frac{-nc_1'(x_1)c_2'(x_2)}{b(n+1)^2} & \frac{c_2'(x_2)^2}{b(n+1)^2} + \frac{q_1(x)c_2''(x_2)}{n+1} & & \frac{c_2'(x_2)c_n'(x_n)}{b(n+1)^2} \\ \vdots & & \ddots & \vdots \\ \frac{-nc_1'(x_1)c_n'(x_n)}{b(n+1)^2} & \frac{c_2'(x_2)c_n'(x_n)}{b(n+1)^2} & \cdots & \frac{c_n'(x_n)^2}{b(n+1)^2} + \frac{q_1(x)c_n''(x_n)}{n+1} \end{pmatrix} \tag{10}$$

$$= \frac{2}{b(n+1)^2}\begin{pmatrix} n^2r_1^2 & -nr_1r_2 & \cdots & -nr_1r_n \\ -nr_1r_2 & r_2^2 & & r_2r_n \\ \vdots & & \ddots & \vdots \\ -nr_1r_n & r_2r_n & \cdots & r_n^2 \end{pmatrix}. \tag{11}$$

Let $z \in \mathbb{R}^n \setminus \{0\}$. Plugging in gives

$$\begin{aligned} z^THz &= z^T\begin{pmatrix} n^2r_1^2z_1 - nr_1r_2z_2 - \cdots - nr_1r_nz_n \\ -nr_1r_2z_1 + r_2^2z_2 + \cdots + r_2r_nz_n \\ \vdots \\ -nr_1r_nz_1 + r_2r_nz_2 + \cdots + r_n^2z_n \end{pmatrix} \\ &= \begin{matrix} n^2r_1^2z_1^2 - nr_1r_2z_1z_2 - \cdots - nr_1r_nz_1z_n \\ -nr_1r_2z_1z_2 + r_2^2z_2^2 + \cdots + r_2r_nz_2z_n \\ \vdots \\ -nr_1r_nz_1z_n + r_2r_nz_2z_n + \cdots + r_n^2z_n^2 \end{matrix} \\ &= (-nr_1z_1 + r_2z_2 + \cdots + r_nz_n)^2 \geq 0, \end{aligned}$$

where the last equality follows from the multinomial theorem. Hence, the Hessian is positive semi-definite, so π_1 and all other firms' profit functions are convex. Industry profits $\pi(x)$ are the sum of convex functions and therefore convex.

Second, suppose all firms have the same linear cost reduction technology $r_1 = r_2 = \cdots = r_n = r < 0$. The objective is to maximize industry profits. The solution of the maximization of a convex function on a convex set lies on the boundary by Theorem 32.2 of Rockafellar (1970). This theorem also implies that there can be only one winner as the simplex is the convex hull of the extreme points $\{e_i : i \in N\}$, where $e_i = (0, \ldots, 0, 1, 0, \ldots, 0)$, with the 1 is at the i^{th} position. Without loss of generality, suppose $\theta_i \leq \theta_j$ for all $j \neq i$. The inequality

$\pi(e_i) \geq \pi(e_j)$ is equivalent to

$$\begin{aligned} &\frac{\left(a - n(\theta_i + r) + \theta_j + \sum_{k \neq i,j} \theta_k\right)^2}{b(n+1)^2} + \frac{\left(a - n\theta_j + \theta_i + r + \sum_{k \neq i,j} \theta_k\right)^2}{b(n+1)^2} \\ \geq &\frac{\left(a - n(\theta_j + r) + \theta_i + \sum_{k \neq i,j} \theta_k\right)^2}{b(n+1)^2} + \frac{\left(a - n\theta_i + \theta_j + r + \sum_{k \neq i,j} \theta_k\right)^2}{b(n+1)^2} \end{aligned} \tag{12}$$

and simplifies to the condition $\theta_i \leq \theta_j$ after routine algebra.

Now I come to the case with asymmetric cost reduction $r_1 \leq r_2 \leq \cdots \leq r_n$. Initial marginal costs are arbitrary. Consumer surplus is strictly increasing in industry output $Q(x)$, hence it is maximized when $Q(x)$ is maximized. Industry production

$$Q(x) = \frac{an - \sum_{i=1}^{n} \theta_i + r_i \cdot x_i}{b(n+1)}$$

is increasing in every x_i. The increase is largest for x_1. There are no interaction effects between the x_i's, thus it is optimal to allocate the full supply to firm 1.

Next, I show that the consumer surplus maximizing license allocation maximizes industry profits when the market size measured by a is large enough. The inequality $\pi(e_1) \geq \pi(e_2)$ is equivalent to an inequality similar to Equation (12) and simplifies to

$$\frac{(n^2+1)\left(r_1^2 - r_2^2\right) - 2(n+1)\left(\theta_2(r_1 + nr_2) - \theta_1(r_2 + nr_1)\right)}{2(n-1)} > (a + \Theta)\left(r_1 - r_2\right),$$

with $\Theta = \sum_i \theta_i$. The left-hand side is independent of a, whereas the right-hand side becomes arbitrarily small for a large enough. Thus, for a large enough industry profits are always maximized by the license allocation in which the firm with the most effective cost reduction wins the full supply.

Finally, let $\theta_1 = \theta_2 = \cdots = \theta_n = \theta$ in addition to $r_1 \leq r_2 \leq \cdots \leq r_n$. I will show that the function $\pi(e_i)$ is strictly decreasing in r_i. All that needs to be done is to take the first derivative of $\pi(e_i)$ with respect to r_i. But first I rewrite industry profits as

$$\begin{aligned} \pi(e_i) &= \frac{(a - n(\theta + r_i) + (n-1)\theta)^2}{b(n+1)^2} + (n-1)\frac{(a - n\theta + (n-1)\theta + r_i)^2}{b(n+1)^2} \\ &= \frac{(a - \theta - nr_i)^2}{b(n+1)^2} + (n-1)\frac{(a - \theta + r_i)^2}{b(n+1)^2}. \end{aligned}$$

The assumption that all firms are active independent of the license allocation requires $a - \theta > 0$. The first derivative of $\pi(e_i)$ with respect to r_i is

$$\begin{aligned}\frac{\partial \pi(e_i)}{\partial r_i} &= \frac{2}{b(n+1)^2}\left((a-\theta-nr_i)(-n)+(n-1)(a-\theta+r_i)\right) < 0 \Leftrightarrow \\ &= -a+\theta+n^2 r_i+(n-1)r_i < 0.\end{aligned}$$

The last inequality is true, because $r_i < 0$ and because $a - \theta > 0$. By symmetry, $r_1 \leq r_i$ implies $\pi(e_1) \geq \pi(e_i)$, showing that industry profits are maximized by the same license allocation as consumer surplus. □

Proposition 5. *Consider the linear Cournot model with inverse demand $P(Q) = a - b \cdot Q$ and bidders with the same strictly convex cost reduction technology, so $c_i(x_i) = \theta_i + \rho(x_i)$ for all $i \in N$ and $\rho'' > 0$. Firm i's profits $\pi_i(x)$ are never concave on X. Industry profits can be convex, but are concave when a is sufficiently large. In any case, consumer surplus is maximized by the license allocation $(1/n, \ldots, 1/n)$. This license allocation is a critical point of industry profits only if $\theta_1 = \theta_2 = \cdots = \theta_n$.*

Proof. Let $c_i''(x_i) = \rho''(x_i) > 0$ for all $x_i \in [0,1]$ and $i \in N$. First, I show that firm i's profit function π_i cannot be concave. Second, industry profits are concave when a is large. Third, I show that industry profits can be convex. Fourth, I show that industry output $Q(x)$ is a concave function and therefore consumer surplus is maximized by the allocation $(1/n, \ldots, 1/n)$. Finally, I look at first order necessary conditions for the maximization of industry profits and find that the license allocation $(1/n, \ldots, 1/n)$ can only maximize $\pi(x)$ if $\theta_1 = \theta_2 = \cdots = \theta_n$.

First, I show that the profit function is never concave. Let $c_i''(x_i) > 0$ for all $x_i \in [0,1]$ and all $i = 1, \ldots, n$. Consider firm 1 without loss of generality. I check the leading minors of the Hessian matrix of $\pi_1(x)$ (Equation (10)) to test for concavity in x. A function is concave if and only if the Hessian is negative-semidefinite. A square matrix is negative-semidefinite if and only if all principal minors of the matrix have alternating sign. In particular, the first leading minor must be non-positive, and the second leading minor must be non-negative. Let the first leading minor be non-positive, i.e.

$$\frac{\partial^2 \pi_1(x)}{\partial x_1^2} \leq 0 \Leftrightarrow nc_1'(x_1)^2 \leq (n+1)c_1''(x_1)q_1(x). \tag{13}$$

The second leading minor

$$\frac{\partial^2 \pi_1(x)}{\partial x_1^2}\frac{\partial^2 \pi_1(x)}{\partial x_2^2} - \left(\frac{\partial^2 \pi_1(x)}{\partial x_1 \partial x_2}\right)^2$$

simplifies to

$$\frac{2nq_1(x)}{n+1}\left(-c_1''(x_1)c_2'(x_2)^2 + nc_1'(x_1)^2c_2''(x_2) - (n+1)c_1''(x_1)c_2''(x_2)q_1(x)\right).$$

Use Inequality (13) to bound the second leading minor from above and observe that this upper bound is strictly less than 0, i.e.

$$\frac{2nq_1(x)}{n+1}\left(-c_1''(x_1)c_2'(x_2)^2 + (n+1)c_1''(x_1)c_2''(x_2)q_1(x) - (n+1)c_1''(x_1)c_2''(x_2)q_1(x)\right) < 0.$$

Hence, both leading minors are negative, implying that π_1 cannot be concave.

Second, I show that industry profits are concave when a is sufficiently large. Industry profits are concave if its Hessian $H_x\pi(x) < 0$ is negative definite. A strongly diagonally dominant matrix is negative definite if all main diagonal elements are negative (Horn and Johnson, 1985). Thus, I have to show that the elements on the main diagonal are negative and that the matrix is strongly diagonally dominant when a sufficiently large. A matrix A is strongly diagonally dominant when $\sum_{j\neq i}|a_{ij}| < |a_{ii}|$ for all $i = 1, \ldots, n$. For the Hessian I take the first partial derivative[15] of

$$\frac{\partial \pi(x)}{\partial x_i} = \frac{2}{n+1}\left(q_i(x)(-nc_i'(x_i)) + \sum_{k\neq i} q_k(x)c_i'(x_i)\right) \tag{14}$$

with respect to x_i and x_j, respectively,

$$\begin{aligned}
\frac{\partial^2 \pi(x)}{\partial x_i^2} &= \frac{2}{n+1}\left(\frac{n^2 c_i'(x_i)^2}{b(n+1)} - nq_i(x)c_i''(x_i) + \sum_{j\neq i}\frac{c_i'(x_i)^2}{b(n+1)} + q_j(x)c_i''(x_i)\right)\\
&= \frac{2}{n+1}\left(\frac{(n^2+n-1)c_i'(x_i)^2}{b(n+1)} + c_i''(x_i)(Q(x) - (n+1)q_i(x))\right) \qquad (15)\\
\frac{\partial^2 \pi(x)}{\partial x_i \partial x_j} &= \frac{2}{n+1}\left(\frac{(-n-2)c_i'(x_i)c_j'(x_j)}{b(n+1)}\right). \qquad (16)
\end{aligned}$$

[15]From the partial derivative of π with respect to x_i it follows that industry profits are strictly increasing in x_i when $q_i(x) \geq q_j(x)$, as $Q(x) - (n+1)q_i(x) < 0$.

The Hessian is then

$$H_x\pi(x) = \begin{pmatrix} \frac{\partial^2\pi(x)}{\partial x_1^2} & \frac{\partial^2\pi(x)}{\partial x_1\partial x_2} & \cdots & \frac{\partial^2\pi(x)}{\partial x_1\partial x_n} \\ \frac{\partial^2\pi(x)}{\partial x_1\partial x_2} & \frac{\partial^2\pi(x)}{\partial x_2^2} & & \frac{\partial^2\pi(x)}{\partial x_2\partial x_n} \\ \vdots & & \ddots & \vdots \\ \frac{\partial^2\pi(x)}{\partial x_1\partial x_n} & \frac{\partial^2\pi(x)}{\partial x_2\partial x_n} & & \frac{\partial^2\pi(x)}{\partial x_n^2} \end{pmatrix}.$$

There are markets for which the Hessian is negative definite. For a sufficiently large, $c_i''(x_i)(Q(x)-(n+1)q_i(x)) < 0$, as $c_i'' > 0$ and $Q(x)-(n+1)q_i(x) = -a+f(x)$, where f is a function independent of a. Thus, for a sufficiently large the elements on the main diagonal are arbitrarily small. What is more, the elements off the main diagonal are independent of a, so the matrix is strongly diagonally dominant for a sufficiently large.

Third, there are markets for which industry profits are convex. Fix an inverse demand function P and consider a sequence of cost function profiles $(c_t)_{t\in\mathbb{N}}$ with point-wise convergence to a linear cost function profile, i.e. for $\tilde{x} \in [0,1]$, $\rho_t'(\tilde{x}) \to r < 0$, $\rho_t''(\tilde{x}) > 0$, and $\rho'' \to 0$. Proposition 4 shows that industry profits are convex in the limit. Along the sequence there are strictly convex cost function profiles for which industry profits are convex.

Fourth, I consider the maximization of industry output $Q(x)$. Consumer surplus is maximized by the license allocation that maximizes $Q(x)$. The Hessian of Q is a diagonal matrix with diagonal element i equal to $-c_i''(x_i)$. Hence, the Hessian of Q is negative definite and $Q(x)$ is concave. Industry production $Q(x)$ is maximized by consider the constrained optimization problem

$$\max_x \sum_{i=1}^n -c_i(x_i) \text{ s.t. } \sum_{i=1}^n x_i \leq 1 \text{ and } 0 \leq x_i \text{ for all } i \in N.$$

The Lagrangian

$$\mathcal{L} = \sum_{i=1}^n -c_i(x_i) + \lambda \left(1 - \sum_{i=1}^n x_i\right) - \sum_{i=1}^n \mu_i x_i$$

leads to the Kuhn-Tucker conditions

$$-c_i'(x_i) - \lambda - \mu_i = 0,\ \mu_i x_i = 0, \text{ for all } i \in N \text{ and } \lambda(1 - x_1 - x_2 - \cdots - x_n) = 0.$$

As the objective function is strictly increasing in every dimension, it must be the case that the feasibility constraint is binding and that therefore $\lambda > 0$. The

Kuhn-Tucker conditions hold for $x_i = 1/n$, $\mu_i = 0$ for all i and $\lambda = -c_i'(1/n) = -\rho'(1/n) > 0$. The concave objective function implies that the license allocation $x = (1/n, \ldots, 1/n)$ is indeed a maximizer of $Q(x)$.

Last, I maximize industry profits π. The first derivative of industry profits with respect to x_i is given in Equation (14). The allocation $\hat{x} = (1/n, \ldots, 1/n)$ is a critical point if the Kuhn-Tucker conditions for the maximization of industry profits are satisfied. These conditions require that

$$\begin{aligned}\frac{\partial \pi(\hat{x})}{\partial x_i} &= \frac{\partial \pi(\hat{x})}{\partial x_j} \Leftrightarrow \\ c_i'(1/n)(-nq_i(\hat{x}) + Q(\hat{x}) - q_i(\hat{x})) &= c_j'(1/n)(-nq_j(\hat{x}) + Q(\hat{x}) - q_j(\hat{x})) \Leftrightarrow \\ q_i(\hat{x}) &= q_j(\hat{x}) \Leftrightarrow \\ c_i(1/n) &= c_j(1/n),\end{aligned}$$

which is only true for all firms if $\theta_1 = \theta_2 = \cdots = \theta_n$. □

Proposition 6. *The VCG auction with bids on licenses is not auction-efficient for all markets $M \in \mathcal{M}$. Let P be log-concave and suppose industry profits are maximized by a no-undersell allocation. The auction is auction-efficient if and only if all firms have a cost function linear in q_i and the same linear cost reduction technology, i.e. all firms $i \in N$ have costs $C_i'(q_i|x_i) = \theta_i + r \cdot x_i$ for $q_i \in [\min_{x \in \bar{X}} q_i(x), \max_{x \in \bar{X}} q_i(x)]$ and $r < 0$.*

Proof. The first statement follows immediately from the second. I will therefore only prove the second statement. Suppose every firm $i \in N$ has costs $C_i'(q_i|x_i) = \theta_i + r \cdot x_i$ for $q_i \in [\min_{x \in \bar{X}} q_i(x), \max_{x \in \bar{X}} q_i(x)]$ and $r < 0$. I will show that the VCG auction with bids on licenses has a dominant strategy that implements the auction-efficient allocation. Proposition 2 shows that every firm has preferences over own licenses. I will show that bidding $B_i(x_i) = \pi_i(x_i, x_{-i}) - \pi_i(0, z_{-i})$ for $(x_i, x_{-i}), (0, z_{-i}) \in \bar{X}$ is a dominant strategy. For this, I first show that $B_i'(x_i) > 0$. As the final allocation is chosen to maximize the sum of bids, the final allocation has no undersell if at least one bidder submits a strictly increasing bidding function. The second step shows that it is weakly dominant strategy to bid truthfully. When all bidders play their weakly dominant strategy, the auction implements the auction-efficient license allocation.

First, I show that the bidding function $B_i(x_i)$ is strictly increasing. Consider the profit maximization problem

$$\max_{q_i} P(Q_{-i}(x_i) + q_i) \cdot q_i - c_i(x_i) \cdot q_i,$$

where $Q_{-i}(x_i)$ is the Cournot continuation equilibrium production of the $n-1$ firms when the license allocation is $(x_i, x_{-i}) \in \bar{X}$. This is a unique number because of the assumptions on the firms' cost functions. Let $V_i(x_i)$ denote the corresponding value function. The envelope theorem (Milgrom and Segal, 2002) implies that

$$V'(x_i) = q_i \cdot P'(Q) \cdot \frac{\partial Q_{-i}(x_i)}{\partial x_i} - c_i'(x_i) \cdot q_i.$$

Vives (1999) shows that aggregate output decreases when x_i is increased, because the best-response functions are decreasing due to the log-concave inverse demand. Hence, profits increase in x_i as both terms are positive. The second term is $-c_i'(x_i) = -r > 0$. The first term $P'(Q) \cdot \frac{\partial Q_{-i}(x_i)}{\partial x_i}$ is the product of two negative expressions. Note that $Q(x)$ is constant on $\bar{X}$ and that the best-response against Q_{-j} has a negative slope. Thus, the aggregate output of the other firms must decrease when firm i receives more licenses.

Now I will show that the bidding function B_i is a dominant strategy. If a bidder uses a strictly increasing bidding function, the final allocation will have no undersell. Let other bidders use the profile of bidding functions B_{-i}. Let $x \in \bar{X}$ denote the no-undersell allocation that is implemented when (B_i, B_{-i}) is played and let $y \in X$ be the license allocation when $(\tilde{B}_i, B_{-i})$ is played, where $\tilde{B}_i$ is any other bidding function of bidder i. If y is a no-undersell allocation, then by the definition of x it is clear that truthful bidding leads to a weakly higher expected utility, i.e.

$$B_i(x_i) - \max_{\tilde{x} \in X} \sum_{j \neq i} B_j(\tilde{x}_j) + \sum_{j \neq i} B_j(x_j) \geq B_i(y_i) - \max_{\tilde{x} \in X} \sum_{j \neq i} B_j(\tilde{x}_j) + \sum_{j \neq i} B_j(y_j) \Leftrightarrow$$
$$\pi_i(x) - \max_{\tilde{x} \in X} \sum_{j \neq i} B_j(\tilde{x}_j) + \sum_{j \neq i} B_j(x_j) \geq \pi_i(y) - \max_{\tilde{x} \in X} \sum_{j \neq i} B_j(\tilde{x}_j) + \sum_{j \neq i} B_j(y_j).$$

If y is an undersell allocation, then profits increase by increasing bidder i's share. This can be done until a no-undersell allocation is reached, so

$$\pi_i(x) + \sum_{j \neq i} B_j(x_j) \geq \pi_i(y_i + 1 - \sum_{j \neq i} y_j, y_{-i}) + \sum_{j \neq i} B_j(y_j) \geq \pi_i(y) + \sum_{j \neq i} B_j(y_j).$$

I showed that truthful bidding is always a best reply, hence it is weakly dominant.

For the other direction of the proposition, let M be a market so that the VCG auction with bids on licenses is auction-efficient. Hence, every bidder has a dominant strategy, which requires that a unique value is assigned to every $x_i \in [0, 1]$. Proposition 2 shows that this is only true when costs are linear in q_i and all firms have the same linear cost reduction technology. □

Proposition 7. *The VCG auction with bids on full license allocations and* $Y_i = X$ *for all* $i \in N$ *is auction-efficient for all* $M \in \mathcal{M}$.

Proof. The bidding language is certainly rich enough if it allows bids on all license allocations, so when $Y_i = X$. Bidding true marginal values is a dominant strategy, as it is a best response against all bidding functions of other bidders. There are many functions so that the bids reflect the true marginal values, but there is only one degree of freedom. I will show that $B_i(x) = \pi_i(x) - \min_{\tilde{x}} \pi_i(\tilde{x})$ is a dominant strategy. Let other bidders use the profile of bidding functions B_{-i}. When bidder i bids $B_i(x)$, then license allocation x is allocated so that

$$B_i(x) + \sum_{j \neq i} B_j(x) \geq B_i(y) + \sum_{j \neq i} B_j(y) \Leftrightarrow$$
$$\pi_i(x) + \sum_{j \neq i} B_j(x) \geq \pi_i(y) + \sum_{j \neq i} B_j(y)$$

for all $y \in X$. Subtracting $\max_{\tilde{x} \in X} \sum_{j \neq i} B_j(\tilde{x})$ on both sides gives bidder i's utility. No other license allocation gives a higher utility, so deviating does not pay off. □

Proposition 8. *There is no auction that is socially efficient for all markets* $M \in \mathcal{M}$.

Proof. Suppose there is an auction that has a dominant strategy that implements the socially efficient license allocation for all $M \in \mathcal{M}$. The following Lemma shows that any $x \in \bar{X}$ can be socially efficient, so one needs $\bar{X} \subseteq Y_i$ for all $i \in N$. I will then use the revelation principle to come to a contradiction.

Lemma 1. *For every* $x \in \bar{X}$ *there is a market* $M \in \mathcal{M}$ *such that* $x \in \arg\max_{\tilde{x} \in X} CS(\tilde{x})$ *and* $x \in \arg\max_{\tilde{x} \in X} \pi(\tilde{x})$, *so* x *is socially efficient.*

Proof. Let $x \in \bar{X}$. It suffices to work with the linear Cournot model. Thus, let $P(Q) = a - b \cdot Q$ and let marginal costs be independent of q_i and equal to $c_i(x_i) = \theta_i + \rho_i(x_i)$. I will first show that there are marginal cost reduction technologies so that x maximizes consumer surplus. Then I will show that the same license allocation is a critical point of industry profits when the initial marginal costs are chosen appropriately. The third step is to choose consumers' demand large enough so that industry profits are a concave function and the necessary conditions are sufficient.

Consumer surplus is increasing in $Q(x)$ and $Q(x)$ is maximized by minimizing the sum of marginal costs. Let m be such that the first m firms have $x_i > 0$ for $i = 1, \dots, m$ and $x_i = 0$ for $i = m+1, \dots, n$. The optimal allocation must satisfy $\rho_1'(x_1) = \rho_2'(x_2) = \cdots = \rho_m'(x_m)$ and $\rho_1'(x_1) = \rho_i'(0) + \mu_i$ for $i > m$, where μ_i is the non-negativity Lagrange multiplier. There are certainly n functions ρ_i with $\rho_i(0) = 0$, $\rho_i' < 0$, and $\rho'' > 0$ for all i.

The second step is to find initial marginal costs so that the license allocation is a critical point of the maximization of industry profits. The necessary first-order condition for firms $i, j \leq m$ is that $\frac{\partial \pi(x)}{\partial x_i} = \frac{\partial \pi(x)}{\partial x_j}$. These partial derivatives are given by Equation (14). The equality condition is equivalent to $q_i(x) = q_j(x)$, which is true if only if $c_i(x_i) = c_j(x_j)$. These $m-1$ necessary conditions have one degree of freedom. If one chooses one θ_i, then the other θ_j are determined by the choice of θ_i. The initial marginal costs θ_i are chosen such that $c_i(1) \geq 0$. One can also set $\theta_i = \theta_1 + \rho_1(x_1)$ for $i = m+1, \dots, n$.

The last step chooses demand a sufficiently large so that all n firms are active in equilibrium independent of the license allocation and that the necessary conditions of the second step are sufficient. The concavity of π was shown in Proposition 5. That all firms are active when a is sufficiently large follows from introspection of Equation (2). □

There are markets in $\mathcal{M}$ for which the auction-efficient license allocation is socially efficient. For these markets the auction maximizes bidders' welfare. By the revelation principle (e.g. Proposition 9.25 of Nisan, 2007), there is an incentive compatible mechanism that also implements the firms' welfare. A mechanism is incentive compatible if it is a dominant strategy to reveal the true profits. The revelation principle also says that the prices are the same in the original auction and the incentive compatible direct mechanism. Note that the domains of the indirect profit functions are connected sets as they are convex. Theorem 9.37 of Nisan (2007) implies that the auction must implement the VCG prices. Hence, the original auction must be the VCG auction with bids on full license allocations. The VCG auction with bids on full license allocations is, however, not socially efficient. This follows from Proposition 5, which shows that the consumer surplus maximizing license allocation is the industry profit maximizing license allocation only if $\theta_1 = \theta_2 = \cdots = \theta_n$. The same is shown basically by Lemma 1, as there is one degree of freedom so that consumer surplus and industry profits are maximized by the same license allocation. □

Proposition 9. *Consider the linear Cournot model with inverse demand* $P(Q) =$

$a - b \cdot Q$, and let all firms have the same strictly convex cost reduction technology. Consider the VCG auction with bids on license allocations. Consumer surplus is non-increasing in the cap $\bar{x} \in (1/n, 1)$. Moreover, with identical initial marginal costs, and the market size a sufficiently large, there exists caps $\bar{x}$ such that the auction outcome maximizes consumer surplus.

Proof. Let the marginal costs be the same strictly convex function, i.e. $c_i(x) = c_j(x)$ and $c_i''(x) = \rho''(x) > 0$ for all i, $j \in N$ and $x \in [0, 1]$. Proposition 5 shows that industry profits $\pi(x)$ have a critical point at the license allocation $(1/n, \dots, 1/n)$. It also shows that industry profits are concave when a is sufficiently large. This implies that the critical point is a global maximum. Note, however, that global concavity is not needed, as it is enough that the Hessian is negative definite at the critical point. The eigenvalue eigendecomposition of the Hessian is available upon request. One can then find caps $\bar{x}$ such that firms can only bid for allocations in a neighborhood around the local maximum, so that the local becomes a global maximum on the restricted domain. □

Proposition 10. *Consider the linear Cournot model with inverse demand $P(Q) = a - b \cdot Q$ and all firms with identical marginal cost functions. Firm n needs to win ϵ in order to be active. When the market size a is sufficiently large, then the VCG auction with bids on license allocations and appropriate caps and set-asides maximizes consumer surplus.*

Proof. I first consider the maximization of industry profits and the role of set-asides, and then the maximization of consumer surplus. In both cases the market size plays a role. For the maximization of industry profits, I have to check the local extrema $(1/(n-1), \dots, 1/(n-1), 0)$ of $\pi(\cdot|n-1)$ and $(1/n, \dots, 1/n)$ of $\pi(\cdot|n)$, respectively. Without set-asides, industry profits under less competition are higher than under more competition, i.e.

$$\pi\left(\frac{1}{n-1}, \dots, \frac{1}{n-1}, 0|n-1\right) > \pi\left(\frac{1}{n}, \dots, \frac{1}{n}|n\right) \Leftrightarrow$$

$$(n-1)\frac{\left(a-(n-1)c\left(\frac{1}{n-1}\right)+(n-2)c\left(\frac{1}{n-1}\right)\right)^2}{bn^2} > n\frac{\left(a-nc\left(\frac{1}{n}\right)+(n-1)c\left(\frac{1}{n}\right)\right)^2}{b(n+1)^2} \Leftrightarrow$$

$$(n-1)(n+1)^2\left(a-c\left(\frac{1}{n-1}\right)\right)^2 > n^3\left(a-c\left(\frac{1}{n}\right)\right)^2.$$

The last inequality is true, because $a - c(1/(n-1) > a - c(1/n)$ and $(n-1)(n+1)^2 > n^3$.

A set-aside $x^{sa} \in [\epsilon, 1/n]$ gives firm n the possibility to win the share of licenses required to be active in the market independent of the license allocation. Hence, under the set-aside, caps as in Proposition 9 and with a sufficiently large market, industry profits have a local and, due to the caps a global, maximum at the license allocation $(1/n, \ldots, 1/n)$. This allocation is implemented in dominant strategies by the VCG auction with bids on allocations.

Consumer surplus is higher with more active firms whenever

$$Q\left(\frac{1}{n}, \ldots, \frac{1}{n}|n\right) > Q\left(\frac{1}{n-1}, \ldots, \frac{1}{n-1}, 0|n-1\right) \Leftrightarrow$$
$$\frac{n\left(a - c(\frac{1}{n})\right)}{b(n+1)} > \frac{(n-1)\left(a - c(\frac{1}{n-1})\right)}{bn}.$$

The last inequality is true when the market size is sufficiently large. □

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