Exercises for T2, Summer term 2019, Sheet 2

1) Photoelectric effect

Light with sufficient energy is able to trigger ejection of electrons from the atomic bonds in a metal via the photoelectric effect. Assuming that electrons start being emitted if the wavelength of the incident light falls below 500 nm, calculate the work function and the corresponding voltage.

2) De Broglie wavelength

Calculate the de Broglie wavelength of a proton with a kinetic energy of 1 eV, 100 eV, 100 keV ($m_p \simeq 938\,\mathrm{MeV}/c^2$). What is the de Broglie-wavelength of a human with a mass of 70 kg, who is moving at $1\,\mathrm{m/s}$? Compare the obtained results with the size of a proton and the size of a human. What is your conclusion concerning the wave character of a human being, compared to that of a proton?

3) De Broglie wavelength of nonrelativistic particles

Write down the general (relativistic) relation between the de Broglie wavelength and the **kinetic energy** $T = E - mc^2$ of a massive particle ($m \neq 0$). Find an approximation for the nonrelativistic case ($T \ll mc^2$).

4) Heisenberg's commutation relation

Show the commutation relation $[\mathbf{Q}, \mathbf{P}] \equiv \mathbf{QP} - \mathbf{PQ} = i\hbar\mathbb{1}$ is true in the operator sense (i.e. acting on any function $f(q)$ of the location variable q) where $\mathbf{Q} \equiv q$, $\mathbf{P} \equiv -i\hbar\frac{\partial}{\partial q}$ and $\mathbb{1}$ is the identity operator.

5) Gaussian integral I

(a) Show that $\int\limits_{-\infty}^{+\infty} dx\, \exp(-ax^2) = \sqrt{\pi/a}$ for a real with $a > 0$.

(b) Calculate $\int\limits_{-\infty}^{+\infty} dx\, x\, \exp(-ax^2)$.

(c) Calculate $\int\limits_{-\infty}^{+\infty} dx\, x^2\, \exp(-ax^2)$.

(d) Calculate $\int\limits_{-\infty}^{+\infty} dx\, x^n\, \exp(-ax^2)$, for an arbitrary natural number n.

6) Gaussian Integral II

(a) Show that $\int\limits_{-\infty}^{+\infty} dx\, \exp(-ax^2) = \sqrt{\pi/a}$ for a complex a with $\mathrm{Re}[a] > 0$.

(b) Calculate $\int\limits_{-\infty}^{+\infty} d^3\vec{\mathbf{k}}\; e^{i\,\vec{\mathbf{k}}\cdot\vec{\mathbf{x}}}\; e^{-\alpha^2\,\vec{\mathbf{k}}^2}$ for α being real.