3. INDIRECT UTILITY & EXPENDITURE

1. Indirect utility
2. Properties of indirect utility
3. Hicksian demand
4. Expenditure function
5. Properties of expenditure function
6. Duality: relation between indirect utility and expenditure
INDIRECT UTILITY

Utility evaluated at the maximum

\[ v(p, m) = u(x^*) \text{ for any } x^* \in x(p, m) \]

Marshallian demand maximizes utility subject to consumer’s budget. It is a function of prices and income.

Substituting Marshallian demand in the utility function we obtain indirect utility as a function of prices and income.
PROPERTIES OF INDIRECT UTILITY

1. Homogeneity of degree zero

2. Strictly increasing in $m$ and decreasing in $p_i \forall i$

3. $v(p, m)$ is quasi-convex

4. Continuous in $p$ and $m$

5. Roy’s identity

$$x_i(p, m) = - \frac{\partial v(p, m)/\partial p_i}{\partial v(p, m)/\partial m}$$
HICKSIAN DEMAND

Consider the dual to the consumer’s problem

\[
\begin{align*}
\min_{x \geq 0} & \quad p \cdot x \\
\text{s. t.} & \quad u(x) \geq u
\end{align*}
\]

**Hicksian demand** (also called **compensated demand**) is the solution to this cost-minimization problem, \( x^h(p, u) \).

Notice the parameters of the cost-minimization problem are prices \( p \) and target utility \( u \).
EXPENDITURE FUNCTION

Expenditure evaluated at its minimum

\[ e(p, u) = p \cdot \tilde{x} \text{ for any } \tilde{x} \in x^h(p, u) \]

Hicksian demand solves the cost-minimization problem. It is a function of prices \( p \) and target utility \( u \).

Substituting Hicksian demand in the expenditure objective we obtain expenditure as a function of \( p \) and \( u \).
PROPERTIES OF EXPENDITURE

1. Homogeneous of degree one in $p$

2. Strictly increasing in $u$ and increasing in $p_i \ \forall i$

3. Concave in $p$

4. Continuous in $p$ and $u$

5. If $u(\cdot)$ is strictly quasi-concave and $e(p, u)$ is differentiable we have Shephard’s lemma

$$\frac{\partial e(p^0, u^0)}{\partial p_i} = x_i^h(p^0, u^0) \quad i = 1, \ldots, n$$
DUALITY

Let \( p \gg 0, \ m > 0, \ u > u(0) \)

1. \( e(p, v(p, m)) = m \)
2. \( v(p, e(p, u)) = u \)

Duality between Marshallian and Hicksian demand

1. \( x(p, m) = x^h(p, v(p, m)) \)
2. \( x^h(p, u) = x(p, e(p, u)) \)