Neoclassical Growth and Commodity Trade

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Abstract

We construct a dynamic Heckscher-Ohlin model in which the initial distribution of production factors across economies makes factor price equalization impossible. The model produces dynamics similar to those of the neoclassical growth model. However, free trade prevents identically parameterized economies from achieving identical steady states. Although poor economies grow faster than rich economies during the transition to the steady state, the former do not catch up with the income per capita levels of the latter. A many-country version of the model exemplifies the open-economy neoclassical growth model’s ability to produce interesting distribution dynamics of income per capita.

Keywords: International Trade, Heckscher-Ohlin, Economic Growth, Convergence.

JEL codes: F1, F4, O4.
1 Introduction

Neoclassical growth is usually modeled under autarky despite the fact that this scenario does not seem to be a good approximation of reality. Trade shares in GDP are remarkably high for a large number of countries. Figure 1 reports the cross-country distribution of the trade share in GDP (exports plus imports over GDP) for 158 non-OPEC countries in 1996.\(^1\) Notice that international trade is quite important for the average country. In the light of these magnitudes, assuming that the evolution of a country’s factor prices (i.e., the incentive to accumulate factors) depends exclusively on the domestic capital-labor ratio seems to be a remarkable leap of faith.\(^2\)

A natural way to introduce international trade into neoclassical growth is by assuming a Heckscher-Ohlin static trade structure, according to which comparative advantage is based on cross-country differences in relative factor endowments. In this setup, free trade makes factor prices depend also on foreign factor endowments. The Heckscher-Ohlin model’s Factor Price Equalization case (henceforth FPE) represents an elegant, albeit extreme, version of this insight: factor prices are equal across countries and only depend on the world’s capital-labor ratio. These unrealistic implications on factor prices, however, question the FPE model’s suitability as a workhorse for studying economic growth in large cross-sections of countries. Although factor prices may not differ that much across countries with similar income per capita levels, casual evidence suggests rich and poor countries exhibit large factor price differences.

An explanation for the lack of FPE in a large sample of countries is that capital-labor ratios are too different.\(^3\) In this case, the international trade equilibrium is characterized by complete specialization on the production side: neither country is able to produce all goods due to differences in factor prices across countries. We denote this as the CS case (for complete specialization), which has received strong empirical support in the empirical trade literature: Davis and Weinstein [7], [8], for example, find that the Heckscher-Ohlin model’s predictions on the net factor content of trade are much more in accordance with the data when they allow for CS.

This paper explores the implications of CS for economic growth. For this purpose, we combine the Ramsey model with a three-good, two-factor Heckscher-Ohlin model under

\(^1\)We report the cross-country distribution of the variable OPENC taken from the Penn World Tables Mark 6.1. We focus on 158 non-OPEC countries and choose the year 1996 to maximize the sample size. See Heston, Summers, and Aten [14].

\(^2\)There are also important theoretical problems with modeling neoclassical growth under autarky. The idea according to which data are consistent with the Solow-Ramsey model as long as one allows different parameters for different countries is basically empty. In such circumstances, the only remaining prediction is that a country’s growth rate should slow down as it grows richer. But the latter is a necessary condition for equilibria to make sense.

\(^3\)The FPE condition requires that the cross-country dispersion of factor endowments be in a certain sense smaller than the dispersion of sectoral capital intensities. See Deardorff [9] for a formal definition. Debaere and Demiroglu [11] and Cuñat [6] carry out empirical implementations of Deardorff’s FPE condition and find it is violated for large samples of countries.
general assumptions on the initial trade regime. In particular, we allow the initial cross-
country distribution of production factors to render worldwide FPE impossible, leading
to CS. The model abstracts from human capital, differences in technologies, productivity
growth, and all sorts of market imperfections, since our main goal is studying the influence
of the trade scenario on the dynamics of countries’ incomes through prices. Being no
closed-form solution available, we solve the model numerically for a given benchmark parameterization and study the transitional dynamics and steady-state values of all variables of interest.

Our setup enables us to frame CS as the ‘standard’ trade regime of a model that
has autarky and FPE for any cross-country distribution of factor endowments as limiting
cases. What distinguishes these three cases is the varying dispersion in the relative factor intensities used in the production of different goods. Concerning the effects of
these regimes on growth, the key difference between the three of them lies in the determination of factor prices. Whereas under autarky and FPE factor prices are determined exclusively by the path of the domestic capital-labor ratio and the world’s capital-labor ratio, respectively, under CS factor prices depend on both domestic and foreign factor endowments.

The model therefore combines the intuitions of the autarky and FPE models. Moreover,
the initial condition leading to CS allows for both effects to play the dominant role
at different development stages. Like under autarky, CS makes a country’s factor prices
depend on its own capital-labor ratio. Therefore, during the transition, the model pro-
duces dynamics similar to those of the neoclassical growth model: the poorer a country,
the faster its growth rate. However, the assumption that countries are identical (but for
their initial conditions) leads to FPE in the long run, where the rate of return to capital,
identical across all countries, is determined by the world’s capital-labor ratio. Poor coun-
tries therefore do not necessarily catch up with rich countries despite being identically parameterized.

Our simulations suggest that the transition towards FPE can be long enough to sustain
the relevance of the CS regime. We obtain a lengthy transition towards FPE despite
assuming a moderate difference between the capital-labor ratios of rich and poor countries,
and moderate values for the elasticity of intertemporal substitution. Larger differences in
factor-endowment ratios and a low elasticity of intertemporal substitution would make the transition even more protracted. The same would occur in the presence of an additional production factor such as human capital, since it is unevenly distributed across countries, and tends to be accumulated more slowly than physical capital.

The CS scenario highlights the different growth performance of countries with very
different capital-labor ratios (or income per capita levels), and subsequently strong differences in their production structures due to comparative advantage. However, countries with similar income per capita levels and production structures are prone to experience
similar factor-price time paths. We introduce this consideration into our model by allowing countries of similar income per capita levels to be under FPE, while remaining completely specialized *vis-à-vis* countries with very dissimilar income per capita levels. A many-country simulation exercise that combines both FPE and CS in this manner yields interesting distribution dynamics, illustrating the richness of the neoclassical growth model under international trade. For example, we can generate scenarios in which the richest among the poor countries diverge from their poorer neighbors, and almost catch up with the rich.

Part of our contribution is also methodological. Our model consists of two heterogeneous representative households (each representing a region or country) that accumulate capital and trade with each other under two possible trade regimes. The capital accumulation process leads eventually to a trade regime switch. This implies that the policy functions for consumption display a particular shape. We numerically approximate the optimal policy functions as implicitly defined by the Euler equations. To cope with their shape, we develop a two-step procedure. For that purpose we adapt to our needs the Galerkin projection method, as described in Judd [15]; the solution procedure seems to be a good compromise between numerical accuracy and computational complexity.

The closest references to our paper are Stiglitz [16], Chen [4] and Ventura [17], who combine the Ramsey and FPE models. A recent paper by Atkeson and Kehoe [2] produces a dynamic two-sector Heckscher-Ohlin model that shows how a poor country’s growth performance and steady state depend on its initial position relative to the rest of the world’s diversification cone, which is assumed in steady state. We assume initial conditions that place all countries far from their steady states, and study the evolution of the trading equilibrium over time, allowing for changes in regime between CS and FPE. Thus, in comparison with Atkeson and Kehoe [2], our model has the potential to describe the dynamics of the entire distribution of countries’ per capita incomes, as well as the distribution of their steady states.

Another important precedent to our work is Deardorff [10], who produces a model in which a steady-state equilibrium with two cones is possible in an overlapping generations framework in which savings are determined by wages. This leads countries to group in ‘clubs.’ In a sense, we are producing the Ramsey-counterpart to Deardorff’s model. As we mentioned above, our model shows that a standard Ramsey model can also generate equilibria in which countries that only differ in initial capital stocks converge to different steady-state levels of income per capita. In comparison with Deardorff [10], we also study the dynamic behavior of countries within the same cone. It is also worth noting that Deardorff’s model leads to steady-state differences in factor prices, whereas our model leads to FPE in the long run.

Finally, the importance of complete specialization has also been studied by Acemoglu

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4See also Bond *et al.* [3].
and Ventura [1] in an endogenous growth framework. They show that even in the presence of linear technologies, de facto diminishing returns to capital can occur because of changes in the terms of trade of completely specialized countries.

The rest of the paper is structured as follows: Section 2 presents a two-region static model of international trade with three possible regimes (autarky, FPE and CS). It discusses how the distribution of factor endowments across countries determines the trade regime in place and how factor prices are determined in each case. In Section 3 we combine the static model with a two-country Ramsey model, and discuss both the functional equations characterizing a recursive competitive equilibrium under perfect foresight and the model’s steady state. In Section 4 we study the dynamics of our variables of interest. We compare the predictions of the CS model with those of the autarky and FPE models. Section 5 analyzes the many-country case by splitting each region in many countries. Section 6 concludes.

2 International Trade

Our static trade model is a relatively simple version of the Heckscher-Ohlin model. Countries only differ in their relative factor endowments. In the presence of differences in capital intensities across sectors, comparative advantage leads capital-abundant (labor-abundant) countries to export capital-intensive (labor-intensive) goods.

Let us initially assume that there are two regions in the world (North and South), indexed by $j \in \{N, S\}$, with identical technologies and preferences, and competitive markets. The world has $k = k_N + k_S$ units of capital and $l = l_N + l_S$ units of labor. We assume $l_N = l_S = 1$ constant. Without loss of generality, let us assume that the North is the capital-abundant region, and that both have positive capital stocks: $k_N > k_S > 0$.

Regions produce a final good $y_j$ with a Cobb-Douglas production function of the form

$$y_j = \phi x_{1,j}^{\alpha/2} x_{2,j}^{1-\alpha} x_{3,j}^{\alpha/2},$$

where $\alpha \in [0, 1]$ and $\phi$ is a positive constant. The final good, which is also the numeraire ($p_j = 1$), is produced out of three intermediate inputs $x_{z,j}$, with prices $p_z$, $z \in \{1, 2, 3\}$. Intermediate goods, in turn, are produced with the following technologies: $y_{1,j} = l_{1,j}$, $y_{2,j} = k_{2,j}^{1/2} l_{2,j}^{1/2}$, and $y_{3,j} = k_{3,j}$. Let us assume that: (i) the final good $y_j$ cannot be traded, whereas intermediates can be traded freely; (ii) there is no international factor mobility.

We can think of each country facing given prices $p_z$ and solving the following problem:

$$\max_{\{\lambda_j, \theta_j\}} \phi x_{1,j}^{\alpha/2} x_{2,j}^{1-\alpha} x_{3,j}^{\alpha/2}$$

($^5 y_z$ distinguishes production of good $z$ from consumption of good $z$, $x_z$.}
subject to

\[ \sum_{z=1}^{3} p_{z} x_{z,j} = \sum_{z=1}^{3} p_{z} y_{z,j} \]  

(2)

\[ y_{1,j} = \lambda_{j} l_{j} \]  

(3)

\[ y_{2,j} = [(1 - \theta_{j}) k_{j}]^{\frac{1}{2}} [(1 - \lambda_{j}) l_{j}]^{\frac{1}{2}} \]  

(4)

\[ y_{3,j} = \theta_{j} k_{j} \]  

(5)

where \( \lambda_{j}, \theta_{j} \in [0, 1] \).

Putting together the desired consumption and production plans of both countries yields the equilibrium, which is unique in the following sense: for any pattern of factor endowments, a unique pricing pattern of goods (and factors) is determined. Also, there is a unique equilibrium pattern of world and regional consumptions of intermediate goods, with consumption ratios the same in every region.\(^6\) Comparative advantage implies that the capital-abundant country is a net exporter (net importer) of capital (labor) services through the commodities traded. However, the static model can lead to two different scenarios. What scenario actually takes place depends on the distribution of factor endowments across North and South:

1. If \( k_{N} \) and \( k_{S} \) are ‘similar enough’, we will have FPE and the countries’ production structures will be indeterminate. That is, there is an infinite number of allocations \((\lambda_{N}, \theta_{N}, \lambda_{S}, \theta_{S})\) compatible with the equilibrium prices. This is the usual Heckscher-Ohlin indeterminacy due to the number of goods being larger than the number of production factors. In this case, all we can say is that the North will export on average capital-intensive goods in exchange for labor-intensive goods. (More precisely, the North will be a net exporter of capital services embodied in commodity trade, and a net importer of labor services.)

2. If \( k_{N} \) and \( k_{S} \) are ‘too diverse’, we will have CS with capital-abundant North producing goods 2 and 3 \((\lambda_{N} = 0)\), and capital-scarce South producing goods 1 and 2 \((\theta_{S} = 0)\). One can show this scenario leads to \((w_{N}/r_{N}) > (w_{S}/r_{S})\); otherwise the North would neither have a comparative advantage in the production of good 3 nor a comparative disadvantage in the production of good 1.

No other scenario is possible for the following reason: first, given that both \( N \) and \( S \) have positive amounts of capital and labor, full employment of resources implies they cannot specialize completely in good 1 one or good 3. Second, CS in good 2 is not possible either, since a region with comparative advantage in this good would also have

\(^{6}\)This is a standard result in international trade theory. See, for example, Dixit and Norman [12] and Dornbusch et al. [13].
a comparative advantage in either of the other goods, due to \((w/r)_N \neq (w/r)_S\). This implies that in the absence of worldwide factor price equalization each region produces two goods. Moreover, in such a scenario we cannot have one region producing goods 1 and 3: with different factor prices across regions, a region cannot have a comparative advantage in the production of both of these two goods.

As we show below, the only information from the trade equilibrium that we need for our dynamic model are each country’s factor prices. The indeterminacy in the FPE equilibrium allocation is therefore not an important problem, but it forces us to focus on factor prices and find out when each of the two scenarios described above applies.

2.1 The Integrated Equilibrium

To understand what we mean by ‘similar enough’ and ‘too diverse’, let us review the concept of integrated equilibrium, which is defined as the resource allocation the world would have if both goods and factors were perfectly mobile internationally. In other words, the integrated equilibrium is the solution to the following closed economy planner’s problem:

\[
\max_{\{\lambda, \theta\}} \phi x_1^{\frac{\theta}{2}} x_2^{1-\alpha} x_3^{\frac{\theta}{2}}
\]

subject to

\[
x_1 = \lambda l
\]

\[
x_2 = [(1 - \theta) k]^{\frac{1}{2}} [(1 - \lambda) l]^{\frac{1}{2}}
\]

\[
x_3 = \theta k
\]

where \(k \equiv k_N + k_S\), \(l \equiv l_N + l_S\), and \(\lambda, \theta \in [0, 1]\). It is easy to show that the solution implies \(\lambda = \theta = \alpha\). The resulting equilibrium sectorial allocation of production factors is therefore: \((k_1, l_1) = (0, \alpha l)\), \((k_2, l_2) = ((1 - \alpha) k, (1 - \alpha) l)\), and \((k_3, l_3) = (\alpha k, 0)\).

Factor prices in the integrated equilibrium depend on world aggregates. In terms of our model, the wage rate \(w\) and the rate of return to capital \(r\) depend, respectively, positively and negatively on the world’s capital-labor ratio \(k/l\):

\[
w = \xi (k/l)^{\frac{1}{2}}
\]

\[
r = \xi (k/l)^{-\frac{1}{2}}
\]

where \(\xi \equiv \frac{1}{2} \phi \alpha^\alpha (1 - \alpha)^{1-\alpha} > 0\). Consequently, the relationship between the factor-price ratio \(\sigma \equiv w/r\) and the world’s capital-labor ratio is positive: \(\sigma = k/l\).

Notice that the capital and labor allocated to each sector are proportional to the corresponding world’s factor endowments; this is due to the Cobb-Douglas assumption. Although it is quite restrictive, it helps us obtain a very simple condition for FPE, as we
Define the FPE set as the set of distributions of factors among economies for which the free-trade equilibrium without international factor mobility achieves the integrated equilibrium’s resource allocation. Intuitively, the FPE set is the set of distributions of factors across economies that enable the latter to achieve full employment of resources while using the techniques implied by the integrated equilibrium. If the vector of production factors lies within the FPE set, the (constrained) trading equilibrium will reproduce the (unconstrained) integrated equilibrium.\(^7\) Factor prices will be equal across countries, and identical to those of the integrated equilibrium.

Consider Figure 2. The dimensions of the box are given by the world endowments \(k\) and \(l\). We measure the allocation of resources to countries \(N\) and \(S\) from origins \(O_N\) and \(O_S\), respectively. The FPE set is depicted by the thick line, which is constructed by aligning the integrated equilibrium’s sectorial allocation vectors from more to less capital-intensive (from both origins). The slope of each vector reflects the capital intensity of the corresponding sector. Distributions of factor endowments across countries outside the FPE set cannot deliver FPE, because they make it impossible for countries to both use the integrated equilibrium’s capital intensities and achieve full employment of resources.

Notice that our modeling strategy allows us to nest the three alternative trade regimes in the same framework. When \(\alpha = 1\), only infinitely labor-intensive good 1 and infinitely capital-intensive good 3 are produced in the integrated equilibrium. This would grant FPE for any distribution of factors across regions.\(^8\) In terms of Figure 2, the FPE set would be the entire box. The smaller \(\alpha\), the less likely FPE. Finally, when \(\alpha = 0\), both regions produce only intermediate good 2, and therefore need not trade with each other. In fact, they behave as if they were closed economies with aggregate production function \(y_j = \phi_k^{1/2} l_j^{1/2}\), \(j = \{N, S\}\). In this sense, the FPE and autarky cases are limiting cases of a more general model that allows for the possibility of CS as the ‘standard’ case.

### 2.2 The Factor Price Equalization Condition\(^9\)

Let us assume \(\alpha \in (0, 1/2)\) for convenience. Consider the following distribution of capital stocks across North and South: \(k_N = (1/2 + \varepsilon)k\), \(k_S = (1/2 - \varepsilon)k\), \(\varepsilon \in (0, 1/2)\). With each region having one unit of labor, \(\varepsilon\) determines differences in relative factor endowments across regions, and therefore whether the FPE condition holds. Figure 2 depicts two possible distributions of production factors across \(N\) and \(S\). The vertical dotted line separates the two regions’ labor endowments.

The variable \(k_{N,FPE}\) denotes the North’s largest capital stock that allows for FPE. One

\(^7\)See, again, Dixit and Norman [12].

\(^8\)This is indeed the case analysed in Ventura [17].

can obtain \(k_{FPE}^N\) by realizing that it is the capital stock that enables the North to produce all of the integrated equilibrium’s production of good 3 (from the integrated equilibrium’s allocation, \(k_3 = \alpha k\)), and the part of the integrated equilibrium’s production of good 2 that employs half the world’s population (from the integrated equilibrium’s allocation, \((k_2/l_2)(1/2)l = (k/l)(1/2)l = (1/2)k\)). Thus, \(k_{FPE}^N = (1/2 + \alpha)k\).

Hence, for \(\varepsilon \in (0, \alpha]\) the FPE condition holds. The short-dashed vectors of Figure 2 represent this case: factor endowments are ‘similar enough’ relative to the integrated equilibrium’s technologies for the integrated equilibrium to be reproduced by the trading equilibrium. For \(\varepsilon \in (\alpha, 1/2)\) the distribution of production factors across North and South instead violates the FPE condition. The long-dashed vectors of Figure 2 represent this case: regions are ‘too diverse’ in their capital-labor ratios and therefore specialize completely.

### 2.3 Complete Specialization

If the factor endowment vectors lie outside the FPE set (i.e., if \(\varepsilon \in (\alpha, 1/2)\)), the trading equilibrium cannot reproduce the integrated equilibrium. This leads factor prices to differ across North and South, which specialize in different ranges of goods according to comparative advantage. In terms of the problem laid out in equations (1) through (5), the CS equilibrium leads to \(\lambda_N = \theta_S = 0\), as we mentioned above.

In the Appendix we solve for the CS competitive trade equilibrium, and obtain the following system of two equations, which yields the factor-price ratios \(\sigma_j\) as functions of the two capital stocks, \(\sigma_j = \sigma_j(k_N, k_S)\):

\[
(1 - \alpha)\sqrt{\sigma_S} - \frac{k_S}{\sqrt{\sigma_S}} = \alpha\sqrt{\sigma_N} \tag{12}
\]

\[
(1 - \alpha)\frac{k_N}{\sqrt{\sigma_N}} - \frac{\sqrt{\sigma_N}}{\sigma_N} = \alpha\frac{k_S}{\sqrt{\sigma_S}} \tag{13}
\]

By manipulating the equilibrium’s pricing equations we can write factor prices as functions of the factor-price ratios, i.e. \(w_j = w_j(\sigma_N, \sigma_S)\) and \(r_j = r_j(\sigma_N, \sigma_S)\). Hence, factor prices are also functions of the capital stocks of North and South: \(w_j = w_j(k_N, k_S)\), and \(r_j = r_j(k_N, k_S)\).

The following results are worth mentioning:

1. The North’s factor-price ratio is greater than that of the South: \(\sigma_N > \sigma_S\) (\(w_N > w_S\) and \(r_N < r_S\)).

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10 This is where the assumption that \(\alpha < 1/2\) applies, since it guarantees \(l_2 > (1/2)l\).

11 To obtain these results, we numerically approximate the solution to (12)-(13), construct the factor prices, and obtain their partial derivatives with respect to \(k_N\) and \(k_S\). In particular, we approximate the solution over a rectangle \(D \equiv [k, \bar{k}] \times [k, \bar{k}] \subset \mathbb{R}^2\) with a linear combination of multidimensional orthogonal basis functions taken from a 2-fold tensor product of Chebyshev polynomials, and choose the coefficients using a simple collocation method. See Judd [15] and Appendix B for more details.
2. The ratio $\sigma_N$ depends positively on $k_N$, and negatively on $k_S$. An increase in $k_N$ creates an excess supply of good 3. This causes $p_3 = r_N$ to decrease. An increase in $w_N$ and a decrease in $r_N$ induce a higher capital-labor intensity in sector 2, helping achieve full employment of resources in the North. An increase in $k_S$ implies a rise in income and spending on all goods for the South. This creates an excess demand of good 3, produced exclusively by the North, and an excess supply of good 2. The latter turns out to have a stronger effect, contributing to a fall in both $r_N$ and $w_N$. The fall in $w_N$ is more pronounced than the fall in $r_N$; this leads to a lower $\sigma_N$, a lower capital-labor intensity in sector 2, and a subsequent transfer of capital from sector 2 to sector 3.

3. The ratio $\sigma_S$ depends positively on both $k_N$ and $k_S$. An increase in $k_S$ requires a rise in $\sigma_S$ (an increase in $w_S$ and a decrease in $r_S$) that induces higher capital-labor intensities to achieve full employment of resources in the South. An increase in $k_N$ causes an excess demand for good 1, produced exclusively by the South, and an excess supply for good 2. This leads to an increase in $p_1 = w_S$, and a decrease in $r_S$. The subsequent increase in $\sigma_S$ produces a higher capital-labor intensity in sector 2, releasing labor towards sector 1.

The results of the CS case are, in a sense, halfway between the autarky and FPE results:

1. Under autarky, factor prices depend exclusively on domestic capital stocks: $r_j = r(k_j), \partial r_j / \partial k_j < 0; w_j = w(k_j), \partial w_j / \partial k_j > 0; j = N, S$.

2. In the FPE case, factor prices depend only on the world’s capital stock: $r_j = r(k), \partial r / \partial k < 0; w_j = w(k), \partial w / \partial k > 0; j = N, S$.

3. In the CS case, each region’s factor prices are affected by both the domestic and foreign capital stocks: $r_N = r_N(k_N, k_S), \partial r_N / \partial k_N < 0, \partial r_N / \partial k_S < 0; w_N = w_N(k_N, k_S), \partial w_N / \partial k_N > 0, \partial w_N / \partial k_S < 0; r_S = r_S(k_N, k_S), \partial r_S / \partial k_N < 0, \partial r_S / \partial k_S < 0; r_S = r_S(k_N, k_S), \partial r_S / \partial k_N < 0, \partial r_S / \partial k_S < 0$. Note that the CS case is not entirely symmetric in the signs of the derivatives. This is due to the fact that regions have different production structures.

3 The Dynamic Model

In this section we combine the static model discussed above with the discrete-time Ramsey model. Each region is populated by a continuum of identical and infinitely lived households, each of measure zero. Being identical, they can be aggregated into a single representative household. A unique homogeneous final good exists, that can be used
for both consumption and investment. The preferences over consumption streams of the representative household in region $j$ can be summarized by the following intertemporal utility function:

$$U_{j,t} = \sum_{s=t}^{\infty} \beta^{s-t} \ln c_{j,s}$$  \hspace{1cm} (14)$$

where $\beta$ is a subjective intertemporal discount factor and $c_{j,t}$ the per-capita consumption level in region $j$ at date $t$.

The representative household maximizes (14) subject to the following intertemporal budget constraint:

$$c_{j,t} + \Delta k_{j,t} = w_{j,t} + (r_{j,t} - \delta) k_{j,t}$$  \hspace{1cm} (15)$$

where $k_{j,t}$ is the current per-capita stock of physical capital in region $j$, $w_{j,t}$ the wage rate, $r_{j,t}$ the rental rate, and $\delta$ the depreciation rate. Factor prices are taken as given by the representative household. Depending on the distribution of capital across regions, factor prices $w_{j,t}$ and $r_{j,t}$ will be determined in the integrated or complete specialization equilibrium.

The first order conditions

$$\beta c_{j,t}(r_{j,t+1} + 1 - \delta) = c_{j,t+1}$$  \hspace{1cm} (16)$$

$$k_{j,t+1} = w_{j,t} + (r_{j,t} + 1 - \delta) k_{j,t} - c_{j,t}$$  \hspace{1cm} (17)$$

and the usual transversality conditions are necessary and sufficient for the representative household’s problem.

### 3.1 Recursive Competitive Equilibrium

A recursive competitive equilibrium for this economy is characterized by equations (16)-(17) together with

$$w_{N,t} = w_{S,t} = \xi \left( \frac{k_t}{2} \right)^{\frac{1}{2}}$$  \hspace{1cm} (18)$$

$$r_{N,t} = r_{S,t} = \xi \left( \frac{k_t}{2} \right)^{-\frac{1}{2}}$$  \hspace{1cm} (19)$$

if $k_{N,t} \leq (1/2 + \alpha) k_t$, and

$$w_{N,t} = \xi \sigma_{N,t}^{\frac{2+\alpha}{2}} \sigma_{S,t}^{-\frac{\alpha}{2}}$$  \hspace{1cm} (20)$$

$$w_{S,t} = \xi \sigma_{N,t}^{\frac{\alpha}{2}} \sigma_{S,t}^{\frac{2-\alpha}{2}}$$  \hspace{1cm} (21)$$

$$r_{N,t} = \xi \sigma_{N,t}^{\frac{\alpha^2}{2}} \sigma_{S,t}^{-\frac{\alpha}{4}}$$  \hspace{1cm} (22)$$

$$r_{S,t} = \xi \sigma_{N,t}^{\frac{\alpha}{2}} \sigma_{S,t}^{\frac{(2+\alpha)}{4}}$$  \hspace{1cm} (23)$$
if \( k_{N,t} > (1/2 + \alpha) k_t \), where the values of \( \sigma_{j,t} \) are implicitly defined by (12)-(13).

If a solution to (16)-(23) exists, the recursive structure of our problem guarantees the former can be represented as a couple of time-invariant policy functions expressing the optimal level of consumption in each region as a function of the two state variables, \( k_N \) and \( k_S \). These policy functions have to satisfy the following functional equations:

\[
\beta c_j (k_N, k_S) (r_j + 1 - \delta) = c_j (k'_N, k'_S) \tag{24}
\]

where:

1. \( k'_j = w_j + (r_j + 1 - \delta) k_j - c_j (k_N, k_S) \);
2. \( w_j = \xi \left( \frac{k^\alpha}{2} \right)^{\frac{1}{2}}, r_j = \xi \left( \frac{k^\alpha}{2} \right)^{\frac{1}{2}} \) if \( k_N \leq (1/2 + \alpha) k \);
3. \( w_N = \xi \sigma_N^{\frac{2+\alpha}{\sqrt{2}}} \sigma_S^{-\frac{\alpha}{2}}, w_S = \sqrt{\frac{\sigma_N}{\sigma_S}} w_N, r_N = \xi \sigma_N^{\frac{2+\alpha}{\sqrt{2}}} \sigma_S^{-\frac{\alpha}{2}}, r_S = \sqrt{\frac{\sigma_N}{\sigma_S}} r_N \) if \( k_N > (1/2 + \alpha) k \);
4. \( r_N' = \xi \left( \frac{k'_N}{2} \right)^{\frac{1}{2}} \) if \( k'_N \leq (1/2 + \alpha) k' \);
5. \( r_S' = \xi \left( \sigma_N' \right)^{\frac{2+\alpha}{\sqrt{2}}} \left( \sigma_S' \right)^{-\frac{\alpha}{2}}, r_S' = \xi \left( \sigma_N' \right)^{\frac{2+\alpha}{\sqrt{2}}} \left( \sigma_S' \right)^{-\frac{\alpha}{2}} \) if \( k'_N > (1/2 + \alpha) k' \).

The values of \( \sigma_j \) and \( \sigma'_j \) are obtained by solving the nonlinear system (12)-(13) numerically. To solve equation (24) numerically, we apply the projection methods described in Judd [15]. Appendix B describes our computational strategy in detail.

We discuss now our benchmark parameterization. In the Appendix we show that, under complete specialization, the parameter \( \alpha \) corresponds to the volume of trade (the value of exports plus the value of imports) over income in the North. To make our choice of \( \alpha \) less arbitrary, we parameterize it according to the average ratio of total trade over GDP, both at current prices, calculated for the US over the 1947:1-2001:4 time horizon, and set \( \alpha = 0.15 \). The initial values for the two regions’ capital stocks are chosen arbitrarily: we set \( k_N = 0.5 \) and \( k_S = 0.1 \). Following Cooley and Prescott [5], we assume \( \beta = 0.949 \) and \( \delta = 0.048 \). Our parameterization is admittedly crude, but our main goal is a purely qualitative comparison among models under different trade regimes.

### 3.2 Steady State

Consider the Euler equation (16) and evaluate it at the steady state:

\[
r_j = r \equiv \frac{1}{\beta} - 1 + \delta \tag{25}
\]

\( \xi \) is calibrated to reproduce a world steady-state capital stock equal to unity in the CS model. Recall that in the long run the model switches to FPE. Our parameterization implies that the time unit is a year.

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12 The scale parameter \( \phi \) is calibrated to reproduce a world steady-state capital stock equal to unity in the CS model. Recall that in the long run the model switches to FPE. Our parameterization implies that the time unit is a year.
Equation (25) pins down the steady-state interest rate as usual in Ramsey-type models. Evidently, FPE has to hold in steady state, since \( r_N = r_S = r \). If FPE applies in steady state, then the equation
\[
r = \xi \left( \frac{k}{2} \right)^{-\frac{1}{2}}
\] (26)
pins down the world-level steady-state capital stock uniquely. It is easy to show that (25) and (26) are enough to uniquely characterize the steady state at the world level.

However, any combination of \( k_N \) and \( k_S \) such that \( k_N + k_S = k \) and FPE holds is compatible with the steady state, and hence (25) and (26) are unable to pin down the steady state at the regional level. Notice, however, that the multiplicity of steady states does not imply the indeterminacy of the optimal consumption plans: once the initial conditions \( k_{j0} \) are specified exogenously, the final steady state to which the system tends in the long run is fully determined and non-degenerate, because (i) the world as a whole is a standard stationary Ramsey economy with a well specified steady state; (ii) the regional policy functions for consumption are unique, and hence imply unique optimal paths for consumption, investment, and capital; (iii) the requirement that \( k_N + k_S = k \) together with the FPE condition jointly imply that \( k_{\min} \leq k_j \leq k_{\max} \) for some \( k_{\max} > k_{\min} > 0 \). In other words, the world reaches a steady state in which Equations (25) and (26) hold, and the cross-region distribution of capital stocks is not degenerate, i.e. both \( k_j \)'s are strictly positive. Such a steady state may be characterized by different values of consumption, income, investment and capital across regions and countries. Notice that in our framework the size of \( k_{\min} \) and \( k_{\max} \) depends exclusively on the size of the FPE region, and hence directly on the value of \( \alpha \).

4 Results

4.1 Autarky

For comparison purposes, we first review the dynamics implied by the standard closed-economy Ramsey model under the same parameterization. The inability to trade yields an autarky model in which economies have the aggregate production function \( y_j = \phi \alpha^\alpha (1 - \alpha)^{1-\alpha} k_j^{1/2} l_j^{1/2} \), \( j = \{N, S\} \). Notice that, but for the constant term \( \alpha^\alpha (1 - \alpha)^{1-\alpha} \), this is the production function of the autarky regime we obtained in Section 2. Figure 3 illustrates the dynamic behavior of the levels and growth rates (denoted by \( \gamma_j \)) of income and capital for both North and South. Given our choice of the initial conditions, the North is in steady state from the very beginning. Therefore it exhibits zero growth rates in all variables. The South experiences positive growth rates initially, but the South-North growth differential falls over time until the South converges both in levels and growth rates to the North. Concerning the South’s transitional dynamics, its speed of convergence is
6.1% per year.

Figure 4 graphs the evolution of the rate of return to capital for both regions: notice the persistent differential in favour of the South. The key role of diminishing returns to capital underlying these results is well known. A low capital stock (with respect to its steady-state level) implies a high marginal productivit y of capital and a high rate of return, inducing an important process of capital accumulation that slows down over time, as the economy becomes richer. North and South exhibit the same parameterization, and consequently identical steady states. Figure 4 shows that, as expected, the capital-income ratios and the investment shares converge in both regions to the same value; however, during the transition the capital-income ratio remains consistently higher in the North and the investment share higher in the South.

4.2 Complete Specialization

With $k_N = 0.5$, $k_S = 0.1$ and $\alpha = 0.15$, we have $\alpha < \varepsilon$. Therefore the initial conditions imply the CS regime. Figures 5 and 6 summarize the dynamics of the CS case. Initial income levels turn out to be quite similar to those under autarky. Notice that the CS-regime lasts for 82 years before converging towards FPE, after which the world does not change regime any more. Unlike in the autarky model, North and South reach different steady-states despite being ruled by identical parameter values. The North reaches a steady-state level of income higher than under autarky, whereas the opposite holds for the South. This result can be related to the different rates of return yielded by the two models: whereas under FPE the North faces a higher rate of return than under autarky (for a given $k_N/l_N$), the South’s rate of return is higher under autarky than under FPE (for a given $k_S/l_S$).

Notice that the model also generates persistent cross-country differences in capital-output ratios and investment shares. Moreover, except for at the very beginning of the transition, there is a positive correlation between income per capita levels and investment ratios. Recall that a similar empirical finding in Mankiw et al. [?] was interpreted in terms of the Solow model’s steady-state predictions: cross-country (parameter) differences in saving rates lead to cross-country differences in steady-state income per capita levels in that model. Ours is an interesting counterpoint to this interpretation: differences in saving rates may arise endogenously simply due to the fact that countries trade and end up having strong differences in capital-labor ratios.

In spite of the lack of convergence in the levels, the model generates convergence in growth rates under both trade regimes. Table 1 reports differences in income levels over time yielded by the autarky and CS models. Notice that at time 1 both models deliver roughly the same difference. Over time, however, the autarky model washes away the whole difference quite rapidly, whereas income differences between North and South
The variation in growth rates of income per capita across North and South displayed by the CS model is smaller than that of the autarky model. Table 1 summarizes the absolute differences between the growth rates of output in the North and the South at a few regularly spaced points in time: at the beginning of the transition, under CS $\Delta \gamma$ is 1.7 times lower than under autarky. In comparison with the autarky case, the North exhibits positive growth rates of income per capita over the transition. Finally, the convergence speed is 6.7% per year in both regions.

The evolution of factor prices implies $N > S$, $w_N > w_S$, and $r_N < r_S$ as long as CS applies. $r_N$ decreases very slowly, whereas $r_S$ falls rapidly in the first periods. Notice that the differential $r_S - r_N$ is smaller and less persistent over time than under autarky. Table 1 summarizes the differences between the interest rates in the North and the South, expressed in percentage points: at the beginning of the transition, when incomes are very similar both for autarky and CS, under CS $r\%\%_\text{CS}$ is 2.2 times lower than under autarky. Sizable cross-country differences in income per capita levels are compatible with small interest rate differentials during the transition and in steady state.

Notice that at time 0 North and South have the same initial capital stocks and very similar levels of income per capita both under autarky and CS. On the other hand, the inequality $r_{\text{CS}}^S > r_{\text{CS}}^N > r_{\text{AU}}^N > r_{\text{AU}}^S$ illustrates the effect of international trade on factor prices. Given that under autarky the interest rate is a function of only the country’s own capital-labor ratio, large differences in capital-labor ratios across countries are translated into large rental-rate differentials. Under CS, in contrast, the equilibrium interest rates of North and South, even if different, must reflect relative scarcities of capital and labor at the world level, and are therefore closer to each other than under autarky.

Even if in the current framework regions reach steady-state levels different from those of the autarky model, international trade is beneficial for both North and South. We calculate the discounted flow of utility, obtained from (14) over a 2000-year horizon, and (not surprisingly) find that both regions obtain the lowest welfare level under autarky. In particular, under autarky North and South obtain respectively $-50.09$ and $-59.27$, while

<table>
<thead>
<tr>
<th>$y_N - y_S / y_S$</th>
<th>$t = 1$</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU</td>
<td>123.61</td>
<td>19.78</td>
<td>4.87</td>
<td>1.33</td>
<td>0.37</td>
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<td>49.54</td>
<td>38.61</td>
<td>36.03</td>
<td>35.34</td>
<td>35.28</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma_N - \gamma_S$</th>
<th>$t = 1$</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
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<tr>
<td>AU</td>
<td>7.60</td>
<td>1.22</td>
<td>0.30</td>
<td>0.08</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>CS</td>
<td>4.60</td>
<td>0.71</td>
<td>0.17</td>
<td>0.04</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$(r_S - r_N)$%</th>
<th>$t = 1$</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>AU</td>
<td>12.58</td>
<td>2.01</td>
<td>0.50</td>
<td>0.14</td>
<td>0.04</td>
<td>0.01</td>
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<tr>
<td>CS</td>
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<td>1.07</td>
<td>0.25</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1: Income, growth, and interest rate differentials.
under free trade they obtain $-49.65$ and $-58.55$.

4.3 Factor Price Equalization

In the introduction, we discarded the FPE case on the basis that FPE does not seem to be a realistic scenario. For the sake of completeness, however, we now discuss the dynamics yielded by the FPE case with the proviso that the comparison with the CS model can not be perfect here: given our initial capital stocks $k_N = 0.5$ and $k_S = 0.1$, we need to enlarge the FPE set to ensure that FPE takes place from the very beginning. We do this by assuming $\alpha = 0.4$. Hence, we compare the CS and FPE trade regimes in the same theoretical framework but under different parameterizations. To minimize the effects of this necessary choice, we recalibrate the value of the scale parameter $\phi$ to make the model reproduce a steady-state capital stock in the world equal to one: this implies that the vale of $\xi$ remains the same under both parameterizations. Given that under FPE the parameter $\alpha$ influences the dynamics of the system only through the size of the FPE set and the value of $\xi$, in the experiment discussed here we are isolating the first effect from the second.

Figures 7 and 8 plot the dynamic behavior of the relevant variables for both North and South. As in the autarky and CS cases, the FPE model yields convergence in growth rates. The speed of convergence is slightly lower than under CS, and equal to 6.1% per year in both countries. Notice, however, that the growth rate differentials generated here are much smaller than under CS or autarky, while the interest rate differentials are, by construction, zero. Concerning the levels, the FPE model is similar to the CS model in the sense that it yields different steady-state income per capita levels for countries with different initial capital-labor ratios. In fact, our FPE model leads to income per capita divergence. Although the South’s growth rate is higher than that of the North, this difference does not make up for the large income gap between the two.

Ventura [17] points out that the predictions of his two-good FPE model (that is, the case implied by $\alpha = 1$ in our model) depend crucially on the value of the elasticity of substitution in the production function of the final good. We have simulated our three-good FPE model for a wide range of elasticities of substitution ($0.1 - 4$). In comparison with the results we report here, we find no remarkable qualitative differences concerning the time paths of both regions’ income per capita levels and growth rates. These results are available upon request.\textsuperscript{13}

\textsuperscript{13}In Ventura [17], an elasticity of substitution higher than unity generates endogenous perpetual growth. In his model, two intermediate goods are produced using only capital or labor. In our model, three intermediate goods are produced, and the second one is produced with both capital and labor, via a Cobb-Douglas technology. Our model does not generate endogenous growth for elasticities of substitution in the range $(0.1 - 4)$. 

5 Many Countries

The division of the world in North and South helps us compare the dynamic behavior of poor and rich economies, but does not tell us much about the dynamics of countries within each region. We address this issue by assuming that North and South consist each of \( n \) countries with identical preferences and technologies. We consider an initial distribution of production factors across North and South identical to the one we assumed in the CS case: \( l_N = l_S = 1 \), \( k_N = (1/2 + \varepsilon)k \), and \( k_S = (1/2 - \varepsilon)k \), \( \varepsilon \in (\alpha, 1/2) \).

Concerning each region’s disaggregation, we assume that countries within each region are under FPE. Without loss of generality, we assume that in both regions countries are ranked from more \((1)\) to less \((n)\) capital abundant. Within each region \( j \), production factors are initially distributed as follows: \((i)\) population is uniformly distributed across countries, so that population in each country is equal to \(1/n \); \((ii)\) the initial capital stocks are uniformly distributed between \( k_{1,j} = (1/n + \varepsilon_j)k_j \) and \( k_{n,j} = (1/n - \varepsilon_j)k_j \), where \( j \in \{N, S\} \), so that \( \sum_{i=1}^{n} k_{i,j} = k_j \). We assume \( \varepsilon_j \) small enough for FPE to hold within each region (or diversification cone).

Rather than solving the corresponding dynamic general equilibrium, we make use of the results obtained for the CS model in the previous section. In particular, we take the paths of income, capital, consumption and prices of each cone and impose that the corresponding variables at the country level be consistent with their region’s integrated equilibrium.\(^{14}\) Obviously, we need to make sure that the cross-country factor distribution satisfies the condition for FPE within each cone, which is discussed in the Appendix. We set \( n = 6 \) and assume an initial cross-country factor distribution that implies \( \varepsilon_N = \varepsilon_S = 0.03 \). After solving for the dynamic paths of our \( 2n \) countries, we check whether the within-cone FPE condition holds.

This many-country model combines the results of the CS and FPE cases discussed above. Figure 9 summarizes the intra-regional evolution of income levels. It is apparent there is convergence in growth rates within each cone. Notice that the within-cone partition of North and South produces additional cross-country variation in growth rates and income levels without higher interest-rate differentials. Within each region, convergence in growth rates does not lead to convergence in income per capita levels. In fact, there is a small increase in the dispersion of income per capita within the North, and a much more remarkable divergence process in the South. However, the worldwide dispersion of income per capita falls, because the South as a whole reduces its distance from the North, as we saw in the CS case.

The model can produce interesting distribution dynamics, since countries with similar (but unequal) initial income per capita levels and production structures may not necessarily follow the same time path. In the steady state, the South’s most capital-abundant

\(^{14}\) The Appendix gives further details about this procedure.
country becomes hardly distinguishable from the North’s most labor-abundant country. The initial income per capita difference between these two countries is reduced over time because of the southern country growing at a much faster rate than the northern country; this is due to the higher return to capital in the South in the early (CS) stages of the transition.

On the other hand, the distance between the South’s richest and poorest countries increases remarkably. Although the poorer country’s growth rate is higher than that of the richest one, this difference is not large enough to reduce the income gap between the two. The intuition here is identical to that of Section 4.3’s worldwide FPE case.

Unlike in the autarky model, shocks can lead countries to different steady states. Assume, for example, that at time 0 we redistribute some capital stock between countries 1 and n in the South cone so that $k_{1,S} = k_{n,S}$ and $k'_{n,S} = k'_{1,S}$. In this case, we just need to relabel the paths of income per capita in Figure 9 correspondingly: country n in the South cone would now achieve a higher steady-state level of income per capita than country 1. Finally, notice also that in this framework a country that is sufficiently small compared to the region it belongs to and suffers a loss in its capital stock, might grow at a higher rate without necessarily experiencing a higher interest rate.

6 Concluding Remarks

Complete specialization is a more realistic trade scenario than autarky or factor price equalization. A very stylized dynamic macroeconomic model that implies complete specialization in its initial conditions yields more realistic dynamics or, at least, overcomes some shortcomings in the neoclassical growth model and the combination of the Ramsey model with FPE. The key insight underlying our model is the determination of factor prices. During the transition, the country with the lower capital-labor ratio has a higher return to capital. In the steady state, factor prices are the same for all countries, and depend only on the world’s capital-labor ratio.

In a strictly neoclassical framework in which countries only differ in initial capital-labor ratios, convergence in growth rates obtains without implying absolute convergence in levels. The CS model can also generate a sizable cross-country variation of growth rates of income per capita without yielding as large interest-rate differentials as in the autarky case. Moreover, it has the potential of producing realistic distribution dynamics. Thus, the Ramsey/CS model seems to be a better benchmark from which to depart when studying the economics of capital accumulation and income per capita growth: if one needs a starting point to assess the importance of human capital or technical progress, then it might be better to start from here rather than autarky.

This paper underlines the importance of international trade for the understanding of economic growth in a world made of open economies and with imperfections in inter-
national capital markets. Further research on trade regimes may help us create more realistic scenarios to analyze the growth performance of countries more accurately.

References


7 Appendix A: Static Equilibrium

This appendix discusses the competitive trade equilibria implied by the FPE and CS scenarios.

7.1 The Integrated Equilibrium

The integrated equilibrium is obtained by considering that the entire world is a single closed economy. The equilibrium conditions can be summarized as follows:

1. Price equal to unit cost:

\[ 1 = c(p_1, p_2, p_3) = \chi p_1^\alpha p_2^{1-\alpha} p_3^\beta \]
\[ p_1 = c_1(w, r) = w \]  
\[ p_2 = c_2(w, r) = 2\sqrt{wr} \]  
\[ p_3 = c_3(w, r) = r \]

where \( \chi = [\phi (\frac{\alpha}{2})^\alpha (1 - \alpha)^{1-\alpha}]^{-1} \) is a positive constant.

2. Consumption shares:

\[ p_1 x_1 = p_3 x_3 \]
\[ p_1 x_1 = \frac{\alpha}{2(1 - \alpha)} p_2 x_2 \]

3. Market clearing:

\[ x_z = y_z \]
\[ \sum_{z=1}^{3} \frac{\partial c_z(w, r)}{\partial w} y_z = l \]
\[ \sum_{z=1}^{3} \frac{\partial c_z(w, r)}{\partial r} y_z = k \]

The unknowns of the problem are \( p_1, p_2, p_3, x_1, x_2, x_3, w, \) and \( r. \) We obtain the following solution for these variables:

1. Factor prices: \( w = \xi \left( \frac{k}{l} \right)^{1/2}, \) and \( r = \xi \left( \frac{k}{l} \right)^{-1/2}, \) where \( \xi = \chi^{-2\alpha-1} \) is a positive constant.

2. Goods prices: \( p_1 = \xi \left( \frac{k}{l} \right)^{1/2}, \) \( p_2 = 2\xi, \) and \( p_3 = \xi \left( \frac{k}{l} \right)^{-1/2}. \) The invariant behavior of \( p_2 \) is due to the symmetry we have imposed on the model.
3. Production of intermediate goods: \( x_1 = \alpha l, \ x_2 = (1 - \alpha) k^1 l^\frac{1}{2}, \) and \( x_3 = \alpha k. \)

4. Finally, we also compute the sectorial allocation of production factors, necessary to determine the FPE condition: \( (k_1, l_1) = (0, \alpha l), \ (k_2, l_2) = [(1 - \alpha) k, (1 - \alpha) l], \) and \( (k_3, l_3) = (\alpha k, 0). \)

### 7.2 Complete Specialization

For \( \varepsilon \in (\alpha, \frac{1}{2}) \), we know that \( y_{1N} = y_{3S} = 0. \) The equilibrium conditions for the two-cone case can be summarized as follows:

1. Price equal to unit cost:

   \[
   1 = c(p_1, p_2, p_3) = \chi p_1^2 p_2^{1-\alpha} p_3^\alpha
   \]

   \[p_1 = c_1(w_S, r_S) = w_S\]  

   \[p_2 = c_2(w_S, r_S) = 2\sqrt{w_S r_S} = c_2(w_N, r_N) = 2\sqrt{w_N r_N}\]  

   \[p_3 = c_3(w_N, r_N) = r_N\]

2. Consumption shares:

   \[p_1 x_1 = \frac{\alpha}{2(1-\alpha)} p_2 (x_2 S + x_2 N)\]  

   \[p_3 x_3 = \frac{\alpha}{2(1-\alpha)} p_2 (x_2 S + x_2 N)\]

3. Market clearing:

   \[\sum_{z=1}^{2} \frac{\partial c_z(w_S, r_S)}{\partial w_S} y_{zS} = 1\]  

   \[\sum_{z=1}^{2} \frac{\partial c_z(w_S, r_S)}{\partial r_S} y_{zS} = k_S\]  

   \[\sum_{z=2}^{3} \frac{\partial c_z(w_N, r_N)}{\partial w_N} y_{zN} = 1\]  

   \[\sum_{z=2}^{3} \frac{\partial c_z(w_N, r_N)}{\partial r_N} y_{zN} = k_N\]

The unknowns of the problem are \( p_1, p_2, p_3, x_1, x_2 S, x_2 N, x_3, w_S, r_S, w_N, \) and \( r_N. \) From the complete specialization equilibrium conditions we obtain the following system
of two equations:

\[(1 - \alpha) \sqrt{\sigma_S} - \frac{k_S}{\sqrt{\sigma_S}} = \alpha \sqrt{\sigma_N} \tag{47}\]

\[(1 - \alpha) \frac{k_N}{\sqrt{\sigma_N}} - \frac{k_S}{\sqrt{\sigma_S}} = \alpha \frac{s}{\sqrt{\sigma_S}} \tag{48}\]

This yields the factor-price ratios \(\sigma_j\) as functions of the two capital stocks \(\sigma_j = \sigma_j (k_N, k_S)\).

1. Factor prices: manipulating the equilibrium’s pricing equations, we can write factor prices as functions of the factor-price ratios. This yields \(w_N = \xi \sigma_N^{2+\alpha} \sigma_S^{-\frac{\alpha}{2}}\), \(w_S = \xi \sigma_N^{\frac{\alpha}{2}} \sigma_S^{2+\alpha}\), \(r_N = \xi \sigma_N^{\alpha+2} \sigma_S^{-\frac{\alpha}{2}}\), and \(r_S = \xi \sigma_N^{\frac{\alpha}{2}} \sigma_S^{-(2+\alpha)}\). Hence, factor prices are also functions of the capital stocks of North and South: \(w_j = w_j (k_N, k_S)\), \(r_j = r_j (k_N, k_S)\).

2. Goods prices: plugging the solutions for \(w_j\) and \(r_j\) into the cost functions, we can also write goods prices as functions of the capital stocks. That is: \(p_1 = w_S (k_N, k_S)\); \(p_2 = c_2 [w_j (k_N, k_S), r_j (k_N, k_S)]\), \(j = N, S\); and \(p_3 = r_N (k_N, k_S)\).

3. Production of intermediate goods: combining the solutions for \(\sigma_j\) and the factor-market clearing conditions yields \(x_{zj} = x_{zj} (k_N, k_S)\).

Given the production structure implied by the CS equilibrium, the balanced trade condition, and our choice of numeraire, the net exports of region \(N\) are as follows:

\[e_{1N} = -\frac{\alpha}{2} xp_1^{\alpha - 1} p_2^{1-a} p_3^{\frac{\alpha}{2}} y_N < 0 \tag{49}\]

\[e_{2N} = \frac{\alpha}{2} \frac{(y_N - y_S)}{p_2} > 0 \tag{50}\]

\[e_{3N} = \frac{\alpha}{2} x p_1^{\frac{\alpha}{2}} p_2^{1-a} p_3^{\frac{\alpha}{2} - 1} y_S > 0 \tag{51}\]

Thus, the North’s volume of trade \(v_N\) is

\[v_N \equiv |p_1 e_{1N}| + |p_2 e_{2N}| + |p_3 e_{3N}| = \alpha y_N. \tag{52}\]

8 Appendix B: Computational Strategy

8.1 Policy functions

Following Judd (1992), we approximate the policy functions for consumption over a rectangle \(D \equiv [k, k] \times [k, k] \in \mathbb{R}_+^2\) with a linear combination of multidimensional orthogonal basis functions taken from a 2-fold tensor product of Chebyshev polynomials. In other
words, we approximate the policy function for cone \( j \in \{N, S\} \) with:

\[
\hat{c}_j (k_N, k_S; a_j) = \sum_{x=0}^{d} \sum_{q=0}^{d} a_j^q \psi_{zq} (k_N, k_S)
\]  

(53)

where:

\[
\psi_{zq} (k_N, k_S) \equiv T_z \left( 2 \frac{k_N - k}{k - k} - 1 \right) T_q \left( 2 \frac{k_S - k}{k - k} - 1 \right)
\]  

(54)

and \( \{k_N, k_S\} \in D \). Each \( T_n \) represents an \( n \)-order Chebyshev polynomial, defined over \([-1, 1]\) as \( T_n (x) = \cos(n \arccos x) \), while \( d \) denotes the higher polynomial order used in our approximation.

We defined the residual functions as:

\[
R_j (k_N, k_S; a_j) \equiv \beta \hat{c}_j (k_N, k_S; a_j) \left( r'_j + 1 - \delta \right) - \hat{c}_j (k'_N, k'_S; a_j)
\]  

(55)

where:

1. \( k'_j = w_j + (1 - \delta + r_j) j - \hat{c}_j (k_N, k_S; a_j) \);  
2. \( w_N = \xi \sigma_{N}^{\frac{2+\alpha}{4}} \sigma_{S}^{-\frac{\alpha}{4}}, w_S = \sqrt{\frac{\sigma_{N}}{\sigma_{S}}} w_N, r_N = \xi \sigma_{N}^{\frac{2+\alpha}{4}} \sigma_{S}^{-\frac{\alpha}{4}}, r_S = \sqrt{\frac{\sigma_{N}}{\sigma_{S}}} r_N \) if \( k_N > \left( \frac{1}{2} + \alpha \right) k \);  
3. \( r_j = r = \xi \left( \frac{k}{2} \right)^{-\frac{1}{2}} \) and \( w_j = w = \left( \frac{k}{2} \right) r_j \) if \( k_N \leq \left( \frac{1}{2} + \alpha \right) k \);  
4. \( r'_N = \xi (\sigma'_N)^{\frac{2+\alpha}{4}} (\sigma'_S)^{-\frac{\alpha}{4}}, r'_S = \xi (\sigma'_N)^{\frac{2+\alpha}{4}} (\sigma'_S)^{-\frac{(2+\alpha)}{4}} \) if \( k'_N > \left( \frac{1}{2} + \alpha \right) k' \);  
5. \( r'_j = r'_j = \xi \left( \frac{k'_j}{2} \right)^{-\frac{1}{2}} \) if \( k'_N \leq \left( \frac{1}{2} + \alpha \right) k' \).

The vectors \( a_j \) can be chosen efficiently using a projection method; in particular, the Galerkin method is well suited to our needs. This method identifies the \( 2 (d + 1)^2 \) coefficients by imposing the following set of orthogonality conditions among \( R_j (k_N, k_S; a_j) \) and the directions \( \{ \psi_{zq} \}^d_{z=0} \):

\[
\int_{k}^{k} \int_{k}^{k} R_j (k_N, k_S; a_j) \psi_{zq} (k_N, k_S) W (k_N, k_S) dk_N dk_S = 0
\]  

(56)

for \( j \in \{N, S\} \) and \( z, q \in \{0, 1, ..., d\} \), where:

\[
W (k_N, k_S) \equiv \left[ 1 - \left( 2 \frac{k_N - k}{k - k} - 1 \right)^2 \right]^{-\frac{1}{2}} \left[ 1 - \left( 2 \frac{k_S - k}{k - k} - 1 \right)^2 \right]^{-\frac{1}{2}}
\]  

(57)

is the multivariate weighting function for which the \( \psi_{zq} \) are mutually orthogonal.

The multivariate integral in (56) can be approximated numerically by using a tensor-product extension of the univariate Gauss-Chebyshev quadrature method. Given \( m >
$d + 1$ Gauss-Chebyshev quadrature nodes in $[k, \bar{k}]$, that correspond to the zeros of $T_m [2 (x - k) / (\bar{k} - k) - 1]$, we can organize them into two (identical) vectors $\{k_{N,i}\}_{i=1}^{m}$ and $\{k_{S,i}\}_{i=1}^{m}$. The conditions in (56) can then be approximated by the following set of 2 $(d + 1)^2$ nonlinear equations:

$$P_{zq}^j (a_j) = \sum_{i=1}^{m} \sum_{l=1}^{m} R_j (k_{N,i}, k_{S,l}; a_j) \psi_{zq} (k_{N,i}, k_{S,l}) = 0$$

where $z, q \in \{0, 1, ..., d\}$. The system (58) can be solved numerically by using any Newton-type algorithm; we adopt Broyden’s method, a variant of the standard Newton’s method that avoids explicit computation of the Jacobian at each iteration.

In our case, the general procedure outlined in the previous paragraph is not directly applicable. The policy functions turn out to be characterized by a particular shape. Under FPE, the policy functions look like a positively sloped hyperplane in $R^3_+$, while under CS they adopt a slightly more curved shape. Any attempt to approximate them using the same polynomials for both regions produces inaccurate results, in particular around the ‘regime’-switching region.

To bypass this problem, we develop a procedure that generates two different sets of coefficients, one for each region. First of all, we define two proper subsets of $D$: $D_{FPE}$, the FPE region, and $D_{CS}$, the CS region where the North is the capital-intensive cone (the policy functions in the other CS region are perfectly symmetric).

The first step in our procedure consists in finding $m_{FPE} \gg d_{FPE} + 1$ quadrature nodes in $[k, \bar{k}]$, and organizing them into the vectors $\{k_{N,i}\}_{i=1}^{m_{FPE}}$ and $\{k_{S,i}\}_{i=1}^{m_{FPE}}$. Then, we isolate the subset of $\{k_{N,i}\}_{i=1}^{m_{FPE}} \times \{k_{S,i}\}_{i=1}^{m_{FPE}}$ that belongs to $D_{FPE}$, obtaining some $\hat{m}_{FPE} < m_{FPE}$ values for $k_N$ and $k_S$. Finally, we solve the following system of equations numerically for the $(d_{FPE} + 1)^2$ elements of $a_{N}^{FPE}$:

$$P_{zq}^N (a_{N}^{FPE}) = \sum_{i=1}^{\hat{m}_{FPE}} \sum_{l=1}^{\hat{m}_{FPE}} R_j (k_{N,i}, k_{S,l}; a_{N}^{FPE}) \psi_{zq} (k_{N,i}, k_{S,l}) = 0$$

where $z, q = 0...d_{FPE}$, since the symmetry of our model guarantees that, under FPE,

$$\hat{c}_S (k_N, k_S) = \hat{c}_N (k_S, k_N; a_{N}^{FPE})$$

Symmetrically, the second step of the procedure consists in finding $m_{CS} \gg d_{CS} + 1$ quadrature nodes in $[k, \bar{k}]$, and isolating the subset of $\{k_{N,i}\}_{i=1}^{m_{CS}} \times \{k_{S,i}\}_{i=1}^{m_{CS}}$ that belongs to $D_{CS}$, obtaining $\hat{m}_{CS} < m_{CS}$ values for $k_N$ and $k_S$. These values are used to solve the system

$$P_{zq}^j (a_j^{CS}) = \sum_{i=1}^{\hat{m}_{CS}} \sum_{l=1}^{\hat{m}_{CS}} R_j (k_{N,i}, k_{S,l}; a_j^{CS}) \psi_{zq} (k_{N,i}, k_{S,l}) = 0$$

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Table 2: Euler equation residuals

<table>
<thead>
<tr>
<th></th>
<th>FPE</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Both</td>
<td>North</td>
</tr>
<tr>
<td>Avg.</td>
<td>1.89e-9</td>
<td>1.59e-6</td>
</tr>
<tr>
<td>Med.</td>
<td>1.47e-10</td>
<td>7.19e-7</td>
</tr>
<tr>
<td>Std.</td>
<td>6.33e-9</td>
<td>3.14e-6</td>
</tr>
<tr>
<td>Max.</td>
<td>7.00e-8</td>
<td>2.02e-5</td>
</tr>
</tbody>
</table>

where $z, q = 0...d_{CS}$, for the $2(d_{CS} + 1)^2$ elements of \ {$a_N^{CS}, a_S^{CS}$}. A necessary condition is that $\tilde{m}_{FPE}^2 > (d_{FPE} + 1)^2$ and $\tilde{m}_{CS}^2 > 2(d_{CS} + 1)^2$.

Out two-step procedure deals with the curved shape of the policy functions while maintaining a good degree of numerical precision. Table 2 summarizes the empirical distribution of the Euler equation residuals in absolute terms, i.e. the values of $|R_j(k_N, k_S, a_j)|$, over 218 equally spaced points in $D_{FPE}$ and 91 equally spaced points in $D_{CS}$. The results are obtained under our benchmark parameterization, which is: $\alpha = 0.15$, $\beta = 0.949$, $\delta = 0.048$, $d_{FPE} = 8$, $d_{CS} = 4$, $k = 0.1$, and $K = 0.9$. Note that the scale parameter $\phi$ has been calibrated to reproduce a world steady-state capital stock equal to unity. As we can see, the size of the residuals is extremely small in the FPE region, and reasonably small in the CS one. The functional equation residuals are of course only an indirect measure of the quality of our approximation, but still a very informative one. Another informative test of the approximation accuracy is the long-run stability of the solution: the approximated system remains in steady state even if the simulation horizon is extended to 10,000 years.

Once the approximated policy functions are available, we choose the initial conditions and simulate the system recursively to generate the artificial time series for all variables of interest by using the appropriate set of policy functions.

8.2 Disaggregation

To disaggregate each cone into $n$ countries, each populated by $1/n$ households, we assume ex-ante that countries in each cone are under FPE. By iterating the country-level intratemporal budget constraint under this assumption, we obtain (for each cone):

$$k_{j,t} = \sum_{s=t}^{\infty} \frac{R_{i,s}}{r_i + 1 - \delta} \left(c_{j,s} - \frac{w_{i,s}}{n}\right) + \lim_{s \to \infty} \frac{R_{i,s}}{r_i + 1 - \delta} k_{j,s+1}$$

(62)

where $j = 1, 2, ..., n$, $R_{i,s} = \prod_{t=t+1}^{s} (r_i + 1 - \delta)^{-1}$, and $R_{i,T} = 1$.

Then we impose the transversality condition and transform the previous equation into
a fully-fledged intertemporal budget constraint:

$$\sum_{s=t}^{\infty} R^s_t c_{j,s} = (r_t + 1 - \delta) k_{j,t} + \frac{1}{n} \sum_{s=t}^{\infty} R^s_t w_s$$

(63)

Finally, by substituting the Euler equation we obtain

$$c_{j,t} = (1 - \beta) \left[ (r_t + 1 - \delta) k_{j,t} + \frac{1}{n} \sum_{s=t}^{\infty} R^s_t w_s \right]$$

(64)

The previous result implies that:

$$c_{1,t} - c_{j,t} = (1 - \beta) (r_t + 1 - \delta) (k_{1,t} - k_{j,t})$$

(65)

where \( j = 2, 3, \ldots, n \). Note that, by definition, \( c_t = \sum_{j=1}^{n} c_{j,t} \), where \( c_t \) is total consumption in each cone; hence:

$$c_{j,t} = c_t - \sum_{i=1, i \neq j}^{n} c_{i,t}$$

(66)

Substituting iteratively (66) into (66) we obtain:

$$c_{1,t} = \frac{1}{n} \left[ (1 - \beta) (r_t + 1 - \delta) \sum_{j=2}^{n} (k_{1,t} - k_{j,t}) + c_t \right]$$

(67)

The remaining consumption levels \( c_{j,t} \), for \( j = 1, 2, \ldots, n \), can now be recovered from (65):

$$c_{j,t} = c_{1,t} - (1 - \beta) (r_t + 1 - \delta) (k_{1,t} - k_{j,t})$$

(68)

The time series for \( c_t \) and \( r_t \), and the initial conditions \( k_{j,0} \) are sufficient to recover the times series for \( c_{j,t}, y_{j,t}, \) and \( k_{j,t} \). Once these series are at hand, we check ex-post that the corresponding static FPE conditions have been satisfied in all periods.

Checking the FPE conditions here is somewhat elaborate. As long as worldwide FPE does not hold, we need to check the FPE condition within each cone. Let us rank countries within a cone from more (1) to less (n) capital abundant:

The South’s FPE condition can be written as follows. Define:

$$k^j_S \equiv \sum_{i=1}^{j} k_{i,S}, \quad l^j_S \equiv \sum_{i=1}^{j} l_{i,S} = \frac{j}{n}, \quad j = 1, 2, \ldots, n - 1$$

(69)

Let \( l_{1S} \) denote the South’s allocation of labor to sector 1, and so on. Then, a necessary and sufficient condition for FPE in the South cone is: for \( j = 1, \ldots, n - 1, \)

$$\begin{cases} 0 \leq k^j_S \leq k_S & \text{if } l^j_S \leq l_{1S} \\ \frac{k_{S \min}}{l_{S \min}} (l^j_S - l_{1S}) \leq k^j_S \leq \frac{k_{S \max}}{l_{S \max}} l^j_S & \text{if } l^j_S > l_{1S} \end{cases}$$

(70)
Similarly, the North’s FPE condition can be written as follows. Define:

\[ k_N^j \equiv \sum_{i=1}^{j} k_{i,N}, \quad l_N^j \equiv \sum_{i=1}^{j} l_{i,N} = \frac{j}{n}, \quad j = 1, 2, \ldots, n - 1 \quad (71) \]

Let \( l_{2N} \) denote the North’s allocation of labor to sector 2, and so on. Then, a necessary and sufficient condition for FPE in the North cone is: for \( j = 1, \ldots, n - 1, \)

\[ \frac{k_{2N}}{l_{2N}} l_N^j \leq k_N^j \leq \frac{k_{3N}}{l_{2N}} l_N^j \quad (72) \]

Once worldwide FPE holds, we combine results in Deardorff (1994) and Cuñat (2001) to check the many-country FPE condition. Let us rank all countries in the world from more (1) to less (2n) capital abundant; define:

\[ k_W^j \equiv \sum_{i=1}^{j} k_{i,W}, \quad l_W^j \equiv \sum_{i=1}^{j} l_{i,W} = \frac{j}{n}, \quad j = 1, 2, \ldots, 2n - 1 \quad (73) \]

Then, a necessary and sufficient condition for FPE is: for \( j = 1, \ldots, 2n - 1, \)

\[
\begin{align*}
0 \leq & \frac{l_W^j}{l_{l_1}} l_W^j \leq k_3 + \frac{k_3}{l_{l_1}} l_W^j & \text{if } l_W^j \in (0, l_1] \\
\frac{k_2}{l_{l_2}} (l_W^j - l_1) \leq & k_W^j \leq k_3 + \frac{k_3}{l_{l_2}} l_W^j & \text{if } l_W^j \in (l_1, l_2) \\
\frac{k_2}{l_{l_2}} (l_W^j - l_1) \leq & k_W^j \leq k & \text{if } l_W^j \in [l_2, l)
\end{align*}
\]

(74)

The previous condition can be rewritten as:

\[
\begin{align*}
0 \leq & k_W^j \leq \alpha k + \frac{k_j}{l_n} & \text{if } j/n \in (0, \alpha l] \\
\frac{k_j}{l_n} \left( \frac{l_n}{l_1} - \alpha l \right) \leq & k_W^j \leq \alpha k + \frac{k_j}{l_n} & \text{if } j/n \in (\alpha l, (1 - \alpha) l) \\
\frac{k_j}{l_n} \left( \frac{l_n}{l_1} - \alpha l \right) \leq & k_W^j \leq k & \text{if } j/n \in [(1 - \alpha) l, l]
\end{align*}
\]

(75)

### 8.3 Autarky

Under autarky, each region behaves as a closed Ramsey economy. The first order conditions for the representative household become

\[
\beta c_t \left( \frac{1}{2} A k_{t+1}^{-\frac{1}{2}} + 1 - \delta \right) = c_{t+1} \quad (76)
\]

\[ k_{t+1} = (1 - \delta) k_t + A \sqrt{k_t} - c_t \quad (77) \]

where \( A \equiv \phi \alpha^\alpha (1 - \alpha)^{1-\alpha}. \)

We approximate the policy function for \( c_t \) over \([k, \bar{k}] \in R^+ \) with a linear combination
of Chebyshev polynomials:

$$\hat{c}(k; a_A) = \sum_{i=0}^{d_A} a_i^A \psi_i(k) \quad (78)$$

where:

$$\psi_i(k) \equiv T_i \left( \frac{2k - k_0}{k_1 - k_0} - 1 \right) \quad (79)$$

To choose a suitable vector $a_A$, we apply the Galerkin method again, defining the residual function as:

$$R(k; a_A) \equiv \beta \hat{c}(k; a_A) \left[ \frac{1}{2} A k' (k; a_A)^{-\frac{1}{2}} + 1 - \delta \right] - \hat{c} [k' (k; a_A); a_A] \quad (80)$$

As before, we obtain $m_A > d_A + 1$ zeros of Chebyshev polynomials in $[-1, 1]$, find the corresponding values in $[k, \bar{k}]$, and organize them into a vector $\{k_i\}_{i=1}^m$. Finally, we solve the following system of equations numerically for $a_A$:

$$P_i(a_A) = \sum_{j=1}^{m_A} R(k_j; a_A) \psi_i(k_j) = 0$$

where $i = 0...d_A$.

Once the approximated policy function is at hand, we simulate the system for each region, using the same parameterization and the same initial conditions as before.
Figure 1: The trade share in GDP for a cross-section of non-OPEC countries in 1996.

Figure 2: The integrated economy.
Figure 3: Income and capital under autarky.

Figure 4: Factor prices, capital-income ratios, and investment shares under autarky.
Figure 5: Income and capital under CS.

Figure 6: Factor prices, capital-income ratios, and investment shares under CS.
Figure 7: Income and capital under FPE.

Figure 8: Factor prices, capital-income ratios, and investment shares under FPE.
Figure 9: Disaggregation: income levels.