

1 The evolution of genomic islands by increased establishment probability of
2 linked alleles

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17

18 **Abstract**

19 Genomic islands are clusters of loci with elevated divergence that are commonly found in

20 population genomic studies of local adaptation and speciation. One explanation for their

21 evolution is that linkage between selected alleles confers a benefit, which increases the

22 establishment probability of new mutations that are linked to existing locally adapted

23 polymorphisms. Previous theory suggested there is only limited potential for the

24 evolution of islands via this mechanism, but involved some simplifying assumptions that

25 may limit the accuracy of this inference. Here we extend previous analytical approaches

26 to study the effect of linkage on the establishment probability of new mutations, and

27 identify parameter regimes that are most likely to lead to evolution of islands via this

28 mechanism. We show how the interplay between migration and selection affects the

29 establishment probability of linked vs. unlinked alleles, the expected maximum size of

30 genomic islands, and the expected time required for their evolution. Our results agree

31 with previous studies, suggesting that this mechanism alone is unlikely to be a general
32 explanation for the evolution of genomic islands. However, this mechanism could occur
33 more readily if there were other pre-adaptations to reduce local rates of recombination or
34 increase the local density of mutational targets within the region of the island. We also
35 show that island formation via erosion following secondary contact is much more rapid
36 than island formation from *de novo* mutations, suggesting that this mechanism may be
37 more likely.

38

39 **Introduction**

40 Studies of natural populations commonly find that genetic divergence is elevated in a
41 number of genomically restricted regions along the chromosomes. These have been
42 dubbed “genomic islands” of speciation (Turner *et al.* 2005), differentiation (Harr 2006),
43 or divergence (Nosil *et al.* 2009), depending on the biological context and the way they
44 are measured. Yet, other studies have not found such islands, although they might have
45 been expected in their respective contexts (Nosil *et al.* 2009; Strasburg *et al.* 2012;
46 Renaut *et al.* 2013). What can be learned about the evolutionary history of a species by
47 studying these genomic islands, and what does their absence tell us? There are many
48 different ways that the interplay between selection and demography can give rise to this
49 characteristic pattern in the genome. Therefore, identifying ways to discount alternative
50 explanations and evaluate the significance of presence, absence, and extent of genomic
51 islands is critical to making inferences about evolutionary history.

52 Mechanistic explanations for the evolution of genomic islands can be categorized
53 by whether clusters arise as: 1) purely the result of drift and demography, 2) neutral by-

54 products of linkage and natural selection with or without on-going gene flow, 3) a direct
55 adaptive response, due to the advantage of physical linkage among loci experiencing
56 divergent natural selection and gene flow (summarised in Table 1), or through some
57 combination of these explanations. These three broad categories reflect differences in the
58 number of selected loci in the island that affect the trait under divergent selection, with
59 either none, one, or several causal loci per island, respectively. We emphasise that
60 explanation 2 can lead to situations where multiple selected loci, each with their own
61 cluster of neutral sites, are located in close proximity along the genome by random
62 chance. This would lead to an empirical pattern very difficult to distinguish from
63 explanation 3 in practice. For conceptual clarity, however, we categorize these
64 mechanisms in terms of the number of causal loci and the type of linkage-mediated
65 benefit among them.

66 Before reviewing these three mechanisms, it is important to situate our approach
67 in the context of the many ways that genetic divergence can be quantified and used to
68 identify genomic islands. The most widely used measures of divergence are F_{ST} (Wright
69 1931, 1943) and related statistics (Nei 1973; 1982; Slatkin 1991; Charlesworth 1998;
70 Excoffier 2007), which express the variance in allele frequency among (partially) isolated
71 populations, normalized by the genetic diversity in an appropriately defined panmictic
72 population. As such, F_{ST} -like statistics are relative measures of divergence. They are
73 sensitive to a variety of processes affecting diversity within and among populations, and
74 therefore tend to be easily confounded. In contrast, absolute measures of divergence such
75 as D_{xy} (Nei & Li 1979) are more specific, but not very sensitive with certain types of
76 data. Here, we formulate our discussion of islands of divergence in terms of F_{ST} . Where

77 necessary, we mention potential confounding factors, and in Table 1 we contrast F_{ST} to
78 patterns including absolute measures that are diagnostic for various mechanisms. For a
79 review of relative and absolute measures and their properties, we refer to Cruickshank &
80 Hahn (2014).

81 Although the action of natural selection through either mechanism 2 or 3 may feel
82 intuitively most parsimonious, it is important to discount neutral explanations in
83 empirical studies (i.e. mechanism 1, Table 1). If a pattern of genomic islands is found
84 along a single gradient or between a single pair of populations, it may be difficult to
85 conclusively rule out the effect of demography. The distribution of coalescence times for
86 neutral alleles becomes much broader under isolation-by-distance or hierarchical
87 structure than under panmixia, so loci that appear to be outliers under an island model
88 may in fact still be consistent with drift (Excoffier *et al.* 2009; Hermisson 2009). Linkage
89 among loci in close proximity on a chromosome could then cause the appearance of an
90 island of extreme F_{ST} . Additionally, rapid population expansion can result in ‘allelic
91 surfing’ due to increased genetic drift at the wave front (e.g. Slatkin & Excoffier 2012).
92 This could likely cause blocks of elevated divergence, especially in regions of low
93 recombination, but has not been extensively studied. However, if the same genomic
94 islands are found along several demographically independent gradients, it is much less
95 likely that the pattern could have arisen purely by drift and demography.

96 Within the second category of explanation, there are several different ways that
97 linkage of neutral loci to an allele under selection can result in elevated divergence along
98 a chromosome. Recurrent bouts of either purifying or positive and spatially homogenous
99 selection can result in reduced genetic variation within populations at regions linked to

100 the loci under selection (Maynard Smith & Haigh 1974; Charlesworth *et al.* 1993), which
101 can inflate relative metrics of divergence, such as F_{ST} (sub-category 2A in Table 1; Nei
102 1973; Charlesworth *et al.* 1997; Nordborg 1997; Cruickshank & Hahn 2014). It is unclear
103 whether alternative, absolute metrics of divergence that are insensitive to variation within
104 populations, such as D_{xy} , are sensitive enough to address this problem, but if genomic
105 islands do not co-occur with locally reduced heterozygosity, then the explanations of
106 purifying selection and global sweeps can be discounted. Alternatively, spatially
107 heterogeneous selection can lead to increased divergence at linked neutral regions. If this
108 occurs as a single selective sweep within a restricted region of the species range (without
109 gene flow), then hitchhiking can decrease heterozygosity and increase divergence at
110 linked sites, but this signature will decay over time (sub-category 2B in Table 1; Maynard
111 Smith and Haigh 1974; Gillespie 2000). When divergent selection operates in the face of
112 sufficiently low gene flow, such local sweeps are never completed and an equilibrium is
113 reached where the change in allele frequency due to selection is opposed by gene flow. In
114 populations of finite size, this recurrent selection against maladaptive gene flow generates
115 a persistent pattern of increased divergence and reduced heterozygosity at linked sites
116 (sub-category 2C in Table 1; Charlesworth *et al.* 1997; Nordborg 1997; Via & West
117 2008; Feder & Nosil 2010). Finally, if locally adapted and previously allopatric
118 populations come into secondary contact, erosion of divergence will occur most rapidly at
119 neutral loci that are less tightly linked to a given adapted locus (sub-category 2D in Table
120 1; Petry 1983; Barton and Bengtsson 1986; Barton and Hewitt 1989).

121 By the third category of explanation, genomic islands evolve as an adaptive
122 consequence of the way recombination mediates the tension between divergent selection

123 and gene flow. If two or more locally beneficial mutations are tightly physically linked,
124 recombination is less likely to break up this favourable combination than when linkage is
125 weak (Fisher 1930; Charlesworth 1979; Lenormand & Otto 2000). This advantage due to
126 linkage can lead to clustered genetic architectures under divergent selection in the face of
127 gene flow, through one of four potential evolutionary routes (category 3 in Table 1): A)
128 increased probability of establishment of linked alleles during initial divergence (Yeaman
129 & Whitlock 2011; Feder *et al.* 2012; Aeschbacher & Bürger 2014); B) competition
130 among genetic architectures as new linked mutations displace older unlinked mutations of
131 equal overall effect (Yeaman and Whitlock 2011); C) competition among genomic
132 architectures, which favours the fixation of rearrangements moving loci into close
133 proximity on a chromosome (Yeaman 2013) or the establishment of segregating
134 inversions that reduce recombination (Noor *et al.* 2001; Rieseberg 2001; Kirkpatrick
135 2006); D) increased persistence time for linked and selected loci under secondary contact
136 following local adaptation in allopatry (Petry 1983; Barton & Bengtsson 1986; Barton &
137 Hewitt 1989). In all of the above cases in category 3, selection at the causal loci would
138 also cause elevated divergence at linked neutral loci.

139 One way to differentiate between explanations of category 2 and 3 is to identify
140 whether, in a single genomic island, there are multiple sites harbouring alleles that
141 contribute to fitness differences between populations. This is expected to occur under the
142 latter but not the former explanation. While it is very difficult to identify which mutations
143 are functionally important, it may be possible to infer their presence from observations
144 about multiple peaks within an island, or through inferences about the maximum
145 expected width of an island under explanations of category 2 vs. 3. Furthermore, we

146 could learn much about a species' evolutionary history by discriminating between the
147 four possible explanations within category 3. In practice, this is difficult because some of
148 the expected statistical signatures from these routes to clustering may be very similar at
149 equilibrium. For instance, at equilibrium, both scenarios 3.A) and 3.D) lead to an
150 expected pattern of F_{ST} peaks surrounded by regions of low F_{ST} . That said, the
151 trajectories to this equilibrium pattern may be quite different. In 3.A), peaks of
152 divergence could be seen as mountains arising against a more or less constant
153 background, whereas in 3.D) regions of reduced divergence are better viewed as valleys
154 eroding between mountains of a (roughly) fixed height. As such, it is helpful to use
155 theoretical methods to investigate the expected properties of these various explanations
156 and examine their relative likelihood.

157 Theoretical arguments have been made suggesting that the higher establishment
158 probability (EP) of linked mutations (explanation 3.A) is unlikely to result in a strong
159 signal of clustering, because the increase in EP is small and extends over a small region
160 of a given chromosome, relative to the size of the unlinked portion of the genome
161 (Yeaman 2013). However, the analysis by Yeaman (2013) discounted the effect of
162 linkage in the regions of parameter space where it is potentially most important, so the
163 inferences made about the likelihood of genomic island evolution via increased EP may
164 be incorrect. Moreover, Yeaman (2013) employed a semi-heuristic approximation to the
165 EP (Yeaman & Otto 2011) that has not been formally tested in the case of multiple linked
166 loci under selection. Here, we first verify this approximation by comparison to an
167 alternative, more formally derived one (Aeschbacher & Bürger 2014). Second, we extend
168 the work of Yeaman (2013) to quantify more comprehensively the potential importance

169 of increased EP at linked mutations as an explanation for the evolution of genomic
170 islands. Specifically, we explore predictions about the expected width of a genomic
171 island, the absolute and relative increase in EP due to linkage between selected sites, the
172 waiting time for island evolution, and the effect of different distributions of mutational
173 effect size on these predictions. Third, we determine how fast neutral divergence that has
174 built up between two loci under spatially divergent selection without gene flow erodes
175 upon secondary contact, and compare this to the time it takes for an equivalent two-peak
176 island to arise by divergent selection with on-going gene flow. Overall, our aim is to
177 refine the theoretical arguments about island evolution, provide some testable predictions
178 about island size, and identify how further study could aim to test the other hypotheses
179 for island evolution, particularly through secondary contact and erosion of genetic
180 divergence around selected loci.

181

182

183 **Model**

184 To explore the likelihood of genomic islands evolving by increased probability of
185 establishment of new linked mutations (explanation 3.A, Table 1), it is necessary to
186 consider both the effect of linkage on the establishment probability (EP) and the rate of
187 occurrence of linked vs. unlinked mutations. To study the effect of linkage on the EP of a
188 *de novo* mutation that occurs near an already established locally adapted allele, we
189 describe a model that incorporates the relevant evolutionary forces (*i.e.* selection, gene
190 flow, recombination, and genetic drift), but is still simple enough to allow for efficient
191 mathematical analysis. We then use simplifying assumptions about the expected

192 distribution of fitness effects of new mutations and the relative size of the mutational
 193 target in linked vs. unlinked regions to parameterize our model and make inferences
 194 about the likelihood of genomic island evolution via increased EP.

195 We consider a discrete-time, monoecious diploid model with continent–island
 196 type migration (Haldane 1930; Wright 1931) and divergent selection at two linked
 197 biallelic loci A and B. Let the alleles at these two loci be A_1 and A_2 , and B_1 and B_2 ,
 198 respectively. We define the migration rate m as the proportion of the island population
 199 replaced by immigrants from the continent each generation, and denote by r the
 200 probability of recombination between the two loci per generation. Assuming additive
 201 interactions across alleles and loci, and ignoring epistasis as well as parental and position
 202 effects in heterozygotes, we write the fitnesses of the nine distinguishable genotypes in
 203 the island population as

$$204 \quad \begin{matrix} & B_1 B_1 & B_1 B_2 & B_2 B_2 \\ \begin{matrix} A_1 A_1 \\ A_1 A_2 \\ A_2 A_2 \end{matrix} & \begin{pmatrix} 1 + a + b & 1 + a & 1 + a - b \\ 1 + b & 1 & 1 - b \\ 1 - a + b & 1 - a & 1 - a - b \end{pmatrix} \end{matrix}, \quad (1)$$

205
 206 where a and b are the selective advantages on the island of alleles A_1 and B_1 relative to A_2
 207 and B_2 , respectively. To enforce positive fitnesses, we require that $0 < a, b < 1$, and $a + b$
 208 < 1 . Unless otherwise stated, we assume that the continental population is fixed for alleles
 209 A_2 and B_2 , whereas alleles A_1 and B_1 are local *de novo* mutations endemic to the island
 210 population. The conditions for establishment and maintenance of one- and two-locus
 211 polymorphisms in the island population have been described before (Bürger & Akerman
 212 2011; Aeschbacher & Bürger 2014). For mathematical convenience, we assume that

213 selection in favour of A_1 is weaker than selection in favour of B_1 ($a < b$). This assumption
 214 implies that allele A_1 cannot be maintained in the island population if allele B_1 is
 215 swamped by gene flow (Haldane 1930; Bürger & Akerman 2011). It is further in
 216 agreement with our principal focus on the effect of linkage on establishment of a weak,
 217 locally beneficial *de novo* mutation arising in the presence of an already existing
 218 migration–selection polymorphism at a background locus. Throughout, we assume that
 219 populations are large enough such that genetic drift can be ignored after an initial
 220 stochastic phase during which *de novo* mutations arise in low absolute numbers. In this
 221 setting, locally beneficial *de novo* mutations have a strictly positive establishment
 222 probability if they can invade in an equivalent fully deterministic model, and *vice versa*
 223 (Bürger & Akerman 2011; Aeschbacher & Bürger 2014). Therefore, if in the following
 224 we say that “ A_1 can be established”, this means both that A_1 has a strictly positive
 225 establishment probability and that it will invade in a corresponding deterministic model.

226 Aeschbacher & Bürger (2014) derived several results for this model that will help
 227 interpret our findings discussed below. We therefore briefly recapitulate them in the
 228 following. First, in a single-locus model (no background locus \mathbf{B}), A_1 can be established if
 229 and only if $m < a$ (cf. Haldane 1930). Second, if $m \geq a$, the additional effect of linkage to
 230 a background locus \mathbf{B} means that A_1 can be established if $m < m^*$, where $m^* =$

231 $\frac{a(b-a+r)}{(a-r)(a-b)+r(1-a)}$ (their Eq. 10). Third, this critical threshold can alternatively be expressed

232 in terms of a critical recombination rate, where linkage must be at least as tight as $r^* =$

233 $\frac{a(a-b)(1+m)}{a(1+2m)-(1+b)m}$ when $m > a / (1 - 2a + b)$ (their Eq. 11). Fourth, if $m > b/(1 - a)$, then no

234 polymorphism will be maintained at locus \mathbf{B} , which automatically implies that A_1 cannot

235 be established because gene flow is too strong.

236 Based on this general model, we derive three approximations for the expected EP
237 of any given new mutation, $\pi = \pi(a, r)$, expressed as a function of a and r , which are
238 discussed in more detail below and in the supplementary materials. To make predictions
239 about the average establishment probability over all available linked mutations, $\bar{\pi}_L$, we
240 integrate $\pi(a, r)$ across the range of possible recombinational distances r at which new
241 mutations could be linked to the background locus, and across the expected Distribution
242 of Fitness Effects (DFE) for values of a . We assume that the DFE is a gamma
243 distribution,

244
$$f_a(k, \bar{a}) = \frac{\left(\frac{k}{\bar{a}}\right)^k}{\Gamma(k)} \bar{a}^{k-1} e^{-\left(\frac{k}{\bar{a}}\right)\bar{a}}, \quad (2)$$

245 with shape parameter k and mean \bar{a} , where $k, \bar{a} > 0$. If $k = 1$, this simplifies to an
246 exponential distribution, $f_a(1, \bar{a}) = \frac{1}{\bar{a}} e^{-a/\bar{a}}$. With a drawn according to equation (2), our
247 assumption of $a < b$ does not necessarily hold; in practice, we therefore set $\bar{a} \ll b$. When
248 integrating over r , we assume a uniform distribution of r between 0 and $1/2$, usually using
249 an upper limit of $r_f = 1/2$, because this represents the threshold to free recombination in a
250 discrete-time model. This approach to integration is biologically akin to assuming that
251 mutations occur with equal probability at all positions within the window of the
252 chromosome where $0 < r < r_f$, and the rate of recombination does not vary along the
253 chromosome. For mutations with $r = 1/2$ on the same chromosome, they are essentially
254 unlinked and are not assumed to contribute to $\bar{\pi}_L$. To represent the average EP of an
255 unlinked mutation, $\bar{\pi}_U$, we multiply $\pi(a, r)$ by the DFE, and integrate over a , setting $r = r_f$

256 = $\frac{1}{2}$. To obtain the approximate size of a genomic island evolving via the benefit of
257 selection at a linked background locus (mechanism 3.A, Table 1), we determine the
258 region of the chromosome over which most new mutations will experience an increased
259 EP via this mechanism. Specifically, we calculate the value of r required to contain 95%
260 of the probability density of $\pi(a, r)$, and refer to this as the 95% window size, or C_{95} .

261 To explore the time scales over which the last step in the evolution of two-peak
262 genomic islands occur under alternative scenarios (explanations 3.A vs. 3.D), we use a
263 combination of stochastic and deterministic theory. We extend our model to bidirectional
264 gene flow and predict the dynamics of neutral divergence using the structured coalescent.
265 Thereby, we replace the neutral migration rate by appropriate rates of effective gene flow,
266 which are a function of the location of the neutral site and the strength of selection at the
267 two loci causing the adaptive peaks (see SI for details).

268

269 **Results**

270 *Analytical predictions for establishment probability*

271

272 *Two-type branching process approximation in discrete time:* The EP under a two-type
273 branching process, $\pi(a, r)$, was derived in Aeschbacher & Bürger (2014), and represents
274 a weighted average over A_1 occurring either on the locally beneficial (B_1) or deleterious
275 (B_2) genetic background (see SI for details). The EP, $\pi(a, r)$, decreases both with the
276 migration rate m and the recombination r , and increases with the selection coefficient a
277 (solid lines in Figure 1). As expected, if migration is sufficiently weak, then A_1 can
278 establish even if unlinked to the background locus B (solid blue line in Figure 1B).

279 To represent the average probability of a new, linked mutation across all possible
280 recombination rates and sizes of selection coefficient, we integrate across both the rate of
281 recombination and the DFE,

282

$$283 \quad \bar{\pi}_L = \int_0^\infty \int_0^{r_f} \pi(a, r) f_a(k, \bar{a}) f_r(0, r_f) dr da, \quad (3)$$

284

285 where $f_r(r_{\min}, r_{\max})$ is a uniform density between r_{\min} and r_{\max} . Other distributions
286 could be used to represent non-homogenous recombination rates or mutation targets, but
287 for simplicity we focus on the uniform case here. We could not find a simple closed form
288 for $\bar{\pi}_L$, so in the following analyses we use numerical integration to investigate how it
289 varies with \bar{a} , b , r , m , and k . The difference between the EP of an average linked vs. an
290 average unlinked mutation of a certain effect size a , i.e. $\bar{\pi}_L^{(r)}(a)$ vs. $\bar{\pi}_U^{(r)}(a)$, increases with
291 m , and above the critical threshold m^* , the EP for an individual unlinked mutation is 0
292 (Figure 2A). Integrating across the DFE, and hence accounting for all possible values of
293 a , shows that the mean EP of unlinked mutations, $\bar{\pi}_U$, decreases more gradually and does
294 not rapidly intersect 0, as it includes the effects of rare large mutations that have non-zero
295 EP (Figure 2B).

296

297 *Slightly-supercritical branching process:* To derive an approximation to the branching-
298 process solution described above, we assume that the selective advantage of the focal
299 mutation and the migration rate are both small relative to the selection coefficient at the
300 background locus and the recombination rate, i.e. that $a, m \ll b, r$. Under these

301 assumptions, it can be shown that the EP of a linked mutation is

302

303
$$\pi(a, r) \approx 2 \frac{a(b+r) - (1+b)mr}{(1+b)(b+r)} \quad (4)$$

304 if $m < a(b+r) / [r(1+b)]$, and 0 otherwise (see SI for derivation). By this
305 approximation, mutations can establish if the recombination rate r is below a critical
306 value given by

307
$$\tilde{r}^- = \frac{ab}{(1+b)m-a}. \quad (5)$$

308 If we further assume that both selection coefficients are much smaller than unity, *i.e.* $0 <$
309 $a \ll b \ll 1$, then this reduces to $\tilde{r}^- = ab/m$, which is identical to the approximate critical
310 recombination rate for invasion of A_1 in a deterministic continuous-time model (see Eq.
311 4.15 in Bürger & Akerman, 2011). This can be further reduced to $\tilde{r}^- \approx b$ when $m \sim a$,
312 which is Barton's rule of thumb that selection against gene flow has an appreciable effect
313 on linked sites only if linkage is sufficiently strong, *i.e.* $r \ll b$ (Barton 2000).

314 Under the assumption of an exponential DFE ($k = 1$), a closed-form solution for the
315 average EP with respect to r and a can be derived by using a Taylor series approximation
316 around $b = 0$ (see SI for details). This yields

317
$$\bar{\pi}_L \approx e^{-\frac{m}{a}} \left\{ \bar{a} + \bar{a}b + bm - 2bm\gamma + 2bm \left[E_i\left(\frac{m}{a}\right) + \ln\left(\frac{\bar{a}}{2bm}\right) \right] \right\} - 2\bar{a}b, \quad (6)$$

318 where $\gamma = 0.577$ is the Euler–Mascheroni constant and $E_i(z) = -\int_{-z}^{\infty} \frac{e^{-t}}{t} dt$ is the
319 exponential integral function. This approximation works well as long as $b \lesssim 0.2$ (see SI
320 and Figure S1 for details), and is very close to the splicing approximation (see below).

321 Both of these approximations overestimate the exact two-type branching process when
 322 either m is large or r is small (Figure 1).

323

324 *Splicing approximation in discrete time*: The rate of increase in frequency of allele A_1
 325 when rare is given approximately by the leading eigenvalue of the Jacobian matrix
 326 describing the stability around the equilibrium where \mathbf{B} is polymorphic and \mathbf{A} is fixed for
 327 A_2 (see SI for details; as per Yeaman and Otto 2011). Because this deterministic rate of
 328 increase in frequency is equivalent to the effect of natural selection in a single-locus one-
 329 population model, this rate can be spliced into the standard probability of fixation for a
 330 new mutation ($\Pr[\text{fix}] = 2s$; Haldane 1927), which yields an approximation to the EP of
 331 A_1 ,

$$332 \quad \pi(a, r) \approx 2 \max\left[0, \frac{2+b-r+m(2a-b-r)+\sqrt{R}}{2(1-a+b)} - 1\right], \quad (7)$$

333 where

$$334 \quad R = (1+m)\{b^2(1+m) + 2b(1-m)r + r[r - m(4 - 4a - r)]\}. \quad (8)$$

335

336 As in the case of the two-type branching process, we must resort to numerical integration
 337 to obtain approximations to the average probability of establishment of linked alleles for
 338 a given selection coefficient a , $\bar{\pi}_L^{(r)}(a)$, and across the entire DFE, $\bar{\pi}_L$. As shown in
 339 Figure 1, the splicing approach (dotted lines) yields an approximation to the EP very
 340 close to the one obtained from the slightly-supercritical branching process assuming
 341 $a, m \ll b, r$ (dashed lines; Eq. 4). Accordingly, it deviates most from the exact two-type
 342 branching process (solid lines; Eq. S3) if a is relatively large and, at the same time, either

343 m is large or r is small. In these cases, the contribution of mutations that occur on the
344 deleterious genetic background B_2 becomes important; both the splicing approach and the
345 slightly-supercritical branching process do not capture this.

346

347 *Island size and establishment probability of linked vs. unlinked mutations*

348 We now use the above approximations to explore the likelihood of genomic islands
349 evolving by mechanism 3.A (Table 1). As outlined above, we integrate both across all
350 possible recombination rates and mutation effect sizes (assuming a gamma-distributed
351 DFE with a mean effect size of \bar{z} , as above) to represent how an average linked vs.
352 unlinked mutation would be affected by the interplay between migration, selection, and
353 recombination. With increasing migration rate in the region of $m \sim \bar{z}$, an increasing
354 fraction of the available mutations become more strongly limited by migration than
355 favoured by selection. This causes a decrease in the mean EP of both linked ($\bar{\pi}_L$) and
356 unlinked ($\bar{\pi}_U$) mutations, but the effect of linkage to the **B** locus mitigates this effect, so
357 that the mean EP decreases faster for unlinked mutations. These broad patterns are
358 illustrated in Figures 3A & B, which show the effect of m on $\bar{\pi}_L$, and Figure 3C, which
359 shows the ratio $\bar{\pi}_L/\bar{\pi}_U$. In each case, there is a pronounced transition in the region of $m \sim$
360 \bar{z} . The decrease in $\bar{\pi}_U$ occurs more rapidly than $\bar{\pi}_L$ because as the migration rate
361 increases, π_U approaches zero for an increasing fraction of unlinked mutations, whereas
362 linked mutations still have non-zero probability of establishment (cf. Figure 2B).
363 Therefore, $\bar{\pi}_L/\bar{\pi}_U$ increases towards infinity if m is increased far beyond \bar{z} , and while
364 linked mutations can still establish, their absolute EP is much reduced.

365 Another parallel effect of increasing migration is to reduce the size of the linked
366 region that can contribute to adaptation. As migration increases, an increasing fraction of
367 the chromosome falls above the critical recombination threshold r^* , such that an
368 increasing fraction of the available linked mutations cannot be established. As a
369 consequence, there is a pronounced decrease in the expected size of the region around the
370 background locus that can contribute to local adaptation if $m > \bar{a}$, as shown by C_{95} , the
371 95% window size (Figure 3D).

372 The shape of the gamma distribution (k) affects the position of the transition zone
373 where i) the EP starts decreasing, ii) the ratio $\bar{\pi}_L/\bar{\pi}_U$ starts increasing, and iii) the size of
374 the 95% window starts decreasing. These transitions occur at lower values of m for $k = 2$
375 (*i.e.* a DFE with relatively more intermediate values of a , compared to $k = 1$) and higher
376 values of m for $k = 0.5$ (*i.e.* a DFE with relatively more extreme values of a , compared to
377 $k = 1$) (Figure S2). Thus, while k does not affect the broad qualitative patterns
378 representing the role of linkage, it does affect the relative importance of linkage at a
379 given value of m (*i.e.* the broad patterns are maintained, but shifted with respect to m with
380 changes in k). Specifically, the average EP of linked mutations, $\bar{\pi}_L$, is higher for smaller k
381 because a larger fraction of the probability density occurs with mutations of larger size,
382 which have proportionally larger EP $\bar{\pi}_L^{(r)}$ (Figure S2B). This is also reflected in the width
383 of the genomic window within which most linked mutations establish: smaller k and
384 hence relatively more mutations of large effect increase the size of the window (C_{95} in
385 Figure S2D). However, put in relation to the average EP of unlinked mutations, $\bar{\pi}_U$, the
386 effect of the shape is compensated for by the fact that unlinked mutations profit
387 proportionally more from a fat right-hand tail (low k) of the input DFE (Figure S2C).

388 It is also worth noting that the EP is lower for higher values of b when m is very
389 low (see SI for details and a derivation of the critical migration rate). This occurs because
390 the contribution to the total fitness made by locus A becomes relatively smaller as b
391 increases (*i.e.*, the marginal fitness ratio of genotype $A_1A_2B.B.$ vs. $A_2A_2B.B.$ decreases
392 with increasing b), which reduces the net advantage of allele A_1 , and hence its EP.
393 However, this effect of the relative magnitude of selection coefficients is only important
394 when the facilitating effect of linkage is relatively small (a large relative to b); when
395 linkage is important, then the EP increases with b .

396

397 *Waiting time for evolution of genomic islands*

398 Taken together, the above results show that linkage becomes critically important for any
399 local adaptation to occur when the migration rate increases beyond the mean selection
400 coefficient ($m > \bar{a}$). However, they also illustrate that in this same region of parameter
401 space, there is a decrease in both the size of the region that can contribute to this
402 adaptation (Figure 3D) as well as the absolute EP of mutations within this region (Figure
403 3A &B). These latter two factors would be expected to greatly increase the waiting time
404 for the first linked mutation to establish, as the mutational target is smaller with small
405 window sizes, and waiting time scales with the inverse of the EP. To obtain a rough
406 approximation to this expected waiting time, we combine the results from Figure 3,
407 calculating $t_L \approx 1/(2N\mu 2C_{95}\bar{\pi}_L)$ and $t_U \approx 1/(2N\mu C_U\bar{\pi}_U)$, for linked and unlinked
408 mutations, where N is the size of the diploid population, μ is the mutation rate per cM per
409 gamete per generation, C_{95} is the 95% window size in cM as defined above, and C_U

410 is the size of the genome that is unlinked to **B**, in cM. We multiply C_{95} by 2 to account
411 for the fact that a linked mutation can occur on both sides of the background locus.
412 Unfortunately, it is difficult to parameterize these equations to make quantitative
413 predictions about the expected waiting time, because we know little about the rate of
414 beneficial mutations. In organisms such as humans, where genome size is on the order of
415 10^9 base pairs (bp), and where there are approximately 10^6 bp/cM and about 10^{-8}
416 mutations/bp/generation, the mutation rate per cM is ~ 0.01 per generation, but it is
417 unclear what fraction of these would be locally beneficial.

418 Given uncertainty about N and μ , and accounting for appropriate mutational
419 target, we scale the waiting times by the inverse of the substitution rate of an average
420 mutation from the same DFE that arises in a completely isolated panmictic population of
421 size N . Because establishment is independent of recombinational distance from the
422 background locus if $m = 0$, this reference mutational target simply amounts to
423 $2C_{95,m=0} = 2 \times 0.95 \times r_f = 0.95$ for $r_f = 0.5$, independently of all the other
424 parameters. Specifically, we approximate the scaled expected waiting times for linked
425 and unlinked mutations as

426

$$427 \quad T_L \approx t_L \left(\frac{1}{4N\bar{\mu}\bar{a}2C_{95,m=0}} \right)^{-1} \approx \frac{4N\bar{\mu}\bar{a}2C_{95,m=0}}{4N\mu C_{95}\bar{\pi}_L} = \frac{0.95\bar{a}}{C_{95}\bar{\pi}_L}, \quad (9a)$$

$$428 \quad T_U \approx t_U \left(\frac{1}{4N\bar{\mu}\bar{a}2C_{95,m=0}} \right)^{-1} \approx \frac{4N\bar{\mu}\bar{a}2C_{95,m=0}}{2N\mu C_U\bar{\pi}_L} = \frac{1.9\bar{a}}{C_U\bar{\pi}_L}. \quad (9b)$$

429

430 This scaling has the advantage of focussing exclusively on the *relative* impact on waiting
431 times of the tension between migration and divergent selection, and therefore seems
432 appropriate for a comparison of linked vs. unlinked locally beneficial *de novo* mutations.

433 Some uncertainty remains about C_U , the size of the unlinked mutational target.
434 However, we can make reasonable choices. For $b = 0.1$ and a C_U of 100 cM, i.e. equal to
435 twice the map distance with respect to which we defined C_{95} , the difference in waiting
436 times for linked and unlinked mutations is negligible if $m < \bar{a}$ (dotted vs. solid curves in
437 Figure 4A). This is expected, because in the limit of no migration, the background locus
438 is fixed for B_1 , and establishment of the focal mutation A_1 does not depend on its
439 recombinational distance from locus B . As m increases above \bar{a} , T_U increases more
440 rapidly than T_L , because linkage now increases the relative establishment probability and
441 the mutational target for linked mutations (C_{95}) is still large (compare Figures 3C and
442 3D). At $m \approx 10\bar{a}$, the increase in T_L becomes log-log linear, while T_U keeps increasing
443 exponentially with m . This is where linkage starts to convey a considerable relative
444 advantage. However, absolute waiting times for linked mutations are at least about 100
445 times higher compared to the case of $m = 0$ (Figure 4A). This log-log linear phase ends if
446 $m \approx b$, at which point gene flow swamps even the beneficial background allele B_1 , and
447 T_L quickly increases to infinity.

448 Realistically, the mutational target size for unlinked mutations is much larger than
449 100 cM in most organisms. As expected, for $C_U > 100$ cM, the waiting time to
450 establishment for unlinked mutations becomes much reduced compared to linked
451 mutations if m is weak enough, suggesting that in this region of parameter space, a strong
452 signature of clustering would be unlikely to evolve. The larger the unlinked target size,

453 the higher the migration rate at which linked mutations are expected to arise earlier than
454 unlinked ones (i.e. where dashed curves intersect with solid ones in Figure 4A). When
455 this occurs, t_L tends to be very long (relative to the waiting time in the absence of
456 migration) except at the highest values of \bar{z} (blue curves in Figure 1A). This illustrates
457 that in the region where a strong signal of clustering is expected via mechanism 3.A, the
458 waiting time for the evolution of this pattern will be very long relative to adaptation in
459 allopatry. The shape of the gamma distribution (k) has a strong effect on absolute waiting
460 times, but does not substantially affect relative difference between linked and unlinked
461 locally beneficial *de novo* mutations (Figure S3). However, there is some effect of b , as
462 the transition to long waiting times with increasing m for linked mutations is sharper and
463 occurs at lower m with small b (Figure 4B).

464

465 *Divergence with gene flow vs. secondary contact*

466 Genomic islands consisting of multiple causal loci can evolve under various mechanisms
467 summarised in category 3 (Table 1). The same amount of divergence is expected at
468 equilibrium in models of secondary contact and *de novo* adaptation (Bürger & Akerman
469 2011; Aeschbacher and Bürger 2014), which makes it challenging to distinguish between
470 these alternative explanations based on empirical observations. However, by studying the
471 dynamics leading to these equilibrium patterns, it may be possible to exclude some
472 mechanisms based on a comparison of associated time scales with the demographic
473 history of populations and species of interest. To illustrate this argument, we concentrate
474 on the competing mechanisms of increased establishment probability of linked mutations
475 in the face of gene flow (explanation 3.A in Table 1) vs. the erosion of neutral divergence

476 upon secondary contact (explanation 3.D). Moreover, we extend our model to allow for
477 symmetric gene flow between the two demes of equal size, as compared to the
478 asymmetric continent–island regime assumed above. We restrict our treatment to
479 genomic islands made up of two causal loci, and we focus on the last evolutionary step
480 needed to complete equivalent two-locus islands at the equilibrium of migration,
481 selection, and genetic drift. By equivalent we mean that the map distance between the
482 two loci **A** and **B** under selection, the migration rate m at parapatric stages, and the
483 selection coefficients a and b are identical in the two scenarios.

484 Under explanation 3.A, we define the last evolutionary step as the rise of a peak
485 of divergence at locus **A** by establishment of a weak, locally beneficial mutation in one of
486 the two demes. Here, we condition on there being an established migration–selection
487 polymorphism at the linked background locus **B**, and on the new mutation being
488 successful. We start counting time when the locally beneficial mutation occurs. As above,
489 we focus on the case where linkage to **B** is essential for establishment of the new
490 mutation. In contrast, under explanation 3.D, we view the erosion of neutral divergence
491 as the last evolutionary step during secondary contact following adaptive divergence at
492 loci **A** and **B** in allopatry (assuming the period of allopatry has been long enough to allow
493 local adaptation, which is not included in the following estimates). Here, we condition on
494 locally adaptive mutations having been fixed before secondary contact, and we start
495 counting time at the onset of secondary contact.

496 In both cases, we ignore mutation at loci **A** and **B**, and we assume that neutral
497 mutation rates are low, so that the infinite-alleles model of mutation holds and F_{ST} can be
498 approximated as

499

500
$$F_{ST} \approx \frac{T_T - T_W}{T_T} \quad (10)$$

501

502 (Slatkin 1991, Wilkinson-Herbots 1998) Here, T_T and T_W are the expected coalescence
503 times between two samples taken at random from the entire population and from within a
504 deme, respectively. These times depend on the local effective population size N_e and the
505 migration rate m in a neutral model. We derive these coalescence times in the SI using
506 classical results for the structured coalescent (Griffiths 1981; Takahata 1988; Notahara
507 1990; Wilkinson-Herbots 1998, 2008, 2012). To account for the effect of selection, we
508 replace m by the appropriate effective rate of gene flow m_e (Petry 1983; Aeschbacher &
509 Bürger 2014; Akerman & Bürger 2014).

510 Under these assumptions, erosion of neutral divergence during secondary contact
511 after adaptive divergence occurs on a much faster time scale than the rise of a new peak
512 in F_{ST} under *de novo* selection with gene flow (Figure 5). With $a = 0.01$, $b = 0.1$, and $m =$
513 0.02 , a potentially identifiable two-peak island is formed already after about $t = 100$
514 generations in the secondary-contact scenario (pale-blue curves in Figure 5A), whereas
515 the peak arising at locus A is barely noticeable at this time; only after about $t = 1000$
516 generations is there a clear second peak (Figure 5B). Under purely deterministic
517 dynamics, i.e. assuming infinitely large local population sizes, the waiting time for the
518 decay of neutral divergence in terms of F_{ST} is well approximated by the inverse of the
519 effective rate of gene flow, $1/m_e$ (SI; Figure S4). Even with a finite local population size
520 N_e , $1/m_e$ seems to provide a good approximation to the time until F_{ST} reaches its
521 equilibrium in the secondary-contact case (Figure 5C). In contrast, it takes much more

522 time for F_{ST} at locus *A* to rise upon occurrence of a locally beneficial mutation in the
523 presence of gene flow (Figure 5D and Figure S5B and S5D).

524 The large differences in time scales over which equivalent genomic islands arise
525 and become potentially detectable suggest that explanation 3.D may be much more likely
526 than 3.A if the two populations or species of interest have diverged relatively recently.
527 Given that the above analysis assumes that a mutation destined to establish has already
528 occurred under 3.A, the total waiting time for mechanism 3.A would be even longer than
529 shown in Figure 5 (as per Figure 4). These differences remain even if adaptive divergence
530 before secondary contact has not been complete, as long as the allopatric phase has not
531 been too short (Figure S5A and S5C). We note that the mutations contributing to local
532 adaptation in allopatry (mechanism 3.D) are not expected to be more clustered than
533 random, unless the underlying mutational target is itself clustered (e.g., due to previous
534 evolution via mechanism 3C). Thus, mechanism 3.D also depends on multiple mutations
535 occurring in close enough proximity to each other that a contiguous island is observed, at
536 least for some time after secondary contact.

537

538 **Discussion**

539 Identification and interpretation of genomic islands is challenging for two interrelated
540 reasons: combinations of evolutionary scenarios can lead to similar patterns of genomic
541 diversity and divergence, and widely used measures of divergence differ in sensitivity
542 and specificity to detect and differentiate between alternative scenarios. The original
543 explanation for island evolution, by which they arise in genomic regions where on-going
544 gene flow is effectively reduced due to divergent selection (Turner et al. 2005), has been

545 challenged and refined to include the various scenarios presented in Table 1. Whereas
546 empirical study will serve to resolve some of these issues, here we have used theoretical
547 approaches to explore the importance of linkage for establishment of new mutations
548 during an initial phase of local adaptation.

549

550 *Evolution of genomic islands by increased establishment probability*

551 There are three main factors that affect the likelihood of genomic islands evolving by
552 increased EP of linked mutations (mechanism 3.A): the relative increase in the EP of
553 linked vs. unlinked mutations, their absolute EP, and the size of the mutational target
554 within the linked region that experiences increased EP. The relative increase in EP for
555 linked vs. unlinked mutations will be greatest when there is very strong selection on a
556 single locus (the **B** locus) accompanied by high migration and weaker selection on other
557 loci (represented by the **A** locus here). If the distribution of fitness effects of new
558 mutations is gamma distributed, linked mutations will have much higher EP relative to
559 unlinked ones when the migration rate exceeds the average selection coefficient
560 substantially ($m \gg \bar{z}$; Figure 3C). However, as the migration rate increases above \bar{z} , the
561 other factors lose importance: Both the absolute EP and the size of the region in which
562 linkage increases EP decrease rapidly (Figure 3A & D). Because the size of the
563 mutational target and the absolute EP both decrease, the waiting time for the
564 establishment of the first linked mutation increases in this region of parameter space. This
565 results in adaptation proceeding at a rate much slower than would occur in the absence of
566 migration (Figure 4). Thus, there is a ‘goldilocks zone’ where the balance between
567 migration and selection is just right for mechanism 3.A: migration is high enough that

568 unlinked mutations have low EP, but not so high that the waiting time for a linked
569 mutation to establish is very long. In other words, this zone is roughly between the
570 migration rate that maximizes the ratio of establishment probabilities in Figure 3C, and
571 the one that minimizes the time in Figure 4). In species with a continuous or stepping-
572 stone pattern of population structure spanning a broad environmental gradient, migration
573 rates may often be too geographically restricted to substantially constrain adaptation,
574 further limiting the potential importance of this mechanism.

575 In light of the sensitivity of this mechanism to the balance between migration and
576 selection, both demography and environment would have to be quite stable over
577 relatively long periods of time for evolution of genomic islands exclusively by increased
578 EP of linked mutations. In such cases, genomic islands would evolve most rapidly in
579 organisms with large population size, as this increases the total number of mutations
580 available for selection and therefore reduces the waiting time. Also, as the average
581 number of crossovers per chromosomes is found to be relatively insensitive to physical
582 chromosome length (Hillers and Villeneuve 2003), we would expect that organisms with
583 larger genomes have higher mutation rates per centimorgan, and that mechanism 3.A
584 might therefore be more important in such organisms. .

585 Two assumptions are implicit in the above discussion: the underlying mutational
586 target for a given polygenic trait is uniformly distributed throughout the genome and the
587 rate of recombination is homogeneous. If there are recombination coldspots around the **B**
588 locus due to segregating inversions or the previous fixation of recombination modifiers,
589 then increased EP due to linkage would extend over a greater physical distance and
590 therefore mutational target. Similarly, if functionally related loci tend to cluster together

591 in the genome (Nützmann & Osbourn 2014), then the rate of decay in the EP per base
592 pair would be unchanged, but the mutational target within this region would be increased.
593 Both of these non-homogeneous features of the genome can co-occur with the loci
594 involved in local adaptation by chance or as a result of previous bouts of local adaptation
595 that favoured the establishment or fixation of such modifiers or rearrangements
596 (mechanism 3.C). In either case, this could considerably increase the potential for the
597 evolution of genomic islands via increased probability of establishment of linked
598 mutations, as suggested by Yeaman (2013).

599 It is worth noting that Figure 4 provides a very rough comparison of the situation
600 for linked vs. unlinked mutations, as it uses the somewhat arbitrary C_{95} to determine the
601 mutational target for linked mutations. This also assumes that the physical distance scales
602 linearly with the rate of recombination, which does not account for the reduction in
603 effective recombination rate when there are even numbers of crossovers between a pair of
604 loci in a given meiosis. Also, by simply contrasting the expected establishment times and
605 ignoring the variances, we do not provide a rigorous assessment of what would constitute
606 a statistically significant signature of an island (even a few clustered loci with many non-
607 clustered ones might be detectable as an island). Finally, our approach ignores the fact
608 that there might be multiple background loci that could independently initialise genomic
609 islands. The latter effect will depend on the DFE, and appropriate treatment of this would
610 require modelling b as being drawn from that DFE, too. Comprehensively addressing
611 these issues is beyond the scope of this paper, but provides an obvious problem to address
612 in the future, and our conclusions throughout should be considered with these caveats in
613 mind.

614

615 *Evolution from new mutations vs. erosion and secondary contact*

616 Whereas mechanism 3.A depends explicitly on the mutation rate, the evolution of islands
617 via erosion following secondary contact (mechanism 3.D) is independent of mutation rate
618 and therefore can proceed much more rapidly (assuming that populations in allopatry
619 have had sufficient time to accumulate the differences that are eroded in secondary
620 contact). Here, we show that island formation via secondary contact and erosion of
621 divergence (mechanism 3.D) will be much more rapid than via establishment of new
622 mutations, even once a mutation that is destined to establish has already occurred (Figure
623 5). We also show that the expected time to island formation via secondary contact and
624 erosion scales approximately with the inverse of the effective migration rate ($1/m_e$; Figure
625 S4) for a wide range of population sizes. While we have not considered adaptation from
626 standing variation here, we suspect that the likelihood of island formation from standing
627 variation is similar to, if not lower than, that for new mutations for the same reasons as
628 we discussed above: in the parameter space where linkage is critical to establishment, the
629 absolute EP is low, so there would need to be many mutations present as standing
630 variation to give a signature. In addition, the fact that these mutations are segregating at
631 some intermediate, albeit potentially low, frequency implies that they have already
632 overcome the curse of stochastic loss. But this initial phase is exactly when linkage is
633 expected to convey the largest relative advantage. Mutations segregating as standing
634 variation may depend much less on this initial benefit. However, further work is
635 necessary to rigorously explore the contrast between standing and *de novo* variation in
636 this context, and special attention should be paid to the cause of standing variation, as

637 migration among populations inhabiting similar environments can introduce pre-adapted
638 variants.

639

640 *The absence of genomic islands*

641 While finding a strong signature of genomic islands can reveal much about the genomic
642 basis of adaptation, the lack of any observable peaks in F_{ST} does not necessarily mean
643 that no adaptation is occurring. While the two-locus models covered here predict that no
644 local adaptation will occur when $m > m^*$, divergent adaptation at the phenotypic level can
645 be maintained at equilibrium through a quantitative genetic response mediated by very
646 small changes in allele frequency coupled with positive covariance among alleles,
647 provided sufficient genetic variance is maintained by mutation (Le Corre & Kremer
648 2003; Yeaman 2015). In such cases, divergence at the underlying loci may be transient,
649 and no signature of islands is expected, nor any significant peaks in F_{ST} . Alternatively, if
650 there are multiple alleles or haplotypes present at a single locus that yield the same
651 locally adapted phenotype, the signature of F_{ST} would likely be greatly reduced,
652 especially at loci linked to the causal selected site(s). Studies of experimental evolution in
653 bacteria commonly find that many different amino acid residues within a given gene
654 evolve in response to the same selective pressure (e.g., Vogwill *et al.* 2014; Bailey *et al.*
655 2015). Similarly, a catalogue of the loci identified in adaptation has found that the same
656 locus often is involved repeatedly with different alleles (Martin & Orgogozo 2013). If
657 several such alleles at a single locus are maintained in a population, the expected
658 signatures of divergence would be weaker. Such patterns could likely evolve much more
659 readily with adaptation from standing variation. While we have restricted our study here

660 to two-deme, two-allele models, further study of more realistic population structures and
661 the possibility of haplotypes of equal fitness is necessary to more completely understand
662 their impact on statistical signatures of adaptation. Finally, it is worth noting that even in
663 cases where we would predict stable genomic islands to form based on our model, in
664 empirical studies these may not be statistically distinguishable from the genomic
665 background due to the highly stochastic nature of the genealogical process, mutation, and
666 recombination. This would be especially problematic in cases where migration between
667 populations is low and F_{ST} at neutral sites is high (Feder & Nosil 2010). In such cases,
668 however, there may still be a genome-wide aggregate signal in the form of a negative
669 correlation between divergence and local recombination rates (Nachman & Payseur 2012;
670 Brandvain *et al.* 2014).

671

672 *Comparison to results of Yeaman (2013)*

673 The analysis presented here uses an approach similar to that of Yeaman (2013),
674 combining approximations for the EP with representations of the size of linked vs.
675 unlinked mutation targets, but extends this analysis and corrects a deficiency in
676 presentation. Because Yeaman (2013) numerically integrated the EP only over the
677 recombination rate but not the DFE, the analysis was restricted to considering the relative
678 advantage of linkage for a single mutation. As such, when migration was high enough
679 that the EP of an unlinked mutation became 0, the ratio of π_L/π_U (referred to there as the
680 DHA, or ‘Divergence Hitchhiking Advantage’) became undefined. This was plotted as a
681 value of 0 in Yeaman’s Figure 1, which is unfortunate as it suggests that the relative
682 advantage of linkage is low in this region of parameter space, when in fact it is infinitely

683 high and mutations of that size could not establish without linkage. Thus, inferences
684 based on Figure 1 from Yeaman (2013) discounted the contribution of potentially the
685 most important region of parameter space (i.e. the shaded region shown here in Figure
686 2A). Here, we correct this oversight by integrating across both recombination and the
687 DFE of possible mutations at the A locus, so that $\bar{\pi}_L$ always includes the contribution of
688 some mutations with non-zero EP (i.e., some mutations have $m^* > m$), and the mean EP,
689 $\bar{\pi}_L$ therefore never becomes 0 (Figure 2B). With this approach, we show how in the
690 region where $\bar{\pi}_L/\bar{\pi}_U$ tends to infinity, the advantage of linkage is counterbalanced by
691 reduced size of mutation target and increased time to establishment. Thus, our analysis
692 leads to the same qualitative conclusions as Yeaman (2013): that mechanism 3.A is
693 unlikely to be a broad explanation for the evolution of genomic islands independently of
694 some other factors affecting the distribution of the mutational target or the local rate of
695 recombination. These conclusions are insensitive to whether we use the splicing approach
696 (as in Yeaman 2013) or the two-type branching process (Aeschbacher & Bürger 2014) to
697 approximate the establishment probability. Although there are quantitative differences
698 between these two approximations if migration is relatively strong or recombination
699 relatively weak (Figure 1), there is good agreement between them in terms of the ratio of
700 establishment probabilities, $\bar{\pi}_L/\bar{\pi}_U$, and the 95% window size, C_{95} (Figure S6).

701

702 *Conclusions and future directions*

703 Understanding the importance of the various mechanisms for the evolution of genomic
704 islands will inevitably require additional theory and more empirical work. Future
705 theoretical work could further explore the role of the DFE, study the effects of variable

706 selection strength at the background locus, and incorporate dominance, epistasis, and
707 other modes of selection (e.g. background selection). This would inform the development
708 of methods for robust inference about the mechanisms underlying genomic patterns of
709 diversity. In parallel, more comprehensive study of the extent of gene flow and
710 hybridization in natural populations is needed, as well as complementary lines of
711 evidence on gene function and phenotypic effect. This will inform the current debate
712 about the importance of gene flow in generating genomic islands (Cruickshank & Hahn
713 2014; Sætre 2014; Feulner *et al.* 2015; Monnahan *et al.* 2015) and help answer a number
714 of open questions: Is on-going gene flow commonly strong enough to make linkage
715 essential for divergence, or does gene flow only modulate evolving or previously existing
716 patterns? Are empirically observed islands produced by one or multiple sites under
717 divergent selection? How commonly do loci contribute to reproductive isolation
718 independent of adaptation? Do these islands reflect regions of reduced recombination as a
719 preadaptation to clustered architectures of traits under divergent selection, or did
720 clustered architectures and reduced local recombination rates evolve in response to such
721 selection? The distinction between preadaptation vs. response to selection may also be
722 seen as a question of timescale. For instance, in sticklebacks that repeatedly colonised
723 post-glacial lakes (Rogers *et al.* 2013), species may have encountered the same type of
724 environmental heterogeneity many times over deep evolutionary time. In such case, what
725 is now a pre-adaptation could be a consequence of previous response to selection for
726 reduced recombination between locally adapting loci. From the theoretical arguments
727 outlined here, it seems unlikely that increased establishment probability due to linkage
728 will provide a broad explanation for the evolution of genomic islands without some sort

729 of preadaptation, either in local recombination rate or the genomic distribution of
730 mutational targets. Comparative genomic studies may provide the best means to identify
731 how commonly such genome-scale changes have shaped the chromosomal landscape
732 upon which local adaptation and reproductive isolation are built.

733

734

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744

745 **References**

- 746 Aeschbacher S, Bürger R (2014) The effect of linkage on establishment and survival of
747 locally beneficial mutations. *Genetics*, **197**, 317–336.
- 748 Akerman A, Bürger R (2014) *The consequences of gene flow for local adaptation and*
749 *differentiation: a two-locus two-deme model*.
- 750 Bailey SF, Rodrigue N, Kassen R (2015) The Effect of Selection Environment on the
751 Probability of Parallel Evolution. *Molecular Biology and Evolution*, **32**, 1436–1448.
- 752 Bank C, Bürger R, Hermisson J (2012) The limits to parapatric speciation: Dobzhansky-
753 Muller incompatibilities in a continent-islands model. *Genetics*, **191**, 845–863.
- 754 Barton N, Bengtsson BO (1986) The barrier to genetic exchange between hybridising
755 populations. *Heredity*, **57 (Pt 3)**, 357–376.
- 756 Barton N, Hewitt GM (1989) Adaptation, speciation and hybrid zones. *Nature*, **341**, 497–
757 503.
- 758 Brandvain Y, Kenney AM, Fligel L, Coop G, Sweigart AL (2014) Speciation and
759 Introgression between *Mimulus nasutus* and *Mimulus guttatus*. *PLoS Genetics*, **10**,
760 e1004410.
- 761 Bürger R, Akerman A (2011) The effects of linkage and gene flow on local adaptation: A
762 two-locus continent-island model. *Theoretical Population Biology*, **80**, 272–288.
- 763 Charlesworth B (1979) Selection on recombination in. , 581–589.
- 764 Charlesworth B (1998) Measures of divergence between populations and the effect of
765 forces that reduce variability. *Molecular biology and evolution*, **15**, 538–543.

- 766 Charlesworth B, Morgan MT, Charlesworth D (1993) The effect of deleterious mutations
767 on neutral molecular variation. *Genetics*, **134**, 1289–1303.
- 768 Charlesworth B, Nordborg M, Charlesworth D (1997) The effects of local selection,
769 balanced polymorphism and background selection on equilibrium patterns of genetic
770 diversity in subdivided populations. *Genetical research*, **70**, 155–174.
- 771 Le Corre V, Kremer A (2003) Genetic variability at neutral markers, quantitative trait loci
772 and trait in a subdivided population under selection. *Genetics*, **164**, 1205–1219.
- 773 Cruickshank TE, Hahn MW (2014) Reanalysis suggests that genomic islands of
774 speciation are due to reduced diversity, not reduced gene flow. *Molecular Ecology*,
775 **23**, 3133–3157.
- 776 Excoffier L, Hofer T, Foll M (2009) Detecting loci under selection in a hierarchically
777 structured population. *Heredity*, **103**, 285–298.
- 778 Feder JL, Gejji R, Yeaman S, Nosil P (2012) Establishment of new mutations under
779 divergence and genome hitchhiking. *Philosophical Transactions of the Royal
780 Society B: Biological Sciences*, **367**, 461–474.
- 781 Feder JL, Nosil P (2010) The efficacy of divergence hitchhiking in generating genomic
782 islands during ecological speciation. *Evolution*, **64**, 1729–1748.
- 783 Feulner PGD, Chain FJJ, Panchal M *et al.* (2015) Genomics of divergence along a
784 continuum of parapatric population differentiation. *PLoS genetics*, **11**, e1004966.
- 785 Fisher R (1930) *The Genetical Theory of Natural Selection*. Clarendon, Oxford.
- 786 Gillespie JH (2000) The neutral theory in an infinite population. *Gene*, **261**, 11–18.
- 787 Griffiths RC (1981) The number of heterozygous loci between two randomly chosen
788 completely linked sequences of loci in two subdivided population models. *Journal
789 of Mathematical Biology*, **12**, 251–261.
- 790 Haldane JBS (1930) A mathematical theory of natural and artificial selection. VI.
791 Isolation. *Mathematical Proceedings of the Cambridge Philosophical Society*, **26**,
792 220–230.
- 793 Harr B (2006) Genomic islands of differentiation between house mouse subspecies. ,
794 730–737.
- 795 Hermisson J (2009) Who believes in whole-genome scans for selection? *Heredity*, **103**,
796 283–284.
- 797 Hillers KJ, Villeneuve AM (2003) Chromosome-wide control of meiotic crossing over in
798 *C. elegans*. *Current Biology*, **13**, 1641–1647.
- 799 Kirkpatrick M (2006) Chromosome Inversions, Local Adaptation and Speciation.
800 *Genetics*, **173**, 419–434.
- 801 Lenormand T, Otto SP (2000) The evolution of recombination in a heterogeneous
802 environment. *Genetics*, **156**, 423–438.
- 803 Lindtke D, Buerkle CA (2015) The genetic architecture of hybrid incompatibilities and

804 their effect on barriers to introgression in secondary contact. *Evolution*, **69**, 1987-
805 2004.

806 López E, Pradillo M, Oliver C *et al.* (2012) Looking for natural variation in chiasma
807 frequency in *Arabidopsis thaliana*. *Journal of Experimental Botany*, **63**, 887–894.

808 Martin A, Orgogozo V (2013) The loci of repeated evolution: a catalog of genetic
809 hotspots of phenotypic adaptation. *Evolution*, **67**, 1235-1250.

810 Maynard Smith J, Haigh J (1974) The hitch-hiking effect of a favourable gene. *Genetical
811 Research Cambridge*, **23**, 23–35.

812 Monnahan PJ, Colicchio J, Kelly JK (2015) A genomic selection component analysis
813 characterizes migration-selection balance. *Evolution*, n/a–n/a.

814 Nei M (1973) Analysis of gene diversity in subdivided populations. *Proceedings of the
815 National Academy of Sciences of the United States of America*, **70**, 3321–3323.

816 Nei M, Li WH (1979) Mathematical model for studying genetic variation in terms of
817 restriction endonucleases. *Proceedings of the National Academy of Sciences of the
818 United States of America*, **76**, 5269–5273.

819 Nachman, MW. and Payseur, BA. (2012) Recombination rate variation and speciation:
820 theoretical predictions and empirical results from rabbits and mice. *Phil. Trans. R. Soc. B*
821 **367**:409–421.

822

823 Noor MAF, Grams KL, Bertucci LA, Reiland J (2001) Chromosomal inversions and the
824 reproductive isolation of species. , **2001**.

825 Nordborg M (1997) Structured coalescent processes on different time scales. *Genetics*,
826 **146**, 1501–1514.

827 Nosil P, Funk DJ, Ortiz-Barrientos D (2009) Divergent selection and heterogeneous
828 genomic divergence. *Molecular Ecology*, **18**, 375–402.

829 Notahara M (1990) The coalescent and the genealogical process in geographically
830 structured population. *Journal of Mathematical Biology*, **29**, 59–75.

831 Nützmann H-W, Osbourn A (2014) Gene clustering in plant specialized metabolism.
832 *Current Opinion in Biotechnology*, **26**, 91–99.

833 Petry D (1983) The Effect on Neutral Gene Flow of Selection at a Linked Locus. , **313**,
834 300–313.

835 Renaut S, Grassa CJ, Yeaman S *et al.* (2013) Genomic islands of divergence are not
836 affected by geography of speciation in sunflowers. *Nature Communications*, **4**,
837 1827.

838 Rieseberg LH (2001) Chromosomal rearrangements and speciation. *Trends in Ecology
839 and Evolution*, **16**, 351–358.

840 Rogers SM, Bowles E, Mee JA The consequences of genomic architecture on ecological
841 speciation in postglacial fishes. , 1–25.

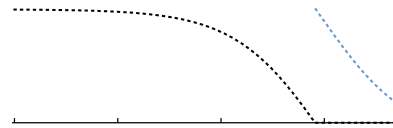
- 842 Sætre G-P (2014) Genome scans and elusive candidate genes: detecting the variation that
843 matters for speciation. *Molecular ecology*, **23**, 4677–8.
- 844 Slatkin M (1991) Inbreeding coefficients and coalescence times. *Genetical Research*
845 *Cambridge*, **58**, 167–175.
- 846 Slatkin M, Excoffier L (2012) Serial founder effects during range expansion: A spatial
847 analog of genetic drift. *Genetics*, **191**, 171–181.
- 848 Strasburg JL, Sherman NA, Wright KM *et al.* (2012) What can patterns of differentiation
849 across plant genomes tell us about adaptation and speciation ? What can patterns of
850 differentiation across plant genomes tell us about adaptation and speciation ?
- 851 Takahata N (1988) The coalescent in two partially isolated diffusion populations.
852 *Genetical Research*, **52**, 213–222.
- 853 Trans P, Lond RS, Barton NH (2000) Genetic hitchhiking Genetic hitchhiking. , 1553–
854 1562.
- 855 Turner TL, Hahn MW, Nuzhdin S V (2005) Genomic Islands of Speciation in *Anopheles*
856 *gambiae*. , **3**.
- 857 Via S, West J (2008) The genetic mosaic suggests a new role for hitchhiking in
858 ecological speciation. *Molecular Ecology*, **17**, 4334–4345.
- 859 Vogwill T, Kojadinovic M, Furió V, MacLean RC (2014) Testing the Role of Genetic
860 Background in Parallel Evolution Using the Comparative Experimental Evolution of
861 Antibiotic Resistance. *Molecular biology and evolution*, **31**, 3314–3323.
- 862 Wilkinson-Herbots H (1998) Genealogy and subpopulation differentiation under various
863 models of population structure. *Journal of Mathematical Biology*, **37**, 535–585.
- 864 Wilkinson-Herbots HM (2008) The distribution of the coalescence time and the number
865 of pairwise nucleotide differences in the “isolation with migration” model.
866 *Theoretical Population Biology*, **73**, 277–288.
- 867 Wilkinson-Herbots HM (2012) The distribution of the coalescence time and the number
868 of pairwise nucleotide differences in a model of population divergence or speciation
869 with an initial period of gene flow. *Theoretical Population Biology*, **82**, 92–108.
- 870 Wright S (1931) Evolution in Mendelian populations. *Genetics*, **16**, 97–159.
- 871 Wright S (1943) Isolation by distance. *Genetics*, **28**.
- 872 Yeaman S (2013) Genomic rearrangements and the evolution of clusters of locally
873 adaptive loci. *Proceedings of the National Academy of Sciences of the United States*
874 *of America*, **110**, E1743–51.
- 875 Yeaman S (2015) (in press) Local adaptation by small-effect alleles. *Am Nat*, **186**.
- 876 Yeaman S, Otto SP (2011) Establishment and maintenance of adaptive genetic
877 divergence under migration, selection, and drift. *Evolution*, **65**, 2123–2129.
- 878 Yeaman S, Whitlock MC (2011) The Genetic Architecture of Adaptation under
879 Migration-Selection Balance. *Evolution*, **65**, 1897–1911.

880 **Figures**

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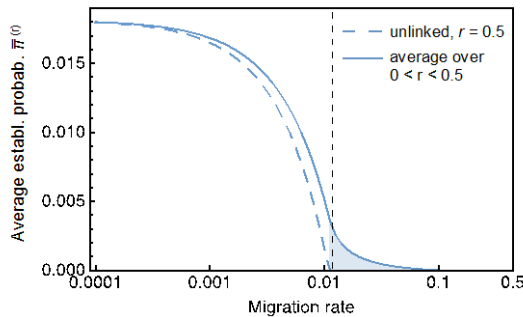
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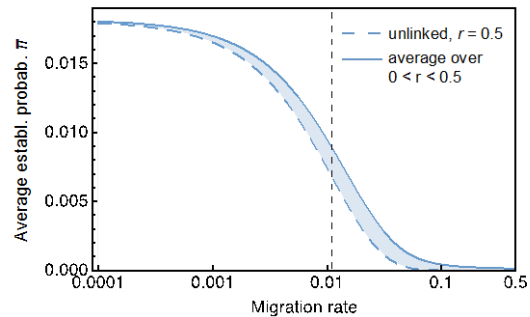
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Figure 1. Comparison of three approximations to the establishment probability of a new linked mutation. The establishment probability based on the two-type branching process (solid lines), a slightly-supercritical two-type branching process assuming $a, m \ll b, r$ (dashed lines), and the splicing approach (dotted lines) as a function of migration rate (A) and recombination rate (B). A) $r = 0.01$; B) $m = 0.01$. In both cases, $b = 0.1$

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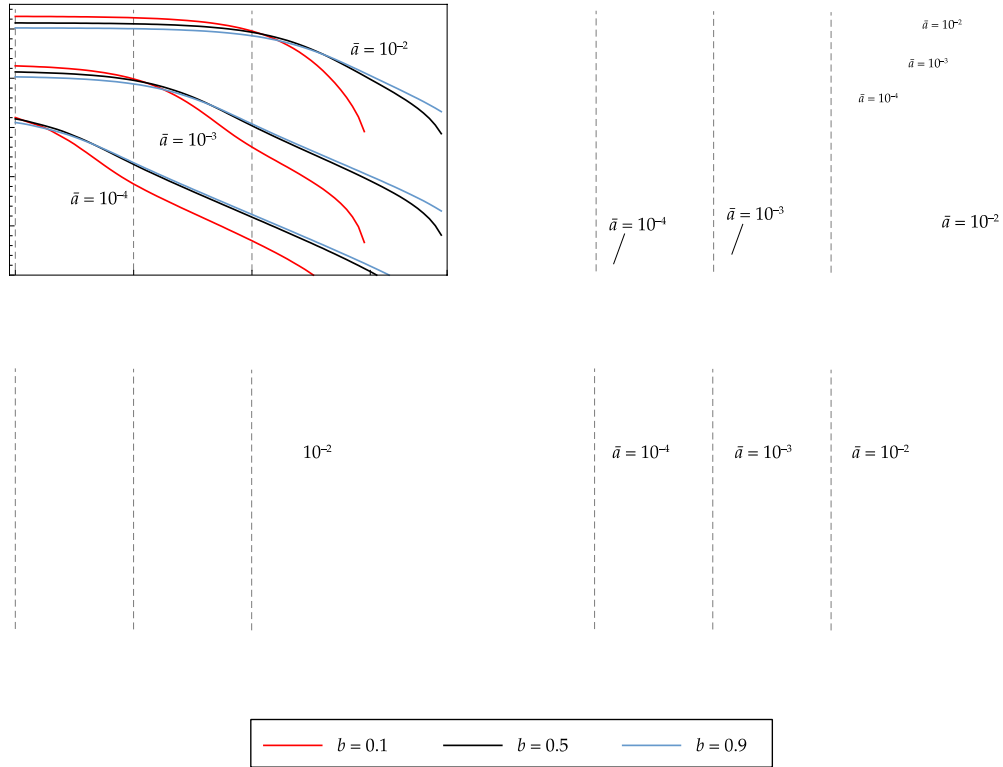
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Figure 2. The effect of linkage on the average establishment probability of locally beneficial mutations. A) Contrast between an unlinked ($\bar{\pi}_U^{(r)}$, dashed curve) and an average linked ($\bar{\pi}_L^{(r)}$, solid curve) mutation of effect $a = 0.01$. B) As in A), but after averaging over an exponential distribution of fitness effects (DFE) with mean $\bar{a} = 0.01$ (dashed for $\bar{\pi}_U$ vs. solid for $\bar{\pi}_L$). In both panels, the selection coefficient at the background locus is $b = 0.1$, and the vertical dashed line indicates the critical migration rate below which a single unlinked mutation of effect $a = 0.01$ can be established. Results are shown for the two-type branching process.

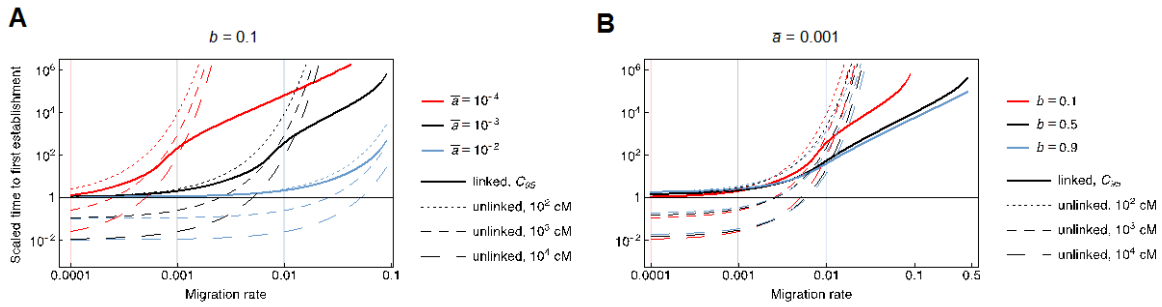


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906 **Figure 3.** The effect of the migration rate on the establishment probability and window
 907 size. The average establishment probability $\bar{\pi}_L$ as a function of the migration rate for
 908 various strengths of selection on the \log_{10} (A) and natural scale (B), and relative to the
 909 average establishment probability of an unlinked mutation, $\bar{\pi}_L/\bar{\pi}_U$ (C). D) The size of the
 910 window within which 95% of all successfully establishing linked *de novo* mutations
 911 occur (C_{95}). All panels show results for the two-type branching process with different
 912 values of \bar{a} and b , and $k = 1$ (exponential DFE).

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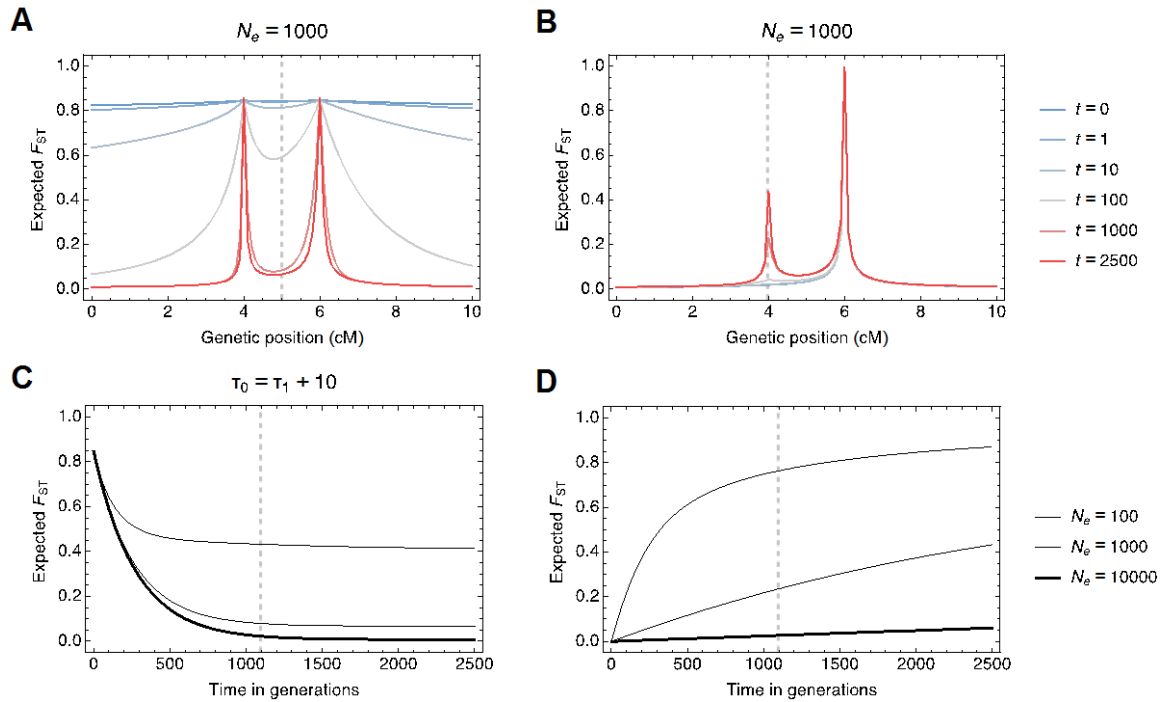
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918 **Figure 4.** Waiting times to the first establishing linked and unlinked locally beneficial
919 mutation. Expected waiting times are shown as a function of the migration rate for linked
920 (solid curves) and unlinked (dashed curves) mutations for various mean selection
921 coefficients \bar{a} at the focal allele (A) and different selection coefficients b at the
922 background locus (B). Times for unlinked mutations are shown assuming three different
923 values of the unlinked mutational target C_U (different degrees of dashing for 100, 1000,
924 and 10000 cM). Waiting times are scaled by the corresponding substitution rate in a
925 completely isolated panmictic population of the same size (see text for details). In both
926 panels, $k = 1$ (exponential DFE; see Figure S3 for other shapes), and predictions are
927 based on the two-type branching process. Light grey lines included to facilitate
928 comparison between parameter combinations.

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Figure 5. Dynamics of F_{ST} upon secondary contact and during the rise of a new selected allele in finite populations. A, B) Divergence between two demes of size $N_e = 1000$ in terms of F_{ST} at a neutral site as a function of its genetic position at various points in time. A) Formation of a two-peak island by erosion of neutral divergence in the neighbourhood of two selected loci A and B positioned at 4 and 6 cM. Before secondary contact, A and B have undergone adaptive divergence over $20 N_e$ generations in allopatry. B) Formation of a two-peak island through the rise of a locally beneficial *de novo* mutation at locus A in the face of gene flow and in the vicinity of a previously established migration–selection polymorphism at B. C, D) Dynamics of F_{ST} at a neutral site in the centre of the two-peak island (C) and at the position of the rising selected locus (D) for different effective population sizes N_e . Vertical lines indicate the inverse of the effective rate of gene flow, $1/m_e$, in the centre of the island, as a deterministic approximation to the waiting time for the erosion of neutral divergence. In all panels, F_{ST} is approximated as a ratio of coalescence times under the appropriate demographic model, with actual migration rates replaced by the deterministic approximation to the effective rate of gene flow, m_e , at the position of the neutral site (see SI for details). Parameters are $a = 0.01$, $b = 0.1$, and $m = 0.02$.