WORKING PAPERS

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2019

Working Paper No: 1902

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

Laboratory Federalism with Public Funds Sharing

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February 2019

Abstract

The theory of laboratory federalism hypothesizes that, in a decentralized multi-jurisdictional system, policies follow an evolutionary learning process with innovation and imitation. This paper studies the role of public funds sharing in such a setting. As a guiding framework we consider a model of decentralized, rich-to-poor redistribution with labor mobility. Uncorrected learning dynamics here lead to a drastic erosion of the welfare state. Suitably designed public funds sharing can correct this failure and may even restore efficiency. Surprisingly, the necessary properties of the sharing scheme for efficiency in the learning model are the same as those that make decentralized Nash play efficient (and vice versa). Public funds sharing, thus, is a powerful corrective device in fiscally decentralized settings for a variety of behavioral modes of government interaction.

Keywords: Evolutionary stability, Laboratory federalism, Labor mobility, Public funds sharing, Redistribution.

JEL Classification: C73, H75, H77.

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†Andreas Wagener has sadly passed away on 29 January, 2019 as we were preparing this manuscript for submission.

1 Introduction

In federal systems, horizontal public funds sharing, also known as fiscal equalization, does not only redistribute resources across jurisdictions but also shapes jurisdictions' incentives in policy decisions. On the negative, such incentive effects show up as moral hazard since relying on interjurisdictional transfers may crowd out the efforts of local governments to generate revenues out of their own tax potential (Quian and Roland, 1998). On the positive, fiscal equalization schemes can serve as Pigouvian mechanisms that help to correct interjurisdictional externalities and potentially restore efficiency (Boadway and Flatters, 1982). This property has found great attention and support both in the theoretical and the empirical literature on fiscal federalism and tax competition (surveyed, e.g., in Boadway, 2004).[1]

The present paper addresses the efficiency-enhancing potential of public funds sharing from a different angle. It offers an evolutionary perspective on fiscal competition, rather than the best-response, Nash equilibrium approach that predominates in the literature. Our approach is motivated by the concept of laboratory federalism which views fiscal descentralization as a Hayekian "discovery procedure" where individual jurisdictions in a decentralized federal system act as "laboratories" in the search for good policies. The proponents of laboratory federalism optimistically hypothesize that fiscally decentralized federal systems converge towards efficiency through a process of innovation and mutual learning (see, e.g., Hayek, 1978; Oates, 1999; 2008; North, 1981; Kollman et al., 2000; Vanberg and Kerber, 1994; Vihanto, 1992; Baybeck et al., 2011).

Here, we want to capture the idea of laboratory federalism explicitly in a model with fiscal descentralization where governments observe the performance of policies adopted elsewhere. Better-performing policies are adopted and tend to spread out at the expense

[1]Early contributions study how federal matching grants can address externalities (e.g., Wildasin, 1989; 1991; DePeter and Myers, 1994). Smart (1998) and Pfingsten and Wagener (1997) extend this idea to general public funds sharing schemes (adding an aggregate budget constraint). Subsequent theoretical studies such as Janeba and Peters (1999), Koethenbuerger (2002), Bucovetsky and Smart (2006), Hindriks et al. (2008), Kotsogiannis (2010), Becker and Kriebel (2017), Sas (2017), or Liesegang and Runkel (2018) discuss and, by and large, corroborate the efficiency-enhancing role of suitably designed public funds sharing; robust empirical evidence is given by Dahlby and Warren (2003), Buettner (2006), Smart (2007), or Egger et al. (2010). Insurance (or stabilization) effects provide an additional rationale for funds sharing; see, e.g., Konrad and Seitz (2003) or Boadway and Hayashi (2004).

of less successful ones. Policy decisions will therefore be driven by *relative* performance comparisons across jurisdictions. This distinguishes the evolutionary perspective from standard Nash play, which relies on absolute payoff maximization.

Game theoretic results on learning and imitation dynamics (Fudenberg and Imhof, 2006; Alós-Ferrer and Schlag, 2009) show that an iterated process of imitation and innovation converges to a so-called evolutionarily stable strategy (ESS) of the underlying stage game. In our context, a policy is called an ESS if, once it has spread out to the entire federation, it cannot be outperformed by any other policy. Unfortunately, the efficiency properties of an ESS are often less positive than hoped for. In fact, standard models of fiscal competition belong to the large class of games where ESS leads to "perfectly competitive" outcomes (Alós-Ferrer and Ania, 2005) in the sense that players behave as if they had no impact on aggregate variables that affect all players simultaneously.[2] For fiscal federalism, this means that governments ignore all effects of their policies on economic variables that are common to the entire federation, such as price levels, equilibrium rates of return, or federation-wide public goods. The rationale is that for relative payoff comparisons among jurisdictions all aggregate effects are irrelevant – precisely because they affect all jurisdictions in the same way.

In many cases, such "aggregate-taking behavior" leads to extremely sharp races to the bottom or over the top, amplifying the inefficiencies already prevailing in best-response play. Consequently, the need for a corrective device in a fiscal federation appears even more urgent from an evolutionary point of view than under Nash play. What then is the role of public funds sharing mechanisms in an evolutionary context? Specifically, given that the funds sharing schemes discussed in the literature are tailored to steer Nash equilibria towards efficiency, what will be their effect if jurisdictions are instead guided by relative performance concerns?

Our answer to these questions is at first glance surprising and, by and large, optimistic. First, public funds sharing mechanisms, as prominently discussed in standard settings of fiscal competition, can also support efficiency under evolutionary pressures. Second, the

[2]Observations of highly inefficient ESS have been made in the literature for standard models of capital tax competition (Sano, 2012; Wagener, 2013; Philipowski, 2015), public infrastructure (Wagener, 2013), or decentralized redistribution (Ania and Wagener, 2016).

characteristics that these mechanisms need to possess to restore efficiency coincide with those for efficient Nash play. Hence, public funds sharing can be a powerful corrective device in decentralized fiscal interaction for different types of government interaction.

The present paper considers an evolutionary version of the classical model of decentralized redistribution from rich to poor with free labor mobility due to Wildasin (1991). Local redistributive policies exert positive interjurisdictional externalities. Consequently, Nash equilibria of decentralized redistribution involve an under-provision of redistribution—the standard decline of the welfare state in the presence of labor mobility. Evolutionary processes based on relative performance fare dramatically worse: redistributive policies break down completely (Ania and Wagener, 2016). The intuition is simple: whenever a jurisdiction lowers the subsidy to the poor, this makes the rich of the jurisdiction better off while worsening the situation of the poor in the entire federation. The latter, aggregate effect cancels out in relative payoff comparisons. What matters is only the positive income effect for the rich. Hence, cutting back transfers to the poor improves a jurisdiction's relative position, triggering an unstoppable erosion of rich-to-poor redistribution.

Now add a public funds sharing scheme that implements transfers across jurisdictions to this setting. Such a scheme can be designed so as to restore efficiency under Nash play, by internalizing externalities of local redistributive policies. Specifically, the sharing scheme needs to compensate jurisdictions for the external costs and benefits that a unilateral change in redistributive policies causes in other jurisdictions (for a formal statement see Proposition 1). Analogously, a public funds sharing scheme under evolutionary play needs to compensate for the *relative* disadvantage of subsidizing the poor. More generous jurisdictions worsen their relative position by reducing the net income of their rich; the federation-wide increase in the consumption of the poor favors all jurisdictions equally, but is irrelevant for relative performance. Thus, to support a positive level of redistribution from rich to poor public funds sharing needs to compensate jurisdictions for the widening gap in the incomes of their rich (see Proposition 2).

Given that the inefficiency is larger and of a different nature in evolutionary instead of Nash play, it would be plausible if public funds sharing required stronger and different mechanisms to restore efficiency in each of the two approaches. Surprisingly, this is not the case: the necessary properties are the same for both settings. The intuition is as follows. Starting in a symmetric Nash equilibrium, consider a unilateral increase in one jurisdiction's subsidy. To the detriment of its rich, the more generous jurisdiction receives a migration inflow and incurs on extra spending on redistribution while the non-deviating jurisdictions will start saving on the provision of social policy—to the benefit of their rich (positive externalities). An efficient funds sharing scheme exactly offsets this income gap that opens between the rich in the deviating and the non-deviating jurisdictions. However, this gap exactly coincides with the gap that an efficient scheme would also have to close under relative performance concerns. Hence, to restore efficiency, a public funds sharing scheme must satisfy the same properties both under Nash and under evolutionary play.

This observation implies that a public funds sharing mechanism that results in the same policy choices when jurisdictions maximize their absolute payoffs and when they maximize their relative payoffs must be taking account of all externalities. Consequently, if a policy in a (corrected) Nash equilibrium is also a (corrected) ESS, then it will indeed be efficient (Proposition 3). This is not to say that Nash and evolutionary play yield the same prediction for any given funds sharing scheme. It means that if the scheme is tailored to make the Nash equilibrium evolutionarily stable, then the equilibrium is efficient.

A positive message from our results for the design of funds sharing is that there is no necessary contradiction in targeting for efficiency under payoff maximization or in the presence of relative performance concerns, which underlie yardstick competition, best practice adoption, imitative learning and other forms of fiscal interaction. In general, it may be possible to have a single fix for all of them. *Mutatis mutandis*, these observations apply to all symmetric frameworks of fiscal interaction with a single policy variable and cross-border externalities due to mobility, such as tax and expenditure competition *à la* Zodrow and Mieszkowski (1986), or decentralized redistribution if the rich players or capital owners (and not the beneficiaries) are mobile (cf. Section 7).

The following important caveat must be added. The results discussed so far refer only to the necessary marginal conditions. We show that a widely discussed and applied public funds sharing scheme, the Representative Tax System (RTS), indeed internalizes all interjurisdictional externalities and satisfies the necessary conditions for efficient policies to be sustained both under Nash and evolutionary play. This picture changes, however, once bigger-than-marginal deviations are considered. Under evolutionary pressures, an RTS may turn the redistribution game into an unstoppable race over the top, with ever higher subsidies (see Example 1 in Section 6). In general, an RTS will be associated with multiple evolutionarily stable solutions, which may or may not contain the efficient one (see Example 2). Sharper predictions can only be made with an explicit dynamic analysis—a topic left for future research.

The rest of the paper is organized as follows: In Section 2 we shortly review the model of decentralized income redistribution with labor mobility due to Wildasin (1991). In Section 3 public funds sharing is added to this basic model. Sections 4, 5, and 6 contain the analysis, respectively studying efficient outcomes, Nash, and evolutionary equilibria. There, we also explore the conditions for efficiency with different public funds sharing schemes. Section 7 concludes and discusses our findings. Proofs are relegated to the Appendix.

2 Decentralized redistribution with perfect mobility

We consider an evolutionary version of the classical model of decentralized rich-to-poor redistribution with free mobility of the poor due to Wildasin (1991).

Basic framework. In an economically integrated area there is a finite number $n \geq 2$ of identical jurisdictions. Each jurisdiction $j \in \{1, \dots, n\}$ is populated by a rich household, who owns the immobile fixed factors of production, and by a number ℓ_j of workers, who are perfectly mobile across jurisdictions. Their total number in the economy is exogenously fixed to $n \cdot \bar{\ell}$.

Each worker inelastically supplies one unit of labor wherever he resides. Production in each jurisdiction follows a Ricardian technology $f(\ell)$ with $f'(\ell) > 0 > f''(\ell)$ for all $\ell \geq 0$. The fixed factors are already embodied in f. To ensure that every jurisdiction is always populated with some workers we assume that $f(0) = 0$ and $f'(0) \to \infty$. Workers in jurisdiction j are paid their marginal product $f'(\ell_j)$ plus a subsidy $s_j \in [0, s_{max}]$ such that their net income and consumption equal

$$c_j = f'(\ell_j) + s_j. \tag{1}$$

Here, s_{max} is an exogenous maximum subsidy level, assumed to be higher than the efficient level introduced below.

The rich in jurisdiction j consumes the residual income at this location plus the net payments, $T_j \in \mathbb{R}$, that j receives through an interjurisdictional public funds sharing scheme:

$$y_j = f(\ell_j) - [f'(\ell_j) + s_j] \cdot \ell_j + T_j. \tag{2}$$

Negative [positive] values of T_j mean that jurisdiction j is a net contributor [beneficiary] of public funds sharing. We will describe sharing mechanisms in more detail below.

Migration equilibrium. Workers are costlessly mobile and will settle wherever their consumption is highest. In a migration equilibrium, consumption levels will be equal across jurisdictions ($c_i = c_j = c$). Formally, given subsidies $\mathbf{s} = (s_1, \ldots, s_n)$, a migration equilibrium is a distribution $(\ell_1(\mathbf{s}), \ldots, \ell_n(\mathbf{s}))$ of workers across jurisdictions with full employment and consumption equalization:

$$\sum_{i=1}^{n} \ell_i(\mathbf{s}) = n \cdot \bar{\ell} \tag{3}$$

$$f'(\ell_i(\mathbf{s})) + s_i = f'(\ell_j(\mathbf{s})) + s_j =: c(\mathbf{s}) \quad \text{for all } i, j \in \{1, \ldots, n\}. \tag{4}$$

Obviously, if two jurisdictions pay the same subsidy they attract equally many poor. Moreover, since all jurisdictions use the same technology, ℓ_j only depends on s_j and the collection of subsidies chosen by the jurisdictions $s_{-j} = (s_1, \ldots, s_{j-1}, s_{j+1}, \ldots, s_n)$, independent of their order. I.e., the number of poor located in jurisdiction j can be expressed as $\ell_j(\mathbf{s}) = \ell(s_j, s_{-j})$, where $\ell(s_j, \cdot)$ is invariant to permutations of the elements of s_{-j}.

Comparative statics. The response of $(\ell_1(\mathbf{s}), \ldots, \ell_n(\mathbf{s}))$ and $c(\mathbf{s})$ to changes in any of the subsidies s_j can be obtained by totally differentiating (3) and (4) with respect to s_j. Specifically, for $i, j = 1, \ldots, n$ and $i \neq j$, an increase in the subsidy in jurisdiction j leads to an inflow of workers into j, to an outflow from every $i \neq j$, and to an economy-wide increase in workers' consumption:[3]

$$
\begin{aligned}
\frac{\partial c}{\partial s_j} &= \frac{1/f''(\ell_j)}{\sum_{k=1}^{n} 1/f''(\ell_k)} > 0, \\
\frac{\partial \ell_j}{\partial s_j} &= -\frac{1}{f''(\ell_j)} \cdot \left(1 - \frac{1/f''(\ell_j)}{\sum_{j=k}^{n} 1/f''(\ell_k)}\right) > 0, \\
\frac{\partial \ell_i}{\partial s_j} &= \frac{1}{f''(\ell_i)} \cdot \frac{1/f''(\ell_j)}{\sum_{k=1}^{n} 1/f''(\ell_k)} < 0.
\end{aligned}
$$

Policy objectives. With respect to political preferences, we follow Wildasin (1991) in assuming that each jurisdiction cares for its social welfare that depends on consumption of the rich (y_j) and of the poor (c) and can be represented by the function

$$U_j = U(y_j, c) \tag{5}$$

with $y_j, c \geq 0$. Since consumption of the poor equalizes across jurisdictions in a migration equilibrium, we drop the subscript on c. We assume that u is strictly quasi-concave with strictly positive partial derivatives $u_y := \partial U/\partial y > 0$ and $u_c := \partial U/\partial c > 0$ everywhere.

3 Public funds sharing

General description. We now add public funds sharing to the economy just described. A funds sharing schemes redistributes resources across jurisdictions depending on their subsidies. At subsidies $\mathbf{s} = (s_j, s_{-j})$ the funds sharing scheme determines transfer payments, $\{T_j(\mathbf{s})\}_{j=1,\ldots,n}$, which can be positive or negative. We assume that the funds

[3] Later on we evaluate effects at symmetric strategy profiles, where $s_1 = \ldots = s_n$ and $\ell_j = \bar{\ell}$ for all j. Then the comparative statics simplifies to

$$\frac{\partial c}{\partial s_j} = \frac{1}{n}, \quad \frac{\partial \ell_j}{\partial s_j} = -\frac{n-1}{n} \frac{1}{f''\left(\bar{\ell}\right)}, \quad \text{and} \quad \frac{\partial \ell_i}{\partial s_j} = \frac{1}{nf''\left(\bar{\ell}\right)}.$$

sharing mechanism has basic anonymity and symmetry properties. In particular, it is represented by a single function

$$T_j(\mathbf{s}) = T(s_j, s_{-j})$$

that is independent of the names of the jurisdictions and robust to permutations of the subsidy values in other jurisdictions, s_{-j}. Focusing on purely horizontal funds sharing without flows of resources to or from other tiers of government in the federation, the sharing mechanisms runs a balanced budget: for all $\mathbf{s}$,

$$\sum_{j=1}^{n} T_j(\mathbf{s}) = 0. \tag{6}$$

Symmetry implies that any two jurisdictions with identical subsidies to the poor, $s_i = s_j = s$, face the same payment if s_{-i} is just a permutation of s_{-j}, i.e., if they face the same collection of subsidies in all other jurisdictions. Moreover, for jurisdictions with identical subsidies the transfer function rewards or punishes changes in the subsidy identically; i.e, if $s_i = s_j = s$ and s_{-i} and s_{-j} are permutations of one another, then

$$\frac{\partial T(s, s_{-i})}{\partial s_i} = \frac{\partial T(s, s_{-j})}{\partial s_j} \quad \text{and} \quad \frac{\partial T(s, s_{-i})}{\partial s_k} = \frac{\partial T(s, s_{-j})}{\partial s_k} \quad \text{for } k \neq i, j. \tag{7}$$

With a balanced budget symmetry also implies $T_j = 0$ for all j whenever $s_j = s$ for all j. Finally, the total change in the transfer payments across jurisdictions must equal zero following any change in any subsidy:

$$\sum_{j=1}^{n} \frac{\partial T_j(\mathbf{s})}{\partial s_k} = 0 \quad \text{for all } k. \tag{8}$$

These properties of T appear plausible given that jurisdictions are identical and no additional funds to support social policy are available in the federation. Our general formulation of public funds sharing is compatible with many different schemes. Two prominent examples, which will also help to illustrate our results, are the following:

Subsidy Equalization Schemes (SES). SES aim at equalizing the subsidy levels across jurisdictions by rewarding above-average subsidy levels and punishing below-average ones. Formally, an SES is represented by:

$$T_j(\mathbf{s}) = \alpha(s_j - \bar{s}(\mathbf{s})) \tag{9}$$

where $j = 1, \ldots, n$, $\bar{s}(\mathbf{s}) = \frac{1}{n}\sum_k s_k$ is the average subsidy level in the economy, and $\alpha > 0$ is a positive parameter that shapes the responsiveness of the funds sharing scheme with respect to deviations from the mean.

Representative Tax System (RTS). RTS is a widely studied and promising way of funds sharing under Nash play (Koethenbuerger, 2002; Bucovetsky and Smart, 2006; Kotsogiannis, 2010; Liesegang and Runkel, 2018; Sas, 2017).[4] Generally, an RTS aims at equalizing tax bases. In the present framework of income redistribution, it amounts to equalizing the volume of redistribution. Formally, an RTS can be represented by:

$$T_j(\mathbf{s}) = \bar{S}(\mathbf{s}) \cdot (\ell_j(\mathbf{s}) - \bar{\ell}) \tag{10}$$

for $j = 1, \ldots, n$, where

$$\bar{S}(\mathbf{s}) = \frac{\sum_k s_k \ell_k(\mathbf{s})}{\sum_k \ell_k(\mathbf{s})} = \frac{1}{n\bar{\ell}} \cdot \sum_{k=1}^{n} s_k \ell_k(\mathbf{s}) \tag{11}$$

is a weighted average of current subsidies with weights equal the number of beneficiaries in each location. This average $\bar{S}$ can be interpreted as a *representative* subsidy level, the one that would yield the same volume of redistribution when applied to *all* poor in the federation. The value of $\bar{S}$ is endogenous to the policies chosen. To illustrate, consider a symmetric situation with identical subsidies and the same number of poor everywhere. If one jurisdiction lowers its subsidy it would make a fiscal gain (lower s_j and lower ℓ_j). The RTS fully redistributes this gain to all other jurisdictions. Hence, the net effect of cutting back subsidies is zero, leaving no incentive for individual jurisdictions to do so.

[4]The idea of RTS and its equalization approach is part of the funds sharing schemes among Canadian provinces (Boadway, 2004) or German local municipalities (Buettner, 2006; Egger et al., 2010).

4 Efficient outcomes

An efficient allocation $\{(y_j^*, \ell_j^*, c^*)\}_{j=1,\ldots,n}$ distributes labor units and consumption levels across jurisdictions so as to maximize total welfare in the federal system. As shown in Appendix A.1, it is characterized by production efficiency and a federation-wide Samuelson condition for c (the equalized consumption level of the poor has the characteristics of a public good to the entire federation). Production efficiency requires that labor yields the same marginal product everywhere. Since all jurisdictions use the same technology, this holds when $\ell_j = \bar{\ell}$ for all j. A symmetric efficient allocation additionally assigns the same consumption levels (y^*, c^*) in all jurisdictions, satisfying $y^* = f(\bar{\ell}) - c^* \cdot \bar{\ell}$ by feasibility and the Samuelson condition

$$\frac{u_c(y^*, c^*)}{u_y(y^*, c^*)} = \bar{\ell}, \tag{12}$$

that equates the (symmetric) marginal rate of substitution between the public good c and the private good y to the federation-wide costs of providing one additional unit of c.
The symmetric efficient allocation does not involve any transfers between jurisdictions. It can be decentralized by having every jurisdiction set its subsidy to $s^* = c^* - f'(\bar{\ell})$. We henceforth call s^* the *efficient subsidy level* and denote the symmetric profile where all jurisdictions choose this subsidy level by $\mathbf{s}^*$.

5 Nash equilibrium with funds sharing

In a migration equilibrium with subsidies $\mathbf{s}$, the payoff to jurisdiction j is given by

$$U_j(\mathbf{s}) \quad = \quad U\left(f(\ell_j(\mathbf{s})) - [f'(\ell_j(\mathbf{s})) + s_j]\ell_j(\mathbf{s}) + T_j(\mathbf{s}), c(\mathbf{s})\right). \tag{13}$$

where $f'(\ell_j(\mathbf{s})) + s_j = c(\mathbf{s})$ for all j. Payoffs are symmetric since all jurisdictions have the same utility and production functions and migration flows, public funds sharing and social welfare of any jurisdiction are invariant to permutations of the subsidies chosen in other jurisdictions.

A vector of subsidies $\mathbf{s}^N = (s_1^N, \ldots, s_n^N)$ is a Nash equilibrium of the decentralized redistribution game if

$$U_j(s_j^N, s_{-j}^N) \geq U_j(s_j, s_{-j}^N)$$

for all $s_j \in S$ and all j. At a Nash equilibrium, each jurisdiction maximizes its own payoffs (including payments out of public funds sharing), taking the subsidies elsewhere as given. In Appendix A.2 we show that an interior symmetric Nash equilibrium satisfies the following condition for all j:

$$\frac{u_c(y^N, c^N)}{u_y(y^N, c^N)} = \bar{\ell} + \frac{1}{\partial c / \partial s_j} \left(s^N \cdot \frac{\partial \ell_j(\mathbf{s}^N)}{\partial s_j} - \frac{\partial T_j(\mathbf{s}^N)}{\partial s_j} \right). \tag{14}$$

In the absence of public funds sharing, when $T_j(\mathbf{s}) = \partial T_j(\mathbf{s})/\partial s_j = 0$ for all $\mathbf{s}$, condition (14) involves $u_c/u_y > \bar{\ell}$. Compared with the efficiency condition (12), this implies an inefficiently low equilibrium value of c^N. This is the classical underprovision result by Wildasin (1991).

In the presence of public funds sharing, conditions (14) and (12) coincide if and only if

$$\frac{\partial T_j(\mathbf{s}^*)}{\partial s_j} = s^* \cdot \frac{\partial \ell_j(\mathbf{s}^*)}{\partial s_j}. \tag{15}$$

The next proposition follows.

Proposition 1. *(i) Without public funds sharing, a symmetric Nash equilibrium,* $\mathbf{s}^N = (s^N, \ldots, s^N)$*, involves inefficiently low subsidies,* $s^N < s^*$*, resulting in an inefficiently low consumption level for the poor,* $c^N < c^*$*.*

(ii) A public funds sharing scheme that supports the efficient subsidy $\mathbf{s}^*$ *as a symmetric Nash equilibrium needs to satisfy condition (15) for all* j*.*

Condition (15) requires that equalization payments be aligned with the migration reactions to unilateral deviations. It asks each jurisdiction to pay an amount equal to the fiscal gain (i.e., saved payouts to the poor) from setting a subsidy below the efficient level. Appendix A.2 shows that a public funds sharing scheme satisfying (15) indeed internalizes all interjurisdictional externalities in the symmetric, efficient Nash equilibrium; i.e.,

$$\sum_{k \neq j} \frac{\partial U_k(\mathbf{s}^N)}{\partial s_j} = 0 \tag{16}$$

for $\mathbf{s}^N = \mathbf{s}^*$ and all j (see also Smart, 1998; Bucovetsky and Smart, 2006). In particular, this implies that

$$\frac{\partial (y_k - y_j)}{\partial s_j} = 0 \tag{17}$$

must hold for any two j, k with $j \neq k$ (cf. equation (A.7)). Hence, equalization payments for an efficient funds sharing scheme must exactly offset the income differential between jurisdictions that result from unilateral deviations from the efficient Nash equilibrium. This observation plays an important role later on when we compare Nash equilibrium and ESS.

We now apply Proposition 1 to the specific funds sharing schemes introduced in Section 3. Details are provided in Appendix A.4.

Corollary 1. *The SES defined by equation (9) supports* $\mathbf{s}^*$ *as a Nash equilibrium only if* $\alpha = -s^*/f''(\bar{\ell})$. *The RTS defined by equations (10) and (11) always satisfies the necessary conditions to support* $\mathbf{s}^*$ *as a Nash equilibrium.*

While an SES can only support efficiency for a suitably chosen parameter α, the second item of Corollary 1 confirms the efficiency-promoting features of an RTS as obtained, for example, by Koethenbuerger (2002), Bucovetsky and Smart (2006) and Kotsogiannis (2010) for tax competition models. The advantage of RTS over SES is that it does not require knowledge of the efficient subsidy or the production technology for its implementation. A RTS just lets s^* emerge as a decentralized equilibrium – but no other subsidy level. By contrast, by suitably choosing α, an SES can support any desired subsidy rate (and not just the efficient one) as a Nash equilibrium.

6 Evolutionary stability

Notation. Evolutionary analysis starts from symmetric situations where some strategy s has spread out to the entire population of players and then considers deviations of the form

$$\mathbf{s} = (r, \overset{m}{\ldots}, r, s, \overset{n-m}{\ldots}, s),$$

where a number m of players (also called mutants) simultaneously deviate to some other strategy r while the remaining $n-m$ players stick to the initial s. The analysis then compares the payoffs of a mutant and a non-mutant. Since the game we are discussing is symmetric, it is not necessary for the analysis to keep track of the order in which strategies are chosen. Moreover, only one alternative strategy is considered at a time. Thus, for notational convenience, we define the functions $\ell^m(r,s)$ and $\ell^{n-m}(r,s)$ as the number of workers located in the m jurisdictions choosing r and in the $n-m$ jurisdictions choosing s, respectively. We also use this kind of notation for the funds sharing payments, $T^m(r,s)$ and $T^{n-m}(r,s)$, as well as for the residual income levels of the rich, $y^m(r,s)$ and $y^{n-m}(r,s)$. Denote by

$$c(r,s) = f'\left(\ell^m(r,s)\right) + r = f'\left(\ell^{n-m}(r,s)\right) + s$$

the resulting consumption level of the poor. The payoff difference between deviators and non-deviators is then given by

$$\varphi(r,s) := U\left(y^m(r,s), c(r,s)\right) - U\left(y^{n-m}(r,s), c(r,s)\right). \tag{18}$$

Roughly speaking, a strategy s is m-stable if for any group of m players that simultaneously deviate to any other strategy $r \neq s$ the payoffs for these mutants falls short of the non-mutants' payoffs. If s resists deviations to any r for any number $m = 1, 2, \ldots, n-1$ of mutants, then s is called globally stable. Global stability is a very stark property, implying that no alternative strategy can destabilize the status quo, no matter how large the group of deviating players is.

Definition of ESS. In this paper, we only consider single deviations, restricting ourselves to $m = 1$. Robustness to single deviations is called *evolutionary stability.* In the context of our model, a subsidy s^E is called a finite-population evolutionarily stable strategy (ESS) if

$$\varphi(s, s^E) \leq 0 \text{ for all } s \in [0, s_{max}] \text{ and } m = 1. \tag{19}$$

In a situation where all jurisdictions set subsidy s^E, a unilateral deviation to some arbitrary s earns lower payoffs than s^E *after deviation* (Schaffer, 1988). Formally, this is equivalent to s^E maximizing the payoff difference, given that all other jurisdictions stick with s^E; i.e., $s^E = \arg\max_s \varphi(s, s^E)$ for $m = 1$.

With a finite number of players, the concepts of ESS and of Nash equilibrium are not related in general. On the one hand, it may be possible to improve payoffs when deviating from an ESS, but in that case the payoffs of non-deviators would improve even more. On the other hand, a strategy played in a symmetric Nash equilibrium is not necessarily an ESS. Deviating lowers the deviator's payoff by definition, but it may reduce the payoffs of the non-deviators even more, resulting in a relative advantage for the deviator.[5]

Characterization of ESS. Small unilateral deviations change relative payoffs at a symmetric profile as follows (see Appendix A.3):

$$\left.\frac{\partial \varphi(r, s)}{\partial r}\right|_{r=s} = u_y(y, c) \cdot \left(\frac{\partial y^m(s, s)}{\partial r} - \frac{\partial y^{n-m}(s, s)}{\partial r} \right). \tag{20}$$

Equation (20) makes the simple but crucial statement that relative payoffs change in the same direction as differences in the incomes of the rich: starting at a symmetric profile, a small unilateral change in the subsidy changes relative payoffs in favor of jurisdictions where the residual income of the rich is higher after deviation.

[5]The concept of finite-population ESS is in the same spirit but behaves differently from the classical concept for a continuum population (Maynard Smith and Price, 1973). The latter is actually a refinement of Nash equilibrium (see, e.g., Vega-Redondo, 1996, Chapter 2).

In Appendix A.3 we show that (20) is equal to

$$\left.\frac{\partial\varphi(r,s)}{\partial r}\right|_{r=s} \;=\; u_y(y,c)\cdot\frac{n}{n-1}\cdot\left(-s\frac{\partial\ell^m}{\partial r}+\frac{\partial T^m}{\partial r}\right). \tag{21}$$

To understand (21) consider a small subsidy increase; recall that this attracts more poor to the deviating jurisdiction. After deviation, the rich in the more generous jurisdiction have relatively higher expenses of redistribution policy by an amount $-s\cdot\frac{n}{n-1}\cdot\frac{\partial\ell^m}{\partial r}$) but they may also get relatively larger payments out of funds sharing amounting to $\frac{n}{n-1}\cdot\frac{\partial T^m}{\partial r}$.[6] The rich in the deviating jurisdiction end up with an income [dis-]advantage over those in non-deviating jurisdictions if the funds sharing scheme more [less] than fully compensates them for the increase in the costs of subsidizing the poor. The next proposition follows.

Proposition 2. *(i) In the absence of public funds sharing, the unique ESS in the decentralized redistribution game is* $s^E = 0$.

(ii) To sustain some positive subsidy level $s > 0$ *as an ESS, the public funds sharing mechanism needs to satisfy:*

$$\frac{\partial T_j(\mathbf{s})}{\partial s_j} = s\cdot\frac{\partial\ell_j(\mathbf{s})}{\partial s_j}, \tag{22}$$

where $\mathbf{s} = (s,\ldots,s)$.

Item (i) of Proposition 2 directly follows from the fact that, in the absence of public funds sharing (i.e., if $T_j(\mathbf{s}) = \partial T_j(\mathbf{s})/\partial s_j = 0$ for all $\mathbf{s}$), expression (21) is strictly negative for $s > 0$ (Ania and Wagener, 2016). Item (i) predicts a long-run breakdown of redistribution in the absence of public funds sharing. Jurisdictions that base their policy choices on relative instead of absolute performance adopt policies that lead to higher incomes for their rich (the common consumption level c of the poor does not affect jurisdictions'

[6] Observe that we are indeed measuring *relative* effects here. For example, starting from a symmetric situation, a marginal subsidy increase imposes on the rich of the m deviating jurisdictions additional costs of $-s\cdot\frac{\partial\ell^m}{\partial r}$, while it saves an amount of $s\cdot\frac{\partial\ell^{n-m}}{\partial r}$ for the rich in any non-mutant jurisdiction. Since, by symmetry and the constant overall population size, we have $(n-1)\cdot\frac{\partial\ell^{n-m}}{\partial r} = -\frac{\partial\ell^m}{\partial r}$, the resulting relative income gap is $-s\cdot\frac{n}{n-1}\cdot\frac{\partial\ell^m}{\partial r}$. A similar argument applies to payments from funds sharing; see (8).

relative positions). The income of the rich in j is, however, higher the lower the subsidies paid to the poor there, which brings in a tendency to cut back on s_j over time.

Item (ii) of Proposition 2 states a condition on a funds sharing scheme to sustain a positive level of redistribution; for this to happen, expression (21) must equal zero. Condition (22) requires that the funds sharing scheme compensates the rich for the relative disadvantage that they would suffer when their jurisdiction increased its subsidy to the poor; as explained, this relative disadvantage comes from the fact that the migration inflow opens a gap between the incomes of the rich in the mutant and the non-mutant jurisdictions.

ESS and Nash Equilibrium. Ideally, we would target $s > 0$ in Proposition 2 to be the efficient level, s^*. Comparing conditions (15) and (22), we observe that the necessary conditions to sustain s^* as a Nash equilibrium and as an ESS actually coincide: funds sharing schemes that restore efficiency have to satisfy the same marginal requirement, irrespective of whether jurisdictions care for absolute or relative payoffs. Obviously, the fact that first-order conditions coincide does not mean that the solutions always coincide and are efficient. In each case, one needs to check second-order conditions and there may be multiple solutions for any of the two equilibrium concepts. Still, our findings imply that whenever an ESS happens to be played in a symmetric Nash equilibrium, the corresponding subsidy must be s^* and *all* externalities must have been neutralized by the funds sharing scheme. The next proposition captures this result formally.

Proposition 3. *If, for the decentralized redistribution game with public funds sharing, subsidy s is both a Nash equilibrium strategy and an ESS, then $s = s^*$.*

At first sight, this result may seem surprising. Technically, it is an instance of a more general observation made by Hehenkamp et al. (2010, Corollary 2). In any symmetric game with differentiable payoffs and compact strategy sets, in order for a Nash equilibrium strategy, s^N, to be an ESS we need to have $\partial U_k(\mathbf{s}^N)/\partial s_j = 0$ for all $k \neq j$, since a Nash equilibrium satisfies $\partial U_j(\mathbf{s}^N)/\partial s_j = 0$ for all j. If s^N is also an ESS, it must then be efficient because it satisfies $\partial/\partial s_j \left(\sum_k U_k(\mathbf{s}^N)\right) = 0$. Since we craft public funds sharing as a steering device towards efficiency, the marginal properties of $T_j(\mathbf{s})$ at $\mathbf{s} = \mathbf{s}^*$ must coincide for evolutionary and for Nash play.

To understand the relationship between Nash equilibrium, ESS, and efficiency in our framework, recall from equation (17) that the sum of all fiscal externalities at a symmetric Nash equilibrium $\mathbf{s}^N = (s^N, \ldots, s^N)$ under an arbitrary public funds sharing scheme is given by $(n-1) \cdot u_y(y^N, c^N) \cdot \frac{\partial}{\partial s_j}\left(y_k(\mathbf{s}^N) - y_j(\mathbf{s}^N)\right)$, where k and j are any pair of jurisdictions (see also the derivation of (A.7) in Appendix A.2). As equation (20) reveals, a necessary condition for strategy s^N to be an ESS as well is $\frac{\partial}{\partial s_j}\left(y_k(\mathbf{s}^N) - y_j(\mathbf{s}^N)\right) = 0$, where j now corresponds to the deviating mutant jurisdiction (cf. expression (A.12)). Therefore, when s^N is both a Nash equilibrium strategy and an ESS, the sum of all fiscal externalities following any unilateral deviation is zero and it must be that $s^N = s^*$.

ESS under a Subsidy Equalization Scheme (SES). Appendix A.4 shows that, once the parameter α is fixed, there is a unique evolutionary equilibrium with an SES. Suitably chosen, α can support any desired subsidy level (including the efficient one) as an ESS.

Corollary 2. *With a SES (9), the unique candidate for an interior ESS is* $s^E = -\alpha f''(\bar{\ell})$.

ESS under a Representative Tax System (RTS). Somewhat surprisingly, an RTS as defined by (10) and (11) does not always support the efficient outcome as an ESS. This appears confusing since an RTS formally satisfies the necessary condition (22) for an efficient ESS at every symmetric profile $\mathbf{s} = (s, \ldots, s)$. In particular,

$$\frac{\partial T_j(\mathbf{s})}{\partial s_j} = (\ell_j(\mathbf{s}) - \bar{\ell}) \cdot \frac{\partial \bar{S}(\mathbf{s})}{\partial s_j} - \bar{S}(\mathbf{s}) \cdot \frac{\partial \ell_j(\mathbf{s})}{\partial s_j} = s \cdot \frac{\partial \ell_j(\mathbf{s})}{\partial s_j}, \tag{23}$$

since at a symmetric profile $\ell_j = \bar{\ell}$ for all j and $\overline{S} = s$. While this means *all* $s \in [0, s_{max}]$ (and in particular s^*) are ESS *candidates* with an RTS, closer inspection reveals that (23) is not always sufficient to characterize a global relative payoff maximum and, even when it is, it does not imply efficiency. The following examples illustrate these points.

Overshooting. Suppose that there are only two jurisdictions ($n = 2$). In Appendix A.5 we show that the payoff difference φ defined in (18) also has a zero second-order derivative at every symmetric profile. Hence, an interior ESS may not exist with an RTS. The next example provides a case in point where the maximization of relative payoffs leads to a permanent increase in the subsidy levels over time. In this sense, an RTS may be an over-powered mechanism with the potential to turn the much feared race to the bottom in rich-to-poor redistribution to an unstoppable race over the top.

Example 1. Consider the production function $f(\ell) = \ell(1-\ell)$. Assume that political preferences are of the Cobb-Douglas type, $U(y,c) = y \cdot c$, and an RTS is in place. For $n=2$, Appendix A.6 shows that $\varphi(r,s) = \Delta(r,s) \cdot c$ with

$$\Delta(r,s) = \frac{(r-s)^3}{8}.$$

We see that Δ is strictly increasing with r everywhere, except at $r = s$. Thus, at every symmetric profile where s is chosen, either jurisdiction can obtain a relative advantage by increasing its subsidy beyond s. This will only stop at the maximum admissible subsidy level, s_{max}, which is the ESS.

Multiple ESS. With more than two jurisdictions, the RTS guarantees that the payoff difference φ is locally concave at every symmetric profile (see Appendix A.5). Thus, *all* $s \in [0, s_{max}]$ (and also s^*) are robust to unilateral *local* deviations with an RTS. However, as the example below illustrates, sufficiently low subsidies can be discarded since they are not robust to larger deviations (still leaving a continuum of ESS). In general, it is unclear whether the discarded subsidy levels include s^*. Hence, not every time that the RTS has the potential to steer Nash play towards efficiency it will also do so under evolutionary play.[7]

Example 2. Consider the same setting as in Example 1 but suppose now that $n=3$. In Appendix A.6 we show that the symmetric efficient solution can be decentralized with $s^* = 0$. In this case, jurisdictions also implement efficient subsidy and consumption levels in a decentralized way at a Nash equilibrium and, by Proposition 2(i), as an ESS. If an RTS is in place, however, the value of equalization payments is given by

$$T_j(\mathbf{s}) = \frac{s_j - \bar{s}}{2}\left(\bar{s} + \frac{3}{2} \cdot \sigma_{\mathbf{s}}^2\right),$$

[7]This does not contradict Proposition 3, since the symmetric Nash equilibrium fails to be ESS here.

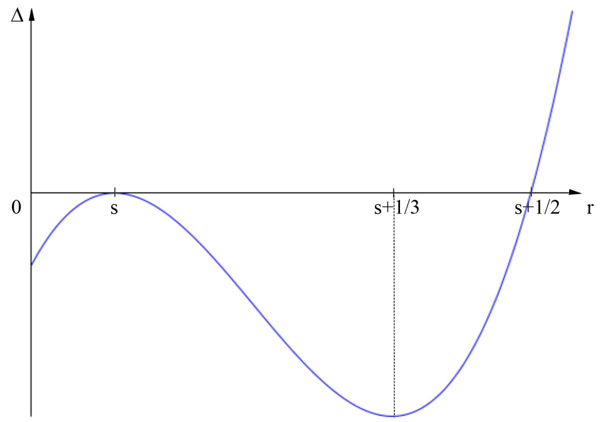

Figure 1: Single deviations from (s, s, s) to (r, s, s).

where $\bar{s}$ is the average and $\sigma^2_{\mathbf{s}}$ the variance of the subsidies chosen at $\mathbf{s}$. This expression makes clear that the volume of RTS transfers rises with the dispersion of the policies chosen. This results in a double rewarding of jurisdictions that considerably raise their subsidy above average. Relative payoffs following a unilateral deviation are given by $\varphi(r, s) = \Delta(r, s) \cdot c$ with

$$\Delta(r, s) = \frac{(r-s)^2\,[2(r-s)-1]}{12}.$$

Recall that $\Delta(r, s)$ is the relative advantage in residual income that any single jurisdiction can reap by unilaterally deviating from a symmetric (s, s, s) to some (r, s, s). The characteristics of Δ are summarized in Figure 1. Confirming (23), every symmetric profile with $r = s$ is a local maximum of Δ. Here the RTS will exactly offset any relative gain or loss of a *small* deviation from the symmetric profile, so that after equalization all jurisdictions are again equally well-off. This suggests that *every subsidy level is a potential candidate ESS*. Notice, however, that at $r = s + \frac{1}{2}$ we also have $\Delta = 0$. It is, thus, possible to have situations where one jurisdiction pays much higher subsidies and gets fully compensated through the RTS, again leaving all jurisdictions equally well-off. If this is feasible, meaning that the compensating jurisdictions can afford the corresponding transfer payments out of their now weakened economic position, the symmetric situation with $r = s$ is not a global maximum of Δ; as Figure 1 illustrates, any jurisdiction can attain a strict relative advantage by deviating to $r > s + 1/2$, whenever this is feasible. As a matter of fact, the

RTS overcompensates a jurisdiction that creates this kind of large disparities in subsidy levels. The analysis in Appendix A.6 shows, however, that this kind of deviations is only feasible for low values of the subsidy, with $s < \frac{1}{8}$; otherwise the non-deviating jurisdictions do not generate enough income to pay the corresponding transfers. It follows that the set of ESS is $S^{ESS} = \left[\frac{1}{8}, s_{max}\right]$. Notice the efficient subsidy level $s^* = 0$ does not belong to S^{ESS} in this example and all ESS are inefficient, indeed involving an over-provision of decentralized redistribution.

7 Discussion and conclusions

Public funds sharing schemes correctly designed to implement efficient Nash equilibria may also be used to steer behavior towards efficiency when policy choices are driven by relative performance comparisons. In a framework of rich-to-poor income redistribution with perfect mobility of the poor, we have argued that evolutionary play based on relative payoffs is a strong source of inefficiency, far more dramatic than Nash play. The fact that the same necessary conditions must hold for funds sharing to restore efficiency in both cases is quite surprising. It shows that public funds sharing can be a powerful corrective device with decentralized fiscal interaction and tax competition under a variety of assumptions on how government play the game.

Dynamics. Although we analyze a one-shot game with relative payoff concerns, the evolutionary stability of fiscal interactions is conceptually inspired by the dynamic idea of laboratory federalism; decentralization is suggested to be superior to policy centralization as it allows for policy experimentation and effective learning of successful policies in repeated interaction over time. Perturbed imitative learning processes can take different forms; yet as long as the most successful policies are followed with positive probability while unsuccessful policies with lower payoffs are discarded, being an ESS is a *necessary* requirement for a policy to be immune against rare experiments and to survive in the long run. However, as our RTS example shows, in general there may be multiple ESS and a further analysis with multiple deviations would be needed to explore their robustness.

Multiple deviations. Our focus on single deviations is for expositional convenience. Propositions 2 and 3 hold irrespective of the number of jurisdictions who simultaneously experiment with a new policy.

Efficiency. In our model of decentralized redistribution, the uncorrected ESS is bound for disaster – and public funds sharing can be a remedy. Other papers, however, show that evolutionary play and imitative learning do not always preclude efficient play. For oligopolistic price competition, Alós-Ferrer et al. (2000) show that evolutionary stability can serve as a selection criterion when there are multiple Nash equilibria. In a context more closely related to this paper, Ania and Wagener (2014) show that when jurisdictions in a federal system try to learn about good policies through imitation and innovation, it will be crucial for the long-run outcome whether the selected policies are sustainable by a simple majority of states in the federation. If the efficient policy happens to be strongly robust in this sense, it will survive in the long run; otherwise, it will be abandoned if enough jurisdictions simultaneously adhere to a more attractive policy experiment. It is an interesting topic for future research how public funds sharing affects such scenarios, both with respect to the set of policies that survives and their efficiency properties.

Knowledge. In contrast to best-response play, which requires players to know the exact mapping from strategies to payoffs, the imitative learning process underlying our analysis only requires observability of past policies and some summary statistic of success; exact knowledge of the economic environment is not needed. From the perspective of laboratory federalism, this makes evolutionary stability a suitable concept to study decentralized fiscal interaction whenever knowledge about the economy (say, about mobility patterns or tax bases) is too limited to allow for best-response behavior. Policy mimicking and occasional innovations turn out to be a viable, though boundedly rational way of policy-making. Against this backdrop, one might question the usefulness of Proposition 2, which argues that an efficient outcome can be implemented in a learning context. After all, it seems to presuppose knowledge of the efficient solution. Moreover, condition (22) uses the reaction of local labor supply to subsidy changes. This is not as severe as it sounds. First, the design and implementation of public funds sharing mechanisms does not in general require any structural knowledge of the economy or specific observabilities. Second, as the SES example shows, any public funds sharing scheme in that class helps to avoid the dismal uncorrected ESS with a complete breakdown of redistribution. Third, even if multiplicity may be an issue, an RTS may support efficiency in evolutionary play—and this is a mechanism that does not presuppose any knowledge of the economic structure. It therefore seems promising to study when exactly can efficiency be reached in evolutionary play without additional information built into the public funds sharing mechanism.

Alternative settings. The present paper focuses on decentralized, rich-to-poor redistribution with labor mobility. This framework is practically relevant (think of international migration), drastically clear in its theoretical predictions, and well-understood in terms of public funds sharing. In addition, the framework is generic in a large class of settings with decentralized fiscal interaction as long as the mobile factors respond to policy changes smoothly. This class encompasses tax competition of various brands, uncoordinated environmental policies, or infrastructural competition. As in the current setting, Nash equilibria are typically inefficient due to fiscal spillovers, public funds sharing may remedy this, and evolutionarily stable strategies lead to aggregate-taking behavior and competitive outcomes (see, e.g., Sano, 2012; Wagener, 2013). *Mutatis mutandis*, our analysis applies to all such scenarios: appropriately designed, public funds sharing schemes can help correct the efficiency failures of evolutionary play.

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Appendix

A.1 Symmetric efficient allocation

An efficient allocation $\{(y_j^*, \ell_j^*, c^*)\}_{j=1,\ldots,n}$ solves the following maximization problem:

$$\begin{array}{ll} \max_{\{(y_j,\ell_j,c)\}_j} & \sum_{j=1}^n \lambda_j u(y_j, c) \\ \text{s.t.} & \sum_{j=1}^n f(\ell_j) = \sum_{j=1}^n (y_j + c \cdot \ell_j) \\ & \sum_{j=1}^n \ell_j = n\bar{\ell}, \end{array} \tag{A.1}$$

where $(\lambda_1, \ldots, \lambda_n)$ is a vector of positive weights. Efficient allocations, thus, maximize total welfare subject to the feasibility constraint (A.1) and the fact that the total number of poor in the federation is constant and equal to $n\bar{\ell}$. Without consequences, the feasibility constraint could be augmented by a balanced funds sharing scheme (with $\sum_j T_j = 0$). The first-order conditions of this problem imply production efficiency with

$$f'(\ell_1) = \ldots = f'(\ell_n) \tag{A.2}$$

and a Samuelson condition of the form

$$\sum_{j=1}^n \frac{u_c(y_j, c)}{u_y(y_j, c)} = n\bar{\ell}. \tag{A.3}$$

Since $f'' < 0 < f'$, condition (A.2) is satisfied if and only if $\ell_1 = \ldots = \ell_n = \bar{\ell}$. Hence, at an efficient allocation output $f(\bar{\ell})$ must be the same in every jurisdiction. The incomes of the rich, y_j, could nevertheless vary across jurisdictions (if $T_j \neq 0$). Confining ourselves to symmetric allocations (where $T_j = 0$ for all j), feasibility requires $y_j = y^* = f(\bar{\ell}) - c^* \cdot \bar{\ell}$ for all j. From (A.3) it then follows that (y^*, c^*) must satisfy $\frac{u_c(y^*,c^*)}{u_y(y^*,c^*)} = \bar{\ell}$.

A.2 Nash equilibrium and efficient funds sharing

An interior Nash equilibrium in pure strategies solves $\frac{\partial U_j(\mathbf{s}^N)}{\partial s_j} = 0$ for all $j = 1, \ldots, n$. Differentiating the payoffs (13) of jurisdiction j with respect to s_j we obtain

$$\frac{\partial U_j}{\partial s_j} = u_y(y_j, c)\left(-s_j \frac{\partial \ell_j}{\partial s_j} - \ell_j \frac{\partial c}{\partial s_j} + \frac{\partial T_j}{\partial s_j}\right) + u_c(y_j, c) \cdot \frac{\partial c}{\partial s_j}. \tag{A.4}$$

If a pure-strategy symmetric equilibrium exists with $s_j^N = s^N$ for all j, it will be associated with $\ell_j = \bar{\ell}$, $T_j = 0$, $c^N = f'(\bar{\ell}) + s^N$ and equal values $y_j = y^N = f(\bar{\ell}) - c^N \cdot \bar{\ell}$ for all j. Rearranging (A.4) we see that a symmetric Nash equilibrium must satisfy

$$\frac{u_c(y^N, c^N)}{u_y(y^N, c^N)} = \bar{\ell} + \frac{1}{\partial c/\partial s_j}\left(s^N \cdot \frac{\partial \ell_j(\mathbf{s}^N)}{\partial s_j} - \frac{\partial T_j(\mathbf{s}^N)}{\partial s_j}\right). \tag{A.5}$$

Supporting an efficient equilibrium with $s^N = s^*$ therefore requires

$$\frac{\partial T_j(\mathbf{s}^*)}{\partial s_j} = s^* \cdot \frac{\partial \ell_j(\mathbf{s}^*)}{\partial s_j}. \tag{A.6}$$

To check that a funds sharing mechanism satisfying (A.6) internalizes all external effects in the efficient equilibrium, consider a unilateral deviation of any j, take derivatives in the payoffs of all other $k \neq j$ with respect to s_j using expression (13), and add up all these derivatives to obtain

$$\begin{aligned} \sum_{k\neq j} \frac{\partial U_k(\mathbf{s}^N)}{\partial s_j} &= \sum_{k\neq j} \left(u_y(y^N, c^N)\frac{\partial y_k}{\partial s_j} + u_c(y^N, c^N)\frac{\partial c}{\partial s_j}\right) \\ &= \sum_{k\neq j} u_y(y^N, c^N) \cdot \left(\frac{\partial y_k}{\partial s_j} - \frac{\partial y_j}{\partial s_j}\right) \\ &= (n-1) \cdot u_y(y^N, c^N) \cdot \frac{\partial (y_k - y_j)}{\partial s_j}. \end{aligned} \tag{A.7}$$

The second line uses the first-order conditions for jurisdiction j in equilibrium and the third line uses symmetry. Consider now any symmetric vector $\mathbf{s} = (s, \ldots, s)$ and let $c = f'(\ell) + s$ be the level of the poor's consumption resulting in the corresponding migration equilibrium. We have that for any $\mathbf{s}$

$$\begin{aligned}
\frac{\partial y_k(\mathbf{s})}{\partial s_j} - \frac{\partial y_j(\mathbf{s})}{\partial s_j} &= \left(f'(\bar{\ell}) - c\right) \cdot \left(\frac{\partial \ell_k}{\partial s_j} - \frac{\partial \ell_j}{\partial s_j}\right) + \frac{\partial T_k}{\partial s_j} - \frac{\partial T_j}{\partial s_j} \\
&= -\frac{n}{n-1}\left(-s \cdot \frac{\partial \ell_j}{\partial s_j} + \frac{\partial T_j}{\partial s_j}\right), \qquad \text{(A.8)}
\end{aligned}$$

where the second line in (A.8) follows from the fact that changes in migration flows as well as in transfers always add up to zero: the total number of workers in the federation is constant ($\sum_k \ell_k = n\bar{\ell}$) and the public funds sharing scheme is self-financing (see (8)). These two identities imply that, respectively, $\partial \ell_k / \partial s_j = -(\partial \ell_j / \partial s_j)/(n-1)$ and $\partial T_k / \partial s_j = -(\partial T_j / \partial s_j)/(n-1)$ for all $k \neq j$. We see that a public funds sharing scheme satisfying (A.6) guarantees that (A.8) and, thus, (A.7) equal zero at $s = s^*$, confirming that it internalizes all interjurisdictional externalities at the symmetric efficient equilibrium.

A.3 Evolutionary stability

We include here the details for the derivation of expressions (20) and (21) in the main text and obtain the first-order conditions for an interior ESS. At any vector of subsidies of the form $\mathbf{s} = (r, s, \ldots, s)$, after a unilateral deviation from s to r, relative payoffs to the deviator as defined by (18) are given by

$$\varphi(r,s) := U\left(y^m(r,s), c(r,s)\right) - U\left(y^{n-m}(r,s), c(r,s)\right), \qquad \text{(A.9)}$$

where

$$\begin{aligned}
y^m &= f(\ell^m) - c(r,s) \cdot \ell^m + T^m \quad \text{and} \qquad \text{(A.10)} \\
y^{n-m} &= f(\ell^{n-m}) - c(r,s) \cdot \ell^{n-m} + T^{n-m} \qquad \text{(A.11)}
\end{aligned}$$

respectively denote the income of the rich in the deviating and in the non-deviating jurisdictions, while $c(r,s) = f'(\ell^m) + r = f'(\ell^{n-m}) + s$ is the resulting consumption level of the poor in a migration equilibrium.

We say that s is an ESS if $\varphi(r,s) \leq 0$ for all r; i.e., if $r = s$ maximizes $\varphi(\cdot, s)$. Taking derivatives in (A.9) with respect to r, we obtain

$$\frac{\partial \varphi}{\partial r} = u_y(y^m, c) \cdot \frac{\partial y^m}{\partial r} - u_y(y^{n-m}, c) \cdot \frac{\partial y^{n-m}}{\partial r} + \frac{\partial c}{\partial r} \cdot \left[u_c(y^m, c) - u_c(y^{n-m}, c)\right].$$

For $r = s$ we have $\ell^m = \ell^{n-m} = \bar{\ell}$ and $T^m = T^{n-m} = 0$. Hence, also $y^m = y^{n-m} = y$ and (marginal) utilities are evaluated at the same value for deviating and the non-deviating jurisdictions. Analogously to (A.8), this implies

$$\begin{aligned} \left.\frac{\partial \varphi}{\partial r}\right|_{r=s} &= u_y(y,c) \cdot \left(\frac{\partial y^m}{\partial r} - \frac{\partial y^{n-m}}{\partial r}\right) \\ &= u_y(y,c) \cdot \left(\left(f'(\bar{\ell}) - c\right) \cdot \left(\frac{\partial \ell^m}{\partial r} - \frac{\partial \ell^{n-m}}{\partial r}\right) + \frac{\partial T^m}{\partial r} - \frac{\partial T^{n-m}}{\partial r}\right) \\ &= u_y(y,c) \cdot \frac{n}{n-1} \cdot \left(-s \cdot \frac{\partial \ell^m}{\partial r} + \frac{\partial T^m}{\partial r}\right). \end{aligned} \tag{A.12}$$

Here, the second line follows after taking derivatives in expressions (A.10) and (A.11) with respect to r and evaluating at $r = s$. The third line follows from (8) and the fact that $\sum_k \ell_k = n\bar{\ell}$.

It, thus, follows that maximization of $\varphi(r,s)$ with respect to r requires $\frac{\partial}{\partial r}(y^m - y^{n-m}) = 0$, which is satisfied for any subsidy $s > 0$ if only if

$$\frac{\partial T^m(s,s)}{\partial r} = s \cdot \frac{\partial \ell^m(s,s)}{\partial r}.$$

A.4 Proof of Corollaries 1 and 2

Proof of Corollary 1. For SES, defined as $T_j = \alpha \cdot (s_j - \bar{s})$, we have that

$$\frac{\partial T_j(\mathbf{s}^*)}{\partial s_j} = \alpha \cdot \left(1 - \frac{1}{n}\right) = s^* \cdot \frac{\partial \ell_j(\mathbf{s}^*)}{\partial s_j}$$

if and only if $\alpha = -\frac{s^*}{f''(\bar{\ell})}$, where we use that $\frac{\partial \ell_j(\mathbf{s})}{\partial s_j} = -\frac{n-1}{n \cdot f''(\bar{\ell})}$ at any symmetric vector $\mathbf{s}$ (cf. Footnote 3).

Instead, the RTS is defined by $T_j = \bar{S}\left(\ell_j - \bar{\ell}\right)$ with $\bar{S} = \frac{1}{n\bar{\ell}} \sum_k s_k \ell_k$. Thus,

$$\frac{\partial T_j(\mathbf{s}^*)}{\partial s_j} = (\ell_j(\mathbf{s}^*) - \bar{\ell}) \cdot \frac{\partial \bar{S}(\mathbf{s}^*)}{\partial s_j} + \bar{S}(\mathbf{s}^*) \cdot \frac{\partial \ell_j(\mathbf{s}^*)}{\partial s_j} = s^* \cdot \frac{\partial \ell_j(\mathbf{s}^*)}{\partial s_j}, \tag{A.13}$$

since at any symmetric $\mathbf{s} = (s, \ldots, s)$ we have $\ell_j = \bar{\ell}$ and $\bar{S} = s$. Condition (15) actually holds for all s and not just for s^*. □

Proof of Corollary 2. It follows from (A.12), the definition of SES, and Footnote 3 that an interior ESS must satisfy

$$\left.\frac{\partial\left(y^m - y^{n-m}\right)}{\partial r}\right|_{r=s} = \frac{n}{n-1}\left(-s \cdot \frac{\partial \ell^m}{\partial r} + \frac{\partial T^m}{\partial r}\right) = \frac{s}{f''(\bar{\ell})} + \alpha = 0.$$

For $s = 0$ this derivative equals $\alpha > 0$. The only value of the subsidy that solves the former equation and the only candidate for an ESS is thus $s = -\alpha \cdot f''(\bar{\ell})$, which can be targeted by an appropriate choice of α. This is indeed an ESS if

$$\left.\frac{\partial^2 \varphi}{\partial r^2}\right|_{r=s} = u_y(y, c) \cdot \left.\frac{\partial^2\left(y^m(r,s) - y^{n-m}(r,s)\right)}{\partial r^2}\right|_{r=s} < 0.$$

Using $\partial c(r,s)/\partial r = -1/n$ for $r = s$ (cf. Footnote 3) and $\ell^m + (n-1)\ell^{n-m} = n\bar{\ell}$ we obtain

$$\left.\frac{\partial^2\left(y^m - y^{n-m}\right)}{\partial r^2}\right|_{r=s} = -\frac{n}{n-1} \cdot \frac{\partial \ell^m}{\partial r} - s \cdot \frac{\partial^2(\ell^m - \ell^{n-m})}{\partial r^2} + \frac{\partial^2(T^m - T^{n-m})}{\partial r^2}. \tag{A.14}$$

Since an SES is linear, the last bracket in this expression is zero and the expression itself reduces to

$$\left.\frac{\partial^2\left(y^m - y^{n-m}\right)}{\partial r^2}\right|_{r=s} = -\frac{n}{n-1} \cdot \left(\frac{\partial \ell^m}{\partial r} + s \cdot \frac{\partial^2 \ell^m}{\partial r^2}\right) \tag{A.15}$$

A sufficient condition for (A.15) to be strictly negative is that ℓ^m is a strictly convex function which holds if $f''' > 0$.[8] □

[8] Note that

$$\left.\frac{\partial^2 \ell^m}{\partial r^2}\right|_{r=s} = \frac{n}{n-1} \cdot \frac{f'''(\bar{\ell})}{\left[f''(\bar{\ell})\right]^2} \cdot \left.\frac{\partial \ell^m}{\partial r}\right|_{r=s}.$$

A.5 ESS with RTS

Expression (23) shows that with an RTS the first-order conditions (22) in Proposition 2(ii) is satisfied for all $s \in [0, s_{max}]$. Here we want to check the second-order conditions, requiring that the payoff difference φ be locally concave. From the proof of Corollary 2 it suffices to check that expression (A.14) is strictly negative; i.e. that the difference $y^m - y^{n-m}$ is locally concave.

Now note that with an RTS we have

$$\frac{\partial^2 \left(T^m - T^{n-m}\right)}{\partial r^2} = \bar{S} \cdot \frac{\partial^2 \left(\ell^m - \ell^{n-m}\right)}{\partial r^2} + 2 \cdot \frac{\partial \bar{S}}{\partial r} \cdot \frac{\partial (\ell^m - \ell^{n-m})}{\partial r} + \left(\ell^m - \ell^{n-m}\right) \cdot \frac{\partial^2 \bar{S}}{\partial r^2}.$$

Evaluating at any symmetric profile with $r = s$, so that $\bar{S} = s$ and $\ell^m = \ell^{n-m} = \bar{\ell}$, and using $\partial \bar{S}/\partial r = 1/n$, this expression reduces to

$$\frac{\partial^2 \left(T^m - T^{n-m}\right)}{\partial r^2} = s \cdot \frac{\partial^2 \left(\ell^m - \ell^{n-m}\right)}{\partial r^2} + \frac{2}{n-1} \cdot \frac{\partial \ell^m}{\partial r}$$

Substituting in (A.14) and rearranging we obtain

$$\left.\frac{\partial^2 \left(y^m - y^{n-m}\right)}{\partial r^2}\right|_{r=s} = -\frac{n-2}{n-1} \cdot \frac{\partial \ell^m}{\partial r},$$

which is zero for $n = 2$ and strictly negative for all $n > 2$.

A.6 Examples

Setting. In our examples, we assume that the production function in each jurisdiction is quadratic, $f(\ell) = \ell(A - b\ell)$, with positive parameters $A, b > 0$. We normalize the total number of mobile poor to one: $\bar{\ell} = 1/n$. To ensure meaningful solutions, we assume that $2b \leq nA \leq 3b$. We consider political preferences of the Cobb-Douglas type: $U_j = y_j \cdot c$. The set of admissible subsidies is common to all jurisdictions and given by $s \in [0, s_{max}]$.

Migration equilibrium. The net income and consumption of workers in jurisdiction j is $c_j = A - 2b\ell_j + s_j$. At subsidies $\mathbf{s} = (s_1, \ldots, s_n)$, a migration equilibrium is characterized by $c_j = c$, which results in

$$\ell_j = \frac{1}{n} + \frac{s_j - \bar{s}}{2b} \tag{A.16}$$

where $\bar{s} = \sum_k s_k/n$ denotes the average subsidy level in the federation. The poor's consumption in all jurisdictions is then given by

$$c = A - 2b/n + \bar{s}. \tag{A.17}$$

The consumption of the rich in location j is given by

$$y_j = b\left(\frac{1}{n} + \frac{1}{2b}(s_j - \bar{s})\right)\left(\frac{1}{n} - \frac{1}{2b}(s_j + \bar{s})\right) + T_j. \tag{A.18}$$

While interjurisdictional transfers T_j may be positive or negative, we restrict y_j to be non-negative. I.e., neither must subsidies be too high to be unaffordable nor must outgoing transfers exceed the value of the net income generated at location j.[9]

Symmetric efficient solution. Since $\bar{\ell} = 1/n$ in this example, the Samuelson condition for a symmetric efficient allocation with Cobb-Douglas utility amounts to $y_j/c = 1/n$. Production efficiency requires that $\ell_j = 1/n$, so that the output per jurisdiction equals $f(1/n)$. Using the feasibility constraint, $y = f(1/n) - c/n$, gives

$$c^* = \frac{1}{2}\left(A - \frac{b}{n}\right).$$

This level can be attained in a decentralized way if all jurisdictions set subsidy

$$s^* = \frac{3b}{2n} - \frac{A}{2}.$$

[9]Even if we are able to define some meaningful upper bound s_{max}, interior solutions for $\ell_j \in [0,1]$ and $y_j \geq 0$ may impose additional restrictions on the parameter values. Furthermore, using expression (A.17), we see that aggregate feasibility requires $\sum_j f(\ell_j) - c = \frac{2b}{n} - \bar{s} - b\sum_j \ell_j^2 \geq 0$. Adding up the squares of expression (A.16) and denoting $\sigma_{\mathbf{s}}^2$ the variance of the subsidies chosen at any vector $\mathbf{s}$, we obtain $\sum_j \ell_j^2 = \frac{1}{n} + \frac{n\sigma_{\mathbf{s}}^2}{4b^2}$. Substituting this expression in the aggregate feasibility condition above, we see that feasible subsidy vectors $\mathbf{s}$ must satisfying $4b\bar{s} + n\sigma_{\mathbf{s}}^2 \leq 4b^2/n$.

Public funds sharing. We consider an RTS, which for $\bar{\ell} = 1/n$ is defined through $T_j = \bar{S}(\ell_j - 1/n)$ with $\bar{S} = \sum_k (s_k \ell_k)$. Using expression (A.16), it is easy to check that

$$\bar{S} = \bar{s} + \frac{n}{2b} \cdot \sigma_{\mathbf{s}}^2,$$

where $\sigma_{\mathbf{s}}^2 = \frac{\sum_k (s_k - \bar{s})^2}{n}$ corresponds to the variance of the subsidies chosen at $\mathbf{s} = (s_1, \ldots, s_n)$. We can see that this results in equalization payments of the form

$$T_j(\mathbf{s}) = \frac{s_j - \bar{s}}{2b} \left(\bar{s} + \frac{n}{2b} \cdot \sigma_{\mathbf{s}}^2 \right). \tag{A.19}$$

Evolutionarily stable strategies. To characterize the ESS, we compute the payoff differential between jurisdictions j and i, which is $U_j - U_i = (y_j - y_i) \cdot c$. As c is a public good, we can work with the income differential $\Delta := y_j - y_i$ as an indicator for the relative payoff advantage to jurisdiction j. In the absence of public funds sharing we have

$$\Delta^0 = y_j - y_i = \frac{s_i^2 - s_j^2}{4b},$$

which obviously decreases with s_j, rendering $s = 0$ the unique ESS irrespective of the number n of jurisdictions.

With an RTS, the income differential becomes

$$\begin{aligned} \Delta = \Delta^0 + T_j - T_i &= \frac{s_i^2 - s_j^2}{4b} + \left(\bar{s} + \frac{n}{2b} \cdot \sigma_{\mathbf{s}}^2 \right) \cdot \left(\frac{s_j - s_i}{2b} \right) \\ &= \frac{s_j - s_i}{2b} \cdot \left\{ \left(\bar{s} - \frac{s_i + s_j}{2} \right) + \frac{n}{2b} \cdot \sigma_{\mathbf{s}}^2 \right\}. \end{aligned} \tag{A.20}$$

Example 1: two jurisdictions

Assuming $n = 2$ and $A = b = 1$, we have $\bar{s} = (s_i + s_j)/2$ and $\sigma_{\mathbf{s}}^2 = (s_j - s_i)^2/4$, so that

$$\Delta = \frac{(s_j - s_i)^3}{8}.$$

This income differential is strictly increasing with s_j everywhere except at $s_j = s_i$, when both jurisdictions choose the same subsidy. Every symmetric profile, however, can be destabilized if one of the jurisdictions increases its subsidy, provided this is admissible. Hence, the unique ESS is $s^E = s_{max}$.

Example 2: three jurisdictions

Now suppose that $n = 3$ and $A = b = 1$. In this case the efficient subsidy $s^* = 0$ is also played in the unique, symmetric Nash equilibrium and as an uncorrected ESS. Starting from a symmetric profile (s, s, s) we consider subsidy vectors of the form $\mathbf{s} = (s_1, s_2, s_3) = (r, s, s)$. From (A.20) we obtain the relative payoff difference of the deviating jurisdiction as

$$\begin{aligned}\Delta(r,s) &= \frac{s^2 - r^2}{4} + \frac{r-s}{2}\left\{\frac{r+2s}{3} + \frac{(r-s)^2}{3}\right\} \\ &= \frac{(r-s)^2\,[2(r-s)-1]}{12}. \end{aligned} \tag{A.21}$$

Note that, for any given s, $\Delta = 0$ if $r = s$ or if $r = \widetilde{r} := s + \frac{1}{2}$ and $\Delta > 0$ if and only if $r > \widetilde{r}$. The first- and second-order partial derivatives of Δ with respect to the subsidy of the deviating jurisdiction are as follows:

$$\begin{aligned}\frac{\partial \Delta}{\partial r} &= \frac{(r-s)\,[r-s-1/3]}{2} \\ \frac{\partial^2 \Delta}{\partial r^2} &= \frac{2(r-s)-1/3}{2}.\end{aligned}$$

Observe that with RTS every symmetric profile with $r = s$ has $\frac{\partial \Delta}{\partial r} = 0$ and $\frac{\partial^2 \Delta}{\partial r^2} < 0$ and is, thus, a local maximum of Δ for any s. However, at $r = \widetilde{r}$ we also have $\Delta = 0$. Thus, $r = s$ is not a global maximum. Using (A.16), (A.18) and (A.19), it can be checked that, at $\mathbf{s} = (\widetilde{r}, s, s)$, we have $\ell_1 = \frac{1}{2}$ and $\ell_2 = \ell_3 = \frac{1}{4}$, while incomes are given by $y_1 = y_2 = y_3 = \frac{1}{3}\left(\frac{1}{8} - s\right)$, equal for all jurisdictions and positive if and only if $s \leq \underline{s} := \frac{1}{8}$.[10] Starting at a symmetric profile with $s < \underline{s}$, if any jurisdiction deviates to $r = \widetilde{r} + \varepsilon$ with $\varepsilon > 0$ sufficiently small, it obtains a strictly positive relative advantage. It follows that $s \in [0, \underline{s})$ are not evolutionarily stable. For $s = \underline{s}$ a deviation to $r = \underline{s} + \frac{1}{2}$ results in exactly

[10] It can be easily checked that subsidy vectors of the form $(s + 1/2, s, s)$ satisfy the aggregate feasibility condition introduced in Footnote 9 if an only if $s \leq 1/8$.

zero residual income in all jurisdictions after deviation. For $s > \underline{s}$ it is not possible to find feasible values of r that exploit the transfer mechanism in this way. All ESS with an RTS, thus, satisfy $s^E \geq \underline{s}$. It follows that the efficient policy $s^* = 0 < \underline{s}$ is not an ESS in this case and laboratory federalism, augmented by an RTS, will systematically result in overly generous redistribution.