WORKING PAPERS

The Price of Capital, Factor Substitutability and Corporate Profits

Philipp Hergovich
Monika Merz

Jänner 2018

Working Paper No: 1801

DEPARTMENT OF ECONOMICS

UNIVERSITY OF VIENNA

The Price of Capital, Factor Substitutability and Corporate Profits

Philipp Hergovich* and Monika Merz†‡

January 16, 2018

Abstract

Technical progress has contributed to a steady decline in the relative price of new capital goods and at the same time facilitated the substitutability between physical capital and labor in output production. This paper studies the quantitative implications that these two changes have for the level and the variability of firms' profits, the capital-to-labor ratio, and also for labor market outcomes when profits arise from rents paid to quasi-fixed factors of production. We embed a CES production function into a model of capital accumulation and competitive search in the labor market, allowing firms to increase their size by hiring multiple workers. We use our model to disentangle the effects of the decline in the relative price of capital and increased factor substitutability. Our analysis identifies each of these two changes as important drivers of the empirically observed rise in the capital-to-labor ratio and in the level and variability of firms' profits. Their overall effect on wages, employment, and the labor share of income is inconclusive, since their respective impact on each of these variables goes in the opposite direction.

Key Words: *factor substitutability, quasi-fixed production factor, competitive search, profits, aggregate trends*
JEL Classification Numbers: *E24,G32,J64*

*Vienna Graduate School of Economics. Email: philipp.hergovich@univie.ac.at

†University of Vienna, CEPR and IZA. Email: monika.merz@univie.ac.at

‡Monika Merz gratefully acknowledges financial support from the *Jubiläumsfonds* of the Austrian National Bank, project no. 16253. We thank seminar participants at the University of Vienna, the Ruhr Graduate School of Economics and the annual meeting 2017 of the Austrian Economic Association for constructive comments. All errors are our own.

1 Introduction

Technological progress has been omnipresent in most industrialized countries and has knowingly caused drastic changes in some economic fundamentals. Among the most noticeable implications of technical progress have been a steady decline in the relative price of new capital goods and a bias towards firms increasingly adopting production technologies that replace labor by machines.

In this paper we explore the role that a steady decline in the relative price of new capital goods or an increased factor substitutability in output production play in explaining three aggregate trends that have prevailed in many OECD countries for several decades: the rise in the capital-to-labor ratio; the steady rise in the level and the variability of corporate profits relative to output; and the decline in the labor share of income. Figure 1 depicts these marked trends for the U.S. economy during the post-WWII period.

We depart from the hypothesis that in the longer run, adjusting labor is more costly for firms than adjusting capital. To implement this hypothesis and lay the ground for exploring our main research question, we develop a dynamic stochastic equilibrium model of a frictional labor market. Firms search for suitable workers by posting job-vacancies and wages, and unemployed workers search for jobs. Firms use capital and labor for producing output with the help of a technology that exhibits a constant elasticity of substitution. Capital can be flexibly adjusted, but expanding labor is subject to search frictions. We calibrate this model to the U.S. economy. The model serves as a lab for disentangling the role that a steady decline in the relative price of new capital goods or an increased factor substitutability play in simultaneously explaining the aggregate trends described above. Via a string of simulation exercises we can distinguish between changing relative factor prices – which correspond to a move along a particular isoquant – and a change in factor substitutability that affects the shape of the isoquant.

We find that under frictional labor markets a rise in the degree of factor substitutability lets firms choose a more capital-intense input mix. The implied decline in labor demand causes wages, employment and subsequently the labor share of income to fall. The fall in the labor share lets corporate profits rise. Also, because firms react more flexibly to exogenous disturbances, the overall variability in the economy including that of corporate profits increases. The implications of a decline in the relative price of new capital are different. Because capital and labor are complements in the production process, the declining relative factor price shifts the input demand away

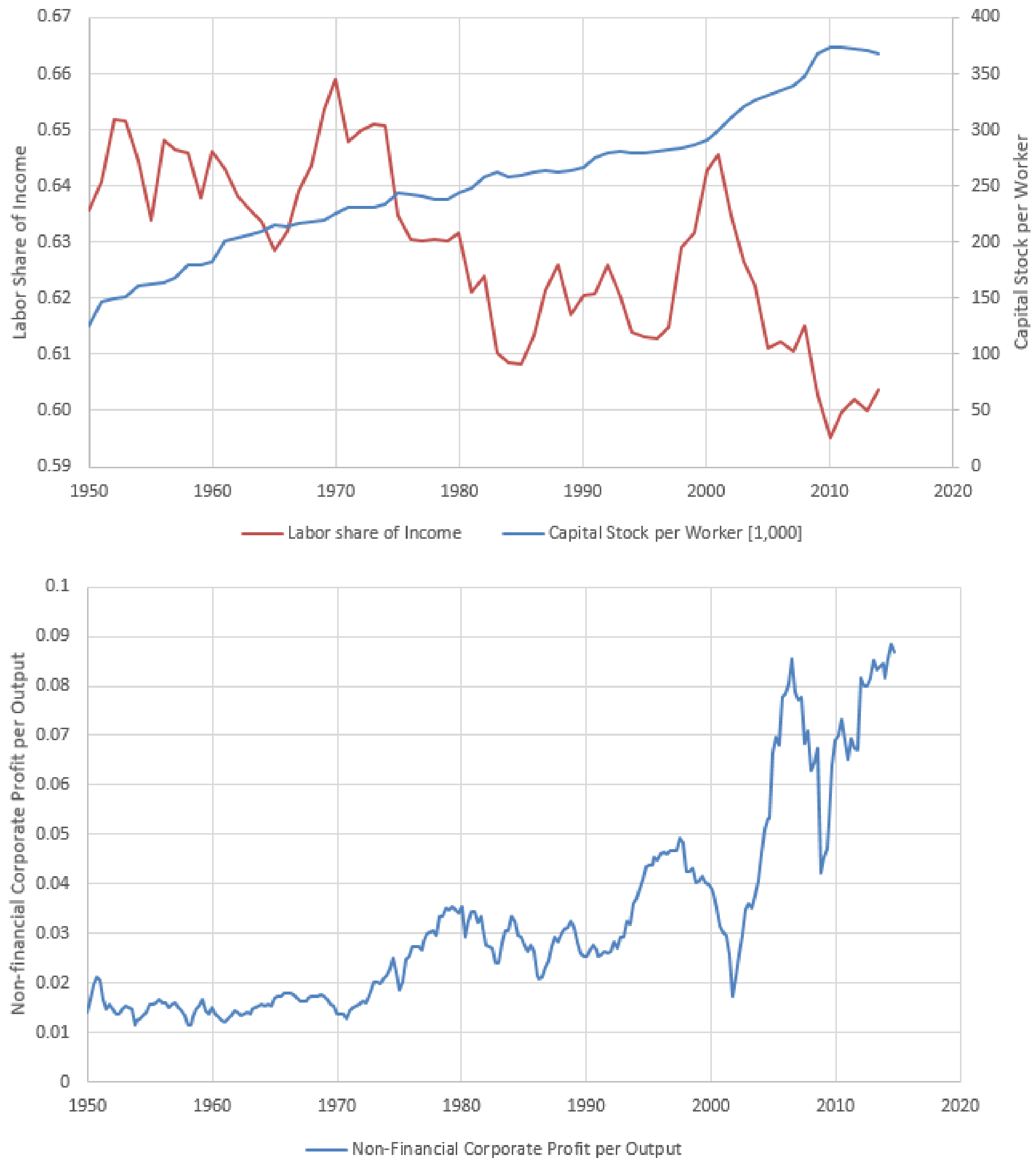

Figure 1: Aggregate Trends in the U.S. Economy

from labor towards capital, but leads to increasing wages, employment, and an increase in the labor share.

Our quantitative results underline the importance of studying the two major implications of technological progress separately, but in an integrated framework that features rather strong frictions in the labor market. While each change considered can help explain all rising trends, only increased factor substitutability generates the observed decline in the labor share. Hence, a possible interpretation of the empirical facts in light of our model is that the implications of increased factor substitutability have quantitatively outweighed those of a decline in the relative price of new capital goods.

Our analysis builds upon and extends in several dimensions an earlier study by Blanchard (1997) who formulated a static general equilibrium model with frictional factor markets and monopolistic competition in the goods market. The primary goal of his cross-country study was to understand the forces at work that had led to diverging trends in unemployment and the labor share of income between some Anglo-Saxon countries including the U.S. and selected countries in continental Europe. Contrary to Blanchard, we exclusively focus on aggregate trends in the U.S. – including those related to corporate profits relative to output –, restrict our analysis to changes in labor demand, and emphasize the importance of relatively strong frictions in the labor market.

The remainder of this paper is structured as follows. Section 2 provides an overview of the literature that is closely related to our work. Section 3 presents our model of competitive search including its calibration. Section 4 includes various simulation exercises and results, while section 5 presents empirical evidence for selected aggregate time-series for the U.S. Section 6 concludes.

2 Related Literature

This paper relates to several strands of literature. First it analyzes questions related to technological progress and its effects on the economy such as e.g. digitization.[1] A recent contribution to the discussion on automation is Acemoglu and Restrepo (2017). The authors empirically study the competition between robots and workers for different tasks. They show that robots have a large and negative effect on employment and wages. We instead do not consider tasks, but rather use an aggregate CES-production function

[1] A cohesive summary of this literature is beyond the scope of this paper, but readers may want to look at Brynjolfsson and Afee (2014) for a general discussion. We instead cover a selection of examples, which all closely relate to our paper.

and study changing factor substitutability by varying the respective parameters. We also find that increased substitutability decreases employment and wages. A direct link between factor substitutability and corporate profits is given in Shim (2015). He empirically determines that firms with higher substitutability are less risky in their profits. Our main result that higher substitutability leads to more volatile profits is in stark contrast to this finding. This is because we model substitutability via a parameter in the CES production function, while Shim uses the "capital-labor-ratio" as proxy for substitutability while working with a Cobb-Douglas production function. However, the substitutability in a Cobb-Douglas production function is by definition unity. We therefore consider our model framework more suitable for the analysis, especially as it nests the production function used in Shim (2015) as a special case. We also do not study the cross-section of heterogeneous firms, but focus on the aggregate economy populated by homogeneous firms.

Our paper also relates to the literature concerned with search frictions in a multi-worker setup. In order to answer our research question we need to abandon the Leontief-type production commonly used in search and matching models, where each firm has one job which can or cannot be filled with one worker. We use a directed search framework instead and allow for firms hiring multiple workers. One of the first papers to study multi-worker-firms in a directed search framework is Hawkins (2013) who employs a theoretical model where firms can commit to a posted wage and attract multiple workers. The author refrains from introducing capital in the production function, which is key for our discussion of substitutability among inputs. The same holds true for the work of Schaal (2015) who links a model with multiple workers per firm to the business cycle and studies the role of uncertainty. Kaas and Kircher (2015) also focus on understanding labor market fluctuations using heterogenous firms that differ in size, while capital is absent from their model. Their findings suggest that firm heterogeneity helps to rationalize why quickly expanding firms offer higher wages. Firm heterogeneity creates sluggish aggregate responses to shocks. The focus of our paper is less on the business cycle and more on understanding long-run trends. All of these aforementioned papers examine details of dynamic adjustment processes with multi-worker firms and for most of our analysis we follow their example. The main difference is that in our model capital is present, and firms are able to substitute among labor and capital. We vary the degree of substitutability, which can be interpreted as long run effects of technological progress. Ours is also, to the best of our knowledge, the first paper to bring together capital and multi-worker firms with directed search.[2]

Finally, our paper adds to the ongoing discussion on the decline of the labor share of income. For decades, the labor share was considered to be constant. More recently a decline in this share has been observed, which has spurred renewed interest in the topic (see e.g. Autor et al. (2017)). Many potential explanations for this phenomenon have been brought forward, including sectoral concentration (Autor et al., 2017), automation and digitization (Arntz et al., 2016), increased markups (Loecker and Eeckhout, 2017) or international trade (Elsby et al., 2013). We focus on the role of technological progress and examine the effects of increased factor substitutability and cheaper capital goods on the labor share. In this respect, our work is closely related to the work by Karabarbounis and Neiman (2013). The authors relate the labor share to the declining price of investment goods and find that lower prices of capital lead to a decline in the labor share. They estimate their model and find an elasticity of substitution between capital and labor of about 1.25. Compared to other estimates (see e.g. Chirinko (2008), or León-Ledesma et al. (2010)) this value is high, but crucial for their results, as it implies that the inputs are substitutes rather than complements. The two cases we examine instead exhibit an elasticity of substitution below one. We show that a decrease in the price of investment goods leads to an increase in the labor share, whereas an increase in factor substitutability lets the share decline.

3 A Model of Competitive Search

Our model economy is populated by a unit mass of identical firms and a unit mass of identical workers. Firms post vacancies and invest in physical capital in order to maximize their profits. Due to labor market frictions, firms cannot hire workers directly, but have to post vacancies at a cost a and a corresponding wage $\tilde{w}$ for as long as the employment relationship lasts. The transition from vacancies to a filled job and from unemployed to employed depends on the number of workers applying to a vacancy and the number of vacancies posted by the firms. Firms can post vacancies in various submarkets and unemployed workers direct their search towards one of those markets, trading off the wage and the chance of getting hired. The interplay of the firms' posting behavior and the workers' application decisions generates labor market tightness, which is defined as the ratio of vacancies to the number of applicants in a market. For ease

[2]With random matching, firms with multiple workers and capital have been studied before. One recent contribution is Gertler et al. (2016).

of exposition the actual matching is depicted by a standard matching function.

3.1 The Firm's Side

We start the detailed description of the model at the firm, because this is our core unit of analysis. There exists a unit mass of identical firms in this economy, which use capital k and labor l to produce a homogeneous output good y. The inputs are transformed into the output good according to a constant elasticity of substitution (CES) production function:

$$y(k_t, l_t, z_t) = z_t \left(\alpha k_t^{\sigma} + (1-\alpha) l_t^{\sigma}\right)^{\frac{1}{\sigma}}, \\ \text{with } \alpha \in (0,1), \ \sigma \in (-\infty, 1]$$

We choose this functional form for two reasons. First, it is more general as it nests the more common Cobb-Douglas function and more importantly, because it allows us to explicitly vary the substitutability of input factors. The elasticity of substitution between k and l depends on the parameter σ and is given by $\frac{1}{1-\sigma}$. As σ is a key model parameter, it is important to understand its effects on the production function. The parameter σ can vary between $-\infty$ and 1. For the limiting case of $-\infty$ the elasticity of substitution equals zero and the production function approaches the Leontief production function with a fixed ratio of input factors. This implies that inputs are perfect complements. For $\sigma = 1$ input factors are perfect substitutes and an isoquant is depicted by a straight line. At $\sigma = 0$ the CES form nests the Cobb-Douglas case.[3] The other parameter entering the production function is α, which governs the capital intensity of production.

Firms can purchase capital at a fixed price p^k per unit. Capital depreciates at a rate δ every period. Because of frictional labor markets, firms can expand their labor force only by posting vacancies v together with a wage rate $\tilde{w}$ in a particular submarket, which is characterized by its respective tightness, θ.[4] For each vacancy posted, the firm has to pay a vacancy posting cost, a. This cost can be thought of as including advertising and training newly hired employees. By assumption, a constant fraction ν

[3] For further discussions on the CES function and its properties see Klump et al. (2012).

[4] We choose wage-posting plus directed search – rather than random search – to avoid the holdup problem a firm would face when making investment decisions. In our competitive search setting, a higher capital stock implies higher wages and also a higher job filling rate. See Acemoglu and Shimer (1999).

of matches breaks up every period. This is the only possibility for a match to end. The firms cannot decide which workers to fire. Thus, the stock of employment l_t is a state variable for the firm in period t.

The fact that firms decide upon the wage offered for a posted vacancy in every period potentially generates a distribution of wages. Since we do not focus on wage dynamics in this paper, we choose to simplify the wage setting process. New hires formed during period t become productive in period $t+1$. These new hires h_t will be paid the posted wage $\tilde{w}_t$. The wage bill that a firm has to pay in period t is given by $l_t w_t$, where w_t denotes a weighted average of the wage paid to continuing workers and new hires from the previous period. In brief, $l_{t+1}w_{t+1} = (1-\nu)l_t w_t + h_t \tilde{w}_t$. We calculate the wage bill in a recursive way, which is described in greater detail in Appendix A. There we show that our recursive formulation is equivalent to keeping track of the entire history of hires and wages. Therefore, w_t is an additional state variable for the firm.

The firm discounts future profits at rate $0 < \beta < 1$. The firm's problem can be summarized as follows:

$$\max_{v_t,\theta_t,\tilde{w}_t,i_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [y(k_t, l_t, z_t) - w_t l_t - i_t p^k - a v_t]$$

subject to

$$\begin{aligned} h_t &= v_t q(\theta_t) \\ l_{t+1} w_{t+1} &= (1-\nu) l_t w_t + h_t \tilde{w}_t \\ l_{t+1} &= (1-\nu) l_t + h_t \\ k_{t+1} &= (1-\delta) k_t + i_t \\ z_{t+1} &= \rho z_t + \epsilon_t,\ \ \epsilon_t \sim \mathcal{N}(0, Var_\epsilon) \end{aligned}$$

This formulation states that firms maximize the expected present discounted value of future profits. Real profits consist of real revenue minus wage payments, investment expenditures and hiring costs. The firm takes as given that the number of newly hired employees equals the posted vacancies multiplied by the job filling rate, the recursive formulation of the wage bill, and the laws of motion for capital, labor and exogenous total factor productivity, z_t. As we elaborate below, in equilibrium two additional constraints must be satisfied, i.e. the optimal application rule for searching workers and a the requirement that the ratio of all job-vacancies to searching workers indeed equals labor market tightness θ in a given submarket.

3.2 The Household's Side

Workers are part of a big family, consisting of a continuum of members normalized to measure 1. Each worker can be employed or unemployed. If unemployed, she chooses to apply to a particular submarket that is characterized by vacancies and the corresponding wage-rate $\tilde{w}_t$. The worker's chances of getting matched depend on the ratio of all vacancies posted to the measure of job seekers in that submarket. If employed, a worker inelastically supplies one unit of labor to the firm and receives a wage w_t in exchange. At the end of each period the family pools all income. This implies that for each individual neither the actual labor market status, nor the individual wage rate in case of employment matter, since all equally share the family's total earnings. We effectively assume full risk-sharing. Moreover, we assume that all agents are risk-neutral and do not save. This is necessary for our recursive wage formulation to be an exact description of the earnings over time.

Unemployed workers will apply for a job only if the value of getting it is the best they can do. This implies they will select the best combination of job finding rate and wage among all the ones offered in equilibrium. Denoting by U the value for an unemployed worker of getting a job the following condition holds:

$$U_t \leq p(\theta_t)\tilde{w}_t + (1 - p(\theta_t))b \tag{1}$$

The value U_t is the value to an unemployed individual who can apply for a job which promises the wage $\tilde{w}$ and a job finding rate $p(\theta)$. U_t exceeds the value of the unemployment benefit b, because firms internalize this condition in their decision problem. If they were to offer just b, one firm could offer a slightly higher wage, thereby attracting all searching workers. Thus, each firm takes U_t as given, although this variable is determined endogenously in equilibrium.[5]

[5] We simplify the problem by abstracting from a continuation value for the unemployed. This makes the worker care only about current wages. However, not applying for a job will decrease the earnings by the household by an entire quarter of the annual wage bill. This loss is big, compared to the chance of a shock that would make it worthwhile for the workers to wait an entire period.

3.3 Matching

In each submarket, job vacancies and searching workers are randomly matched. We capture this process by a standard Cobb-Douglas matching function $m(u_t, v_t)$, which we assume to exhibit constant returns to scale:

$$m(u, v) = Bv^{\gamma}u^{(1-\gamma)} \ , \ B > 0 \tag{2}$$

where $\gamma \in [0, 1]$ is the elasticity of total matches with respect to vacancies, and B governs the efficiency of the matching process.

Dividing the number of matches by the measure of searching workers yields the job-finding rate $p(\theta)$ results, whereas dividing it by the number of vacancies delivers the job filling rate for the firm, $q(\theta)$. A firm posting vacancies v_t can expect to attract $h_t = v_t q(\theta_t)$ new workers.

3.4 Labor Market Equilibrium

Each firm enters period t with its stock of capital k_t, its workforce l_t, the firmwide wage level w_t which it pays to every employee, and the realization of the exogenous aggregate shock process z_t. Those variables form its state vector (k_t, l_t, w_t, z_t).

When maximizing the expected present discounted value of future profits, the firm takes into account the laws of motion for each of its state variables and also the job application rule for searching workers given by equation (1). Substituting in the various laws of motion, we can summarize the firm's problem with the help of the following Lagrangian.[6]

$$\begin{aligned}\mathcal{L} = \max_{\theta_t, k_{t+1}, l_{t+1}, w_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \{ y(k_t, l_t, z_t) - w_t l_t - [k_{t+1} - (1-\delta)k_t]p^k - a\frac{l_{t+1} - (1-\nu)l_t}{q(\theta_t)} \} \\ + \lambda_t \left[U_t - (1 - p(\theta_t))b - p(\theta_t)\frac{l_{t+1}w_{t+1} - (1-\nu)l_t w_t}{l_{t+1} - (1-\nu)l_t} \right]\end{aligned}$$

The first-order-necessary conditions that need to be satisfied in equilibrium are given by

[6] For an alternative complete formulation of the problem see Appendix B.

$$\frac{\partial}{\partial\theta_t}: a(l_{t+1}-(1-\nu)l_t)\frac{q'(\theta_t)}{q(\theta_t)^2}+\lambda_t p'(\theta_t)\left[b-\frac{l_{t+1}w_{t+1}-(1-\nu)l_t w_t}{l_{t+1}-(1-\nu)l_t}\right]=0$$

$$\frac{\partial}{\partial k_{t+1}}: p^k=\beta\left[\frac{\partial y(k_{t+1},l_{t+1},z_{t+1})}{\partial k_{t+1}}+p^k(1-\delta)\right]$$

$$\frac{\partial}{\partial l_{t+1}}: -\frac{a}{q(\theta_t)}+\lambda_t(-p(\theta_t))\frac{(1-\nu)l_t[w_t-w_{t+1}]}{(l_{t+1}-(1-\nu)l_t)^2}$$

$$+\beta\left[\frac{\partial y(k_{t+1},l_{t+1},z_{t+1})}{\partial l_{t+1}}-w_{t+1}+a\frac{(1-\nu)}{q(\theta_{t+1})}+\lambda_{t+1}\{-p(\theta_{t+1})\frac{(l_{t+2}(1-\nu)[w_{t+2}-w_{t+1}]}{(l_{t+2}-(1-\nu)l_{t+1})^2}\}\right]=0$$

$$\frac{\partial}{\partial w_{t+1}}: \lambda_t(-p(\theta_t))\frac{l_{t+1}}{l_{t+1}-(1-\nu)l_t}+\beta\left[-l_{t+1}+\lambda_{t+1}\{p(\theta_{t+1})\frac{(1-\nu)l_{t+1}}{l_{t+2}-(1-\nu)l_{t+1}}\}\right]=0$$

As all firms are identical and so are all workers, their respective behavior can be summarized by that of a representative agent. Note that the representative firm continues to react to changes in the economy in a competitive way. Our competitive search setup in this particular environment reduces the many possible submarkets to a single market.

We close the model by enforcing the requirement that in equilibrium, the ratio of posted vacancies to the measure of unemployed workers needs to equal labor market tightness, $\frac{v}{1-l}=\theta$. Substituting v_t by $\frac{l_{t+1}-(1-\nu)l_t}{q(\theta_t)}$, and exploiting algebraic properties of our matching function, we get the following expression as additional equilibrium condition:

$$\theta_t=\left(\frac{l_{t+1}-(1-\nu)l_t}{B(1-l_t)}\right)^{\frac{1}{\gamma}} \tag{3}$$

In order to reach a steady state, we need a vector $(l^*,k^*,w^*,\theta^*,\lambda^*)$[7] which solves the system given by the 4 F.O.N.C.s plus equation (3). In equilibrium the value U is determined by the optimal values for wages and labor market tightness plugged into condition 1 with equality.[8]We then solve the model around the deterministic steady state by second-order perturbation.

[7]Stars denote equilibrium values.

[8]For further discussion on the solution process of labor-search models see Rogerson et al. (2005)

4 Quantitative Analysis

4.1 Calibration

As the model cannot be solved analytically, calibration becomes an important matter. The model has a variety of parameters which need to be determined. We take certain values from the literature in order to achieve comparability. We perform robustness checks to ensure that these values are not driving the results. The crucial parameters are calibrated in order to match empirical targets, which are important when talking about factor substitutability and its implications for firms and workers.

We calibrate the model to quarterly data from the US economy in order to enable comparability with related papers and because of data availability. The full parametrization of the model is given in Table 1.

Parameter	Interpretation	Value	Target
α	Capital intensity	0.7914	Labor share 60%
σ	Substitutability parameter	-3/2	Elasticity of substitution 0.4
p^k	Price of capital	1	Normalization
γ	Matching function elasticity	0.5	Standard
B	Matching Efficiency	0.8	Unemployment rate 7%
b	Unemployment benefit	0.9	Replacement ratio 60%
a	Vacancy posting cost	4	$p(\theta) = 0.99$
β	Discount factor	0.975	Standard
δ	Depreciation rate of capital	0.026	Depreciation rate of capital
ν	Separation rate	0.075	Labor turnover

Table 1: Baseline Calibration

One of our central questions is what happens to firm profit and other key variables like employment and investment, if a firm is able to substitute more easily among capital and labor. To address this question, we vary the parameter σ, which directly relates to the elasticity of substitution between capital and labor. As a baseline value, we pick $\sigma = -\frac{3}{2}$, which corresponds to an elasticity of substitution of 0.4. This value lies at the lower end of what the literature deems plausible.[9] We will change the parameter σ to $-\frac{2}{3}$ to model increased substitutability and study its effects. We use the range provided by Chirinko (2008) as a guideline for one of the experiments we perform in the context of our model.

[9]For a survey of these values see Chirinko (2008). He argues that empirical estimates of the elasticity of substitution range from 0.4 to 0.6.

The parameter α which enters the production function is central to the problem, as the technology available to the firm is key to our analysis. This parameter amounts to an additional degree of freedom in the production function, which we have to tackle in our analysis.[10] We calibrate α, to ensure that the model outcomes are comparable across alternative calibrations. In a standard neoclassical model with a Cobb-Douglas production function and no frictions, the parameter α corresponds to the share of capital. We will handle α in a similar manner, by using it to calibrate a labor share of 60%. When studying what happens to the labor share, especially when the price of capital decreases, we adjust α such that the output level remains constant.

We normalize the price of capital, p^k, to one. This price governs the rate at which a firm can turn its output good into next period's capital. In our comparative statics exercises, we will consider what happens when we lower this price, thereby rendering investment of the firm more productive. At a price equal to one, the output good produced by the firm can simply be used as next period's capital. When lowering p^k, we implicitly make the technology via which output can be turned into capital more efficient. A falling price of investment goods might cause similar effects as increased factor substitutability. Whether it is cheaper to invest in capital, or if the capital stock can perform more tasks is hard to distinguish in reality, as both effects occur simultaneously. In our model, we can separate these two effects.

We set the efficiency parameter B of the matching function to target an unemployment rate of 7% and choose the unemployment benefit to match a replacement ratio equal to 0.6. The replacement ratio is defined as unemployment benefit b relative to the equilibrium wage.

The vacancy posting cost a is chosen such that a worker's job finding rate of the worker close to 0.99, the rate implied by the monthly rate of 0.34 which Shimer (2005) reports.

The remaining parameters are set to values commonly found in the literature. Many have a clear economic interpretation. Shimer (2005) shows that around 3.42 % of workers in the US labor force leave their jobs each month. So we set ν equal to 0.075 for a period of three months to account for the transitions made within a quarter. The depreciation rate of 0.026 reflects the empirical equivalent. Although not explicitly targeted, our set of calibrated parameter values implies a plausible value for the cost of hiring. Blatter et al. (2012) report this value to be between 10 to 17 weeks of wage payments. The value in our baseline-calibration is 16.7 weeks.

[10] For a discussion of the term *normalization* see e.g. León-Ledesma et al. (2010).

4.2 Results

Table 2 summarizes the findings for the level of our variables of interest and selected shares of output under the baseline calibration and for the case where the parameter σ is increased from $-3/2$ to $-2/3$. This increase corresponds to a rise in the elasticity of substitution among input factors from 0.4 to 0.6.

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -2/3$ $p^k = 1$
k	7.0470	8.0188
l	0.9299	0.8984
w	1.5260	1.3196
θ	1.5475	0.6875
$q(\theta)$	0.6431	0.9648
$p(\theta)$	0.9952	0.6633
v	0.1085	0.0698
y	2.3651	1.9724
π	0.3290	0.3011
u	0.07	0.10
profit share	0.1391	0.1527
labor share	0.6	0.6
investment share	0.0775	0.1057
hiring cost share	0.1834	0.1416
α	0.7914	0.5298

Table 2: Steady State Results

Such a rise makes production more capital-intensive while conditions for the worker worsen. The job finding rate $p(\theta)$ declines, and so do employment and wages. The firm spends more on investment and less on hiring, which can be seen by the decrease in the hiring cost share, which is the costs of hiring divided by output. As the firm produces with greater capital intensity it uses less labor and also posts fewer vacancies. At the same time output declines. By construction, the labor share remains constant, but the profit share increases. The profit share of 13% is a little bit higher than what we observe in the data and it further increases when factor substitutability rises.

Overall the increased substitutability among input factors benefits the firms via higher profits. Workers suffer from lower wages and higher unemployment.

4.2.1 A Lower Price of Capital

As documented in detail by Gordon (1990) the relative price of investment goods has steadily declined for decades. Figure 2 illustrates this ongoing trend. In this section, we explore the quantitative effects of a decline in p^k under our baseline scenario ($\sigma = -3/2$) and under an increased degree of factor substitutability ($\sigma = -2/3$). Note that a lower price of new capital can be interpreted as firm's improved ability to produce investment goods from the output good. Table 3 reports the results from our numerical experiment. A lower price p^k leads to more capital and labor used in the economy under the two regimes of factor substitutability we consider. In both regimes, capital and labor exhibit a relatively low elasticity of substitution, and thus are complements. Also at the lower price of capital a rise in factor substitutability lets employment and wages decrease and the profit share increase. Again, a higher degree of substitutability leads to lower overall output.

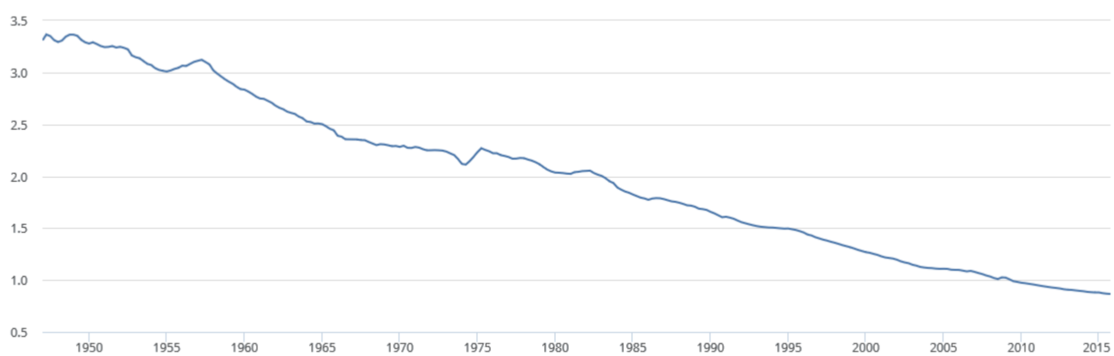

Figure 2: Relative Price of Investment Goods. *Source: FRED St. Louis*

In sum, when increased factor substitutability occurs together with lower prices of capital in a world of frictional labor markets, the only reliable statement we can make is that the extent of capital in use increases.

4.2.2 A Decline in the Labor Share

In all of the previous experiments, we recalibrated the parameter α to keep the labor share at 60%. This was done in accordance with the famous empirical facts presented in Kaldor (1961). One of these facts states that the labor share is constant over long periods of time. As can be seen in Figure 3, this share actually shows a declining trend over time.[11] Of course, a declining share of GDP accruing to labor implies

[11] The same holds true for other OECD countries (compare Autor et al. (2017)).

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -3/2$ $p^k = 0.7$	$\sigma = -2/3$ $p^k = 0.7$
k	7.047	9.0327	10.8953
l	0.9299	0.9415	0.9152
w	1.5260	1.6596	1.34089
θ	1.5475	2.2787	1.0228
$q(\theta)$	0.6431	0.53	0.791
$p(\theta)$	0.9952	1.2076	0.8091
v	0.1085	0.1332	0.0868
y	2.3651	2.6043	2.149
π	0.329	0.3443	0.3142
u	0.07	0.0585	0.0848
profit share	0.1391	0.1322	0.1462
labor share	0.6	0.6	0.6
investment share	0.0775	0.0631	0.0923
hiring cost share	0.1834	0.2074	0.1615
α	0.7914	0.8099	0.5351

Table 3: Steady State Results

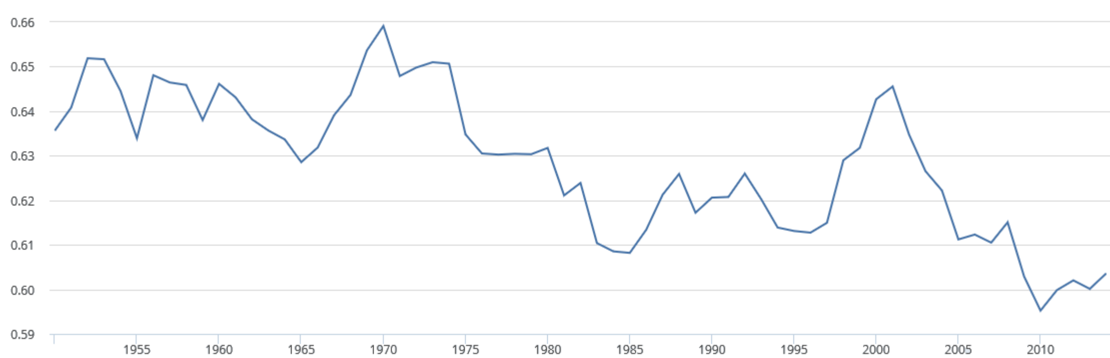

Figure 3: Labor Share in the US. *Source: FRED St. Louis*

that other factors gain from these developments. There exist a variety of possible explanations for why the labor share might decline. Among these reasons are technical change (Karabarbounis and Neiman (2013)), trade globalization ((Elsby et al., 2013)), or superstar firms (Autor et al. (2017)). Our paper contributes by illustrating the separate role played by increased factor substitutability and a declining price of capital in explaining the decline of the labor share.

In what follows we explore how the labor share of GDP reacts to a decline in the price of capital, and to an increase in factor substitutability. Towards this end we recalibrate

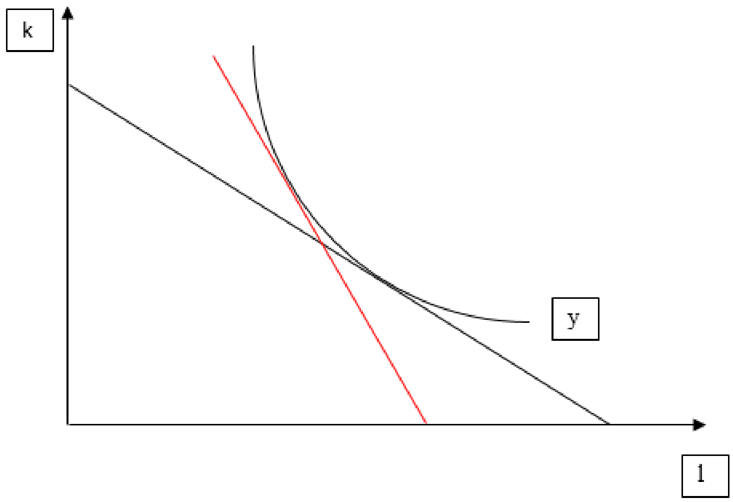

Figure 4: A Change in the Relative Price of Capital

α to keep steady-state output constant. First, we consider a change in the relative price of capital, and illustrate the implications for a firm's demand for production factors in Figure 4. In that figure, the slope of the cost line equals the negative ratio of input factor prices, i.e. the ratio between the wage rate w and the price of capital, p^k. A drop in the price of capital increases the steepness of the cost line which is depicted in red. That is because cheaper capital increases the firm's demand for capital and also for labor. A drop in p^k endogenously leads to a rise in the wage rate. A higher wage is needed to attract more workers. We observe that the point of tangency moves to the left, resulting in a higher capital-labor ratio and a more capital-intensive production.[12] The rise in the overall capital intensity of the US economy in production is consistent with evidence from US data.[13]

A decrease in the price of capital to $p^k = 0.7$ causes the firm to use more capital and renders production more capital-intensive. At the same time total expenses for investment drop, because of the price decline by 30%. Employment and wages increase, which results in a rise of the labor share.

In a separate set of exercises we vary σ, the parameter that governs factor substitutability. In particular, we change it from $-3/2$ to $-2/3$, rendering input factors more substitutable. Due to frictions in the labor market, the firm decides to increasingly

[12]We refrain form illustrating the case of increased substitutability, because it would alter the shape of the production function too much, since σ and α change substantially.

[13]See Appendix D.4.

replace labor by capital. The decline in labor demand lets wages decrease. The labor share subsequently drops by around 4 percentage points. The drop in the job finding rate for unemployed workers underlines the worsened situation for the factor labor. Table 4 presents the results of our two experiments in greater detail and contrasts them to our earlier findings.

Variable	$\sigma = -3/2$ $p^k = 1$	$\sigma = -3/2$ $p^k = 0.7$	$\sigma = -2/3$ $p^k = 1$
k/l	7.5782	8.7749	11.0589
w	1.5260	1.5603	1.4487
θ	1.5475	1.7220	1.1889
$q(\theta)$	0.6431	0.6096	0.7337
$p(\theta)$	0.9952	1.0498	0.8723
v	0.1085	0.1148	0.0941
y	2.3651	2.3651	2.3651
π	0.3290	0.3023	0.3898
profit share	0.1391	0.1278	0.1648
labor share	0.6	0.6157	0.564
investment share	0.0775	0.0623	0.1119
hiring cost share	0.1834	0.1942	0.1592
α	0.7914	0.7828	0.5827

Table 4: Steady State Results with Constant Output

We conclude that cheaper investment goods cannot be the sole source for the empirically observed decrease in the labor share in many countries, since it would imply an increase in employment and wages, and thus in the labor share. On the other hand, increased factor substitutability tends to depress this share. Our exercise has emphasized the importance of explicitly distinguishing between these two forces at work when trying to understand the implications of technological progress on firms' profits and related labor market variables, including the labor share of income.

4.3 Changes in Variability

In what follows, we will investigate whether increased factor substitutability dampens or increases the variability of profits. We consider a stochastic environment where the firms face shocks to total factor productivity (TFP). We assume TFP to follow an AR-(1) process with a persistence parameter of 0.9. Increments are normally distributed with mean zero and a standard deviation equal to 0.007.

Variable	Mean	Coeff. of Variation
k	7.0341	0.0566
l	0.9293	0.0077
w	1.5275	0.0293
θ	1.5686	0.2157
i	0.1829	0.9716
v	0.1087	0.1343
y	2.3660	0.1285
π	0.3287	0.7667
profit share	0.1318	0.7215
labor share	0.6078	0.1012
investment share	0.0764	0.9620
hiring cost share	0.1838	0.0511

Table 5: Variability for $\sigma = -3/2$

We do a second-order approximation around the deterministic steady state of our model. The stochastic results for our baseline calibration are given in Table 5. It displays the approximated mean and the coefficient of variation. The mean is calculated in a stochastic environment, where TFP shocks are present. Because we use a second-order approximation, the means do not equal their corresponding steady state values, due to the asymmetries introduced by search frictions. We also compute the coefficient of variation, which is defined as the standard deviation divided by the mean of the variable. This statistic can be used to judge how much one variable reacts compared to another one.

While capital is more volatile than employment, the volatility of profit and the profit share is an order of magnitude larger than that of capital. Also investment and the investment share are very volatile, which is consistent with empirical evidence, as investment is the most volatile component of GDP.[14] We will use the coefficient of variation for comparing the variability of particular variables across different model parametrizations.

We ask how the volatility changes under different regimes of factor substitutability. We consider what happens when we increase σ from $-3/2$ to $-2/3$ and even further to $\sigma = 3/10$. The results are depicted in Table 6.

With increased factor substitutability the firm more flexibly reacts to stochastic fluctuations in aggregate productivity and primarily adjusts the factor which is less costly to vary. In our model, there is no adjustment friction on capital, so the firm

[14] See e.g. `https://fredblog.stlouisfed.org/2015/08/gdp-components-volatility/`

	$\sigma = -2/3$		$\sigma = 3/10$	
Variable	Mean	Coeff. of Var.	Mean	Coeff. of Var.
k	8.0045	0.0928	8.3277	0.2657
l	0.8972	0.0140	0.7667	0.0608
w	1.3196	0.0292	1.0675	0.0290
θ	0.7042	0.2763	0.1139	0.5277
i	0.2081	1.5195	0.2165	3.9201
v	0.0702	0.1809	0.0242	0.4174
y	1.9739	0.1426	1.3781	0.2297
π	0.3008	1.1406	0.2453	3.5039
profit share	0.1458	1.1427	0.1631	3.8283
labor share	0.6092	0.1082	0.6156	0.1481
investment share	0.1030	1.5340	0.1529	4.0536
hiring cost share	0.1418	0.0748	0.0678	0.2581

Table 6: Variability for Higher Values of σ

reacts more strongly in capital. The volatility of investment increases in the degree of factor substitutability. The variability of labor also rises, which can be seen when comparing the coefficient of variations across simulations. The variability, as measured by the coefficient of variation increases for all variables, rendering the economy more volatile. The mechanism behind this is as follows. As firms can now more easily substitute between capital and labor, the labor market frictions become less important, as the firms can now more effectively use two margins of adjustment. Suppose a negative shock hits the economy. If the production technology were Leontief, hiring less workers would entail a risk because it is difficult to adjust the workforce once productivity increases again. With greater factor substitutability, the firm posts less vacancies as it can react in capital if the economy recovers.

5 Empirical Evidence

We are now in a position to discuss how our model predictions compare to their real world counterparts. We do so mainly for illustrative purposes and as a plausibility check of our model. While our model replicates the empirically observed negative relationship between the profit share and the labor share it has difficulties explaining the behavior of investment. This is because we abstract from financing issues and corporate debt while focusing on the effects of factor substitutability on firms' profits and the labor market.

We take US time-series data on key economic variables and compare their statistical moments to their counterparts generated by our model. A central equation in all of our discussion is firms' profits defined as follows:

$$\begin{aligned}\pi_t =& y_t - w_t l_t - i_t p^k - v_t a \\ \frac{\pi_t}{y_t} =& 1 - \frac{w_t l_t}{y_t} - \frac{i_t p^k}{y_t} - \frac{v_t a}{y_t}\end{aligned}$$

Once we allow for errors ϵ_t that we assume to be normally distributed, we can estimate the following econometric model:

$$\frac{\pi_t}{y_t} = \alpha_0 + \alpha_1 \frac{w_t l_t}{y_t} + \alpha_2 \frac{i_t p^k}{y_t} + \alpha_3 v_t a + \epsilon_t \tag{4}$$

Most of our data originates from the FRED database.[15] We take aggregate time series of real GDP, non-financial corporate profits, investment and labor share directly from this database.[16] Each series comes at a quarterly frequency and covers the period from the first quarter of 1947 to the last quarter of 2016. We construct investment share and profit share by dividing the respective variables by contemporaneous GDP.

For vacancies we use an updated version of the data constructed by Barnichon (2010), which we downloaded directly from the author's website. The data are an index of open vacancies over the labor force and have been constructed from the "Help-Wanted-Index" which only relies on job openings printed in newspapers and the online Help-Wanted Index.[17]

We perform OLS regressions and present the estimation results in Table 7. The table displays the following specifications. In column (1) we estimate the regression model from equation (4). The coefficients of the labor share and the job openings each are negative. The coefficient of the investment share is significantly positive, which is unexpected given the definition of profits in our model, as there investment directly reduces profits. The coefficient of the investment share remains negative when we we use its first lag in column (2). This is done to control for potential lags between actual

[15] For a detailed description see Appendix D

[16] We take real GDP instead of non-financial value added, to enable comparision with our discussion on the dividend share in Appendix E

[17] As we do not have any data for the vacancy posting costs a, which we assume to be constant, the estimate of α_3 will actually be $\frac{\alpha_3}{a}$. However, we will also not divide vacancies by GDP, because normalizing the relatively constant index of vacancies by GDP would impose downward trends in this variable.

	(1)	(2)	(3)
Labor share	-0.635***	-0.640***	-0.108***
	(0.0304)	(0.0291)	(0.0222)
Investment share	0.261***		0.0250
	(0.0341)		(0.0146)
Job openings	-0.249***	-0.248***	-0.0506
	(0.0745)	(0.0745)	(0.0260)
L.Investment share		0.265***	
		(0.0335)	
L.Profit share			0.864***
			(0.0331)
Constant	41.33***	41.55***	7.205***
	(2.171)	(2.085)	(1.475)
Observations	264	263	263
Adjusted R^2	0.859	0.863	0.980

Notes: The dependent variable is profit share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7: Regression Results, 1947Q1-2016Q4

investment and the implied increase in revenue.

We detect autocorrelation in the residuals using the Breusch Godfrey-Test and therefore include the first lag of profit share in column (3). The coefficient of the investment share becomes insignificant, while the coefficients of labor share remains strongly negative and the hiring cost share barely fails to be significant at the 5% level.[18] This is in line with the predictions of our model. Investment share and profit share are empirically highly positively correlated, because of two reasons. Firstly, there is a discrepancy between the definition of profit in the model and that in the data. Profits in the model represent economic profits accruing through rents, while in the data corporate profits are defined as revenues minus costs. Investment expenditures do not constitute costs in this sense, because the firm still owns the capital and only the depreciation of capital lowers profits.[19] Secondly, and perhaps more importantly, our model assumes that firms' current period's retained earnings are uset to cover investment expenditures. This stands in sharp contrast to how firms in reality pay for their investments, which might include debt or additional equity. This is in line with the arguments made by Danthine and Donaldson (2002), who use the idea that wage payments enjoy seniority over dividend and other payment, which is why the labor share and profits are negatively correlated.

We can also compare the simulation results from our model to the correlations observed in the data. The results are presented in Table 8. This is a common exercise in the business cycle literature. When targeting first moments, second moments are used to determine the goodness of fit of a model. Keep in mind that our model was not primarily designed to explain the business cycle, but rather to study the effects of different degrees of input substitutability on profits and labor market variables.

When we compare the correlations over the full length of our time series we get a similar picture as in the regressions. Investment and profit share are positively related.

With the perturbation techniques applied in Dynare, we are also able to get an approximation of the theoretical correlations among the variables. We present the results in Table 9. Our simulation results are fairly close to their theoretical counterparts, so 100 simulations seem to be sufficient.

As discussed earlier, our theoretical model is not designed to match the business cycle, but to study how different degrees of input substitutability affect the profit of the firm. Nevertheless our model closely matches the correlation between the labor share

[18]It is, however, significantly negative if we use real GDP instead of non-financial real GDP.

[19]We control for this by using dividends as dependent variable in Appendix E. The positive correlation remains.

	Model $\sigma = -3/2$				US Data			
	Profit	Labor	Hiring	Investm.	Profit	Labor	Hiring	Investm.
Profit	1				1			
	(0)							
Labor	-0.1365	1			-0.2731	1		
	(0.0477)	(0)						
Hiring	-0.8016	-0.4786	1		0.4351	-0.0157	1	
	(0.0289)	(0.0055)	(0)					
Investm.	-0.912	-0.2807	0.9745	1	0.6040	-0.2725	0.7105	1
	(0.0119)	(0.0191)	(0.0044)	(0)				

Notes: The model has been simulated 100 times for the exact same number of periods as data points are available (264). All data are HP-filtered, with a smoothing factor of 1600.

Table 8: Correlations Between Various Shares

$\sigma = -1.5$	Profit	Labor	Hiring	Investm.
Profit	1			
Labor	-0.1442	1		
Hiring	-0.7999	-0.4761	1	
Investm.	-0.9108	-0.2770	0.9748	1

Table 9: Theoretical Correlations

and the investment share. These two variables are key elements of the firm's decision of their input mix. It also replicates a positive correlation between the hiring cost share and the investment share, although the correlation is higher than in the data. A reason for this may be lumpy investment, related to fix costs, which are not present in the model.

Because the data and our model use different definitions for profit, the discrepancies are little surprising. In reality, firms tend to invest and hire new employees in good times when profits are high. In our model, hiring more people will decrease contemporaneous profits, while the gains only materialize in the next period. In reality firms can use debt or issue new equity to finance investments, a possibility our model does not capture.

5.1 Sub-Periods

When inspecting the time series of profit shares presented in Figure 5 different regimes stand out. In the beginning the share is almost flat, until it picks up at the beginning of the 1970s. From 2000 onwards, we see strong variability in the rate. We subdivide the entire period according to these observations. The first period ranges from 1951 to 1970, where the start is determined by data availability and the end coincides with the end of the NBER recession in 1970. The second period lasts until the burst of the dotcom bubble in 2000, while the last sub-period ranges from 2001 to the end of 2016.

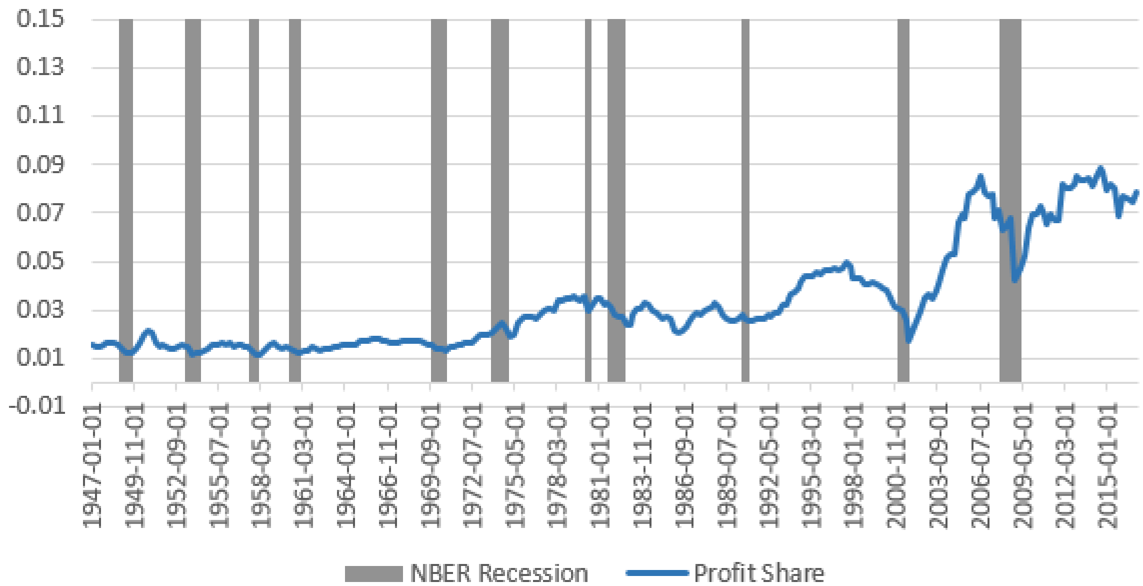

Figure 5: Non-Financial Profit Share in the US. *Source: FRED St. Louis*

All estimation results are reported in Appendix C. Each table relates to a specific sub-period. We briefly summarize the main findings below. The coefficient associated with the labor share remains consistently negatively correlated with profit share and even increases in magnitude. This means that the tradeoff between profit share and labor share becomes stronger over time. While the investment share has a significantly negative effect on profit share in the period 1971-2000, this effect turns positive in the period 2001-2016. The variable job openings is not significant when running regressions per period. We now use fewer observations for each regression, thus standard errors tend to be bigger.

A different way to control for changes in the underlying regimes is to use dummy variables. We therefore run a regression over the entire length of the sample and control for the different regimes with time period dummies. The results are presented in Table 10. We observe that the labor share has a significant and negative effect on the profit share, while the investments share is insignificant. The negative coefficient on the job openings is significant at the 10% level. The time dummies do not enter with a significant coefficient, indicating that the observed relationships are stable over the entire time period.

	(1)
L.Profit share	0.860***
	(0.0322)
Labor share	-0.103***
	(0.0249)
Investment share	0.0184
	(0.0179)
Job openings	-0.0388
	(0.0208)
Period1	0.0354
	(0.0492)
Period2	0.0997
	(0.130)
Constant	6.932***
	(1.624)
N	263
adj. R^2	0.980

Notes: See Table 7

Table 10: Regression over the Full Sample Period with Time Dummies

	Profit	Labor	Hiring	Investment
	1951Q1 - 1970Q4 (80 obs.)			
Profit	1			
Labor	-0.6285	1		
Hiring	0.5649	-0.1566	1	
Investment	0.8161	-0.5914	0.5154	1
	1971Q1- 2000Q4 (120 obs.)			
Profit	1			
Labor	-0.2550	1		
Hiring	0.5688	0.0026	1	
Investment	0.4732	-0.198	0.8027	1
	2001Q1-2016Q4 (64 obs)			
Profit	1			
Labor	-0.3476	1		
Hiring	0.6789	0.1798	1	
Investment	0.8022	-0.1161	0.827	1

Notes: All variables except for hiring are expressed relative to output.

Table 11: Empirical Correlations by Sub-Periods

To sum up, there is a clear negative relationship between the labor share and the profit share. This result is robust across alternative specifications and is consistent with the results generated by our theoretical model.

5.1.1 Correlations

In addition to performing a regression analysis, we can compare the correlations between the time series we observe in the data to their model counterparts. If we split up the time series into the three periods previously described, we get the correlation matrices observed in Table 11. The first correlation we look at is the one between profit share and labor share. It exhibits an inverted U-shape over time. While it is strongly negative in the beginning, it grows less negative in the second period, only to become negative again from 2000 onwards.

A similar pattern can be observed for our model. When the degree of substitutability increases, the correlation between the profit share and the labor share grows more strongly negative. This is because a higher wage bill lowers the profit of the firm, but then the firm can more easily rely on capital in output production. However, these results should be taken with a grain of salt, because the post 2000 sample period is relatively short and includes the Great Recession.

We also see that the correlation between the labor share and the investment share

	Profit	Labor	Hiring	Investment
Profit	1			
Labor	-0.226	1		
Hiring	-0.7963	-0.3863	1	
Investment	-0.923	-0.1662	0.9577	1

Notes: See Table 11.

Table 12: Theoretical Correlations for $\sigma = -2/3$

has turned less negative over time, which can be interpreted as evidence for skill-biased technological growth.[20] As firms invest more, the labor share does not decline by as much as it used to, because one still needs better qualified people with higher wages to handle the newly installed technologies.[21] Our model replicates the positive correlation between hiring and investment. This happens because of the complementarities between capital and labor. The correlations between these two empirical series increases over time, which is consistent with what happens in our model under increased substitutability. With higher substitutability, the firm chooses a more capital-intensive input mix, thus increasing the marginal product of an additional worker. After positive productivity shocks, it thus pays to hire more workers and increase profits.

6 Conclusions

We have developed a dynamic stochastic equilibrium model of a frictional labor market where firms search for suitable workers by posting job-vacancies and wages, and unemployed workers search for jobs. Firms use capital and labor for producing output with the help of a technology that exhibits a constant elasticity of substitution. Capital can be flexibly adjusted, but expanding labor is subject to search frictions. We have calibrated this model to the U.S. economy. We have used it to disentangle the role that a steady decline in the relative price of new capital goods or an increased factor substitutability play in explaining three ongoing trends that have prevailed in many industrialized countries: the rise in the capital-to-labor ratio; the steady rise in the level and the variability of firms' profit-to-output ratio; and the decline in the labor share of income.

Our quantitative results underline the importance of studying the two changes con-

[20]See Krusell et al. (2000)

[21]As different skill levels are beyond the scope of this paper, we will refrain from exploring these results in greater detail.

sidered separately. While each change can help explain the rising trends, only the rise in the factor substitutability generates the observed decline in the labor share. Hence, a possible interpretation of the facts through the lense of our model is that the implications of increased factor substitutability quantitatively outweigh those of a decline in the relative price of new capital goods.

Our model of firms using capital and labor for output production while operating in frictional labor markets is rich yet tractable enough to lend itself to various extensions so that it can help us study closely related issues in macro/labor, or labor/finance. The implicit assumption that firms use retained earnings to pay for investment renders a counterfactual negative correlation between investment expenditures and profit shares. A natural next step therefore could be to allow firms to take on debt, thereby choosing their capital structure and make this choice dependent on the structure of the labor market. When combined with firm heterogeneity, this framework can be the analytical basis for studying the cross-sectional implications for the level and variability of return on equity as examined by Shim (2015).

References

Acemoglu, D. and P. Restrepo (2017): "Robots and Jobs: Evidence from US Labor Markets," NBER Working Paper 23285, National Bureau of Economic Research, Inc.

Acemoglu, D. and R. Shimer (1999): "Holdups and Efficiency with Search Frictions," *International Economic Review*, 40, 827–849.

Arntz, M., T. Gregory, and U. Zierahn (2016): "The Risk of Automation for Jobs in OECD Countries: A Comparative Analysis," Tech. rep.

Autor, D., D. Dorn, L. F. Katz, C. Patterson, and J. V. Reenen (2017): "The Fall of the Labor Share and the Rise of Superstar Firms," NBER Working Paper 23396, National Bureau of Economic Research, Inc.

Barnichon, R. (2010): "Building a composite Help-Wanted Index," *Economics Letters*, 109, 175–178.

Blanchard, O. J. (1997): "The Medium Run," *Brookings Papers on Economic Activity*, 28, 89–158.

Blatter, M., S. Muehlemann, and S. Schenker (2012): "The Costs of Hiring Skilled Workers," *European Economic Review*, 56, 20–35.

Brynjolfsson, E. and A. M. Afee (2014): *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies*, W.W. Norton & Company.

Chirinko, R. S. (2008): "Sigma: The Long and Short of It," *Journal of Macroeconomics*, 30, 671–686.

Danthine, J.-P. and J. B. Donaldson (2002): "Labour Relations and Asset Returns," *Review of Economic Studies*, 69, 41–64.

Elsby, M., B. Hobijn, and A. Sahin (2013): "The Decline of the U.S. Labor Share," *Brookings Papers on Economic Activity, Fall.*

Gertler, M., C. Huckfeldt, and A. Trigari (2016): "Unemployment Fluctuations, Match Quality, and the Wage Cyclicality of New Hires," NBER Working Paper 22341, National Bureau of Economic Research, Inc.

Gordon, R. J. (1990): *The Measurment of Durable Goods Prices*, National Bureau of Economic Research.

Hawkins, W. B. (2013): "Competitive Search, Efficiency, And Multiworker Firms," *International Economic Review*, 54, 219–251.

Kaas, L. and P. Kircher (2015): "Efficient Firm Dynamics in a Frictional Labor Market," *American Economic Review*, 105, 3030–3060.

Kaldor, N. (1961): "Capital Accumulation and Economic Growth," in *The Theory of Capital*, ed. by F. Lutz and D. C. Hague, New York: St. Martin's Press.

Karabarbounis, L. and B. Neiman (2013): "The Global Decline of the Labor Share," NBER Working Papers 19136, National Bureau of Economic Research, Inc.

Klump, R., P. McAdam, and A. Willman (2012): "The Normalized CES Production Function: Theory And Empirics," *Journal of Economic Surveys*, 26, 769–799.

Krusell, P., L. E. Ohanian, J.-V. RÌos-Rull, and G. L. Violante (2000): "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis," *Econometrica*, 68, 1029–1054.

León-Ledesma, M. A., P. McAdam, and A. Willman (2010): "Identifying the Elasticity of Substitution with Biased Technical Change," *American Economic Review*, 100, 1330–1357.

Loecker, J. D. and J. Eeckhout (2017): "The Rise of Market Power and the Macroeconomic Implications," NBER Working Paper 23687, National Bureau of Economic Research, Inc.

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market-A Survey," *Journal of Economic Literature*.

Schaal, E. (2015): "Uncertainty and Unemployment," Tech. rep.

Shim, K. H. (2015): "The Substitutability of Labor: Implications for the Cross Section of Stock Returns," Tech. rep.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, 25–49.

A Recursive Wages

In a competitive search framework where firms post wage contracts, firms can decide to offer different wages in different periods. This can be caused by shocks, which will affect the optimal wage posted by the firm and can create a wage dispersion within a firm. To avoid keeping track of the entire wage distribution, we use the following recursive formula:

$$w_{t+1}l_{t+1} = w_t l_t(1-\nu) + \tilde{w}_t h_t$$

To show that this formulation is equivalent in terms of the total wage bill to keeping track of the entire wage history of wages posted by the firm, consider a firm in period t with l_t employees at a wage rate w_t. It hires h_t new employees at a wage rate $\tilde{w}_t$, while ν of the existing workforce leave the firm. For the firm it doesn't make a difference whether it pays a new wage rate w_{t+1} to all of its employees in period $t+1$, which are made up by $l_t(1-\nu)+h_t$ or whether it pays $(1-\nu)l_t$ of its employees a wage w_t and the other h_t receive $\tilde{w}_t$. As all earnings are pooled due to the big family assumption, also the household only cares about the total wage bill. We can now simply shift back the time index by one period, and are in the same situation as before, because w_t and l_t are state variables for the firm. We thus have shown that the recursive formulation of wages allows us to calculate the posted wages in a consistent way.

B An Alternative Formulation of the Firm's Problem

This is an alternative formulation of the problem, where all the laws of motion are written as constraints. It makes for a nice distinction between the choice variables of the firm in period t $(v_t, \theta_t, \tilde{w}_t, i_t)$, and the endogenous state variables in the next period. However, the resulting system of equations is more complicated, but eventually determines the same equilibrium.

$$
\begin{aligned}
\mathcal{L} = \max_{v_t,\theta_t,\tilde{w}_t,i_t} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t [& z_t y(k_t, l_t) - w_t l_t - i_t p^k - a v_t] \\
& + \lambda_1 \left[U_t - (1 - p(\theta_t))b - p(\theta_t)\tilde{w}_t\right] \\
& + \lambda_2 \left[l_{t+1} w_{t+1} - (1 - \nu) l_t w_t - v_t q(\theta_t)\tilde{w}_t\right] \\
& + \lambda_3 \left[l_{t+1} - (1 - \nu) l_t - v_t q(\theta_t)\right] \\
& + \lambda_4 \left[k_{t+1} - (1 - \delta) k_t - i_t\right]
\end{aligned}
$$

Differentiating with respect to the 4 choice variables and next period's endogenous state variables leads to the following nonlinear system of equations. As we have 4 Lagrange multipliers we denote their time indices by superscripts rather than subscripts.

$$
\begin{aligned}
\frac{\partial}{\partial v_t} :& - a - \lambda_2^t q(\theta_t)\tilde{w}_t - \lambda_3^t q(\theta_t) = 0 \\
\frac{\partial}{\partial \theta_t} :& \lambda_1^t [p'(\theta_t) b - p'(\theta_t)\tilde{w}_t] - \lambda_2^t v_t q'(\theta_t)\tilde{w}_t - \lambda_3^t v_t q'(\theta_t) = 0 \\
\frac{\partial}{\partial \tilde{w}_t} :& - \lambda_1^t p(\theta_t) - \lambda_2^t v_t q(\theta_t) = 0 \\
\frac{\partial}{\partial i_t} :& - p^k - \lambda_4^t = 0 \\
\frac{\partial}{\partial w_{t+1}} :& \lambda_2^t l_{t+1} + \beta[-l_{t+1} - \lambda_2^{t+1}(1 - \nu) l_{t+1}] = 0 \\
\frac{\partial}{\partial l_{t+1}} :& \lambda_2^t w_{t+1} + \lambda_3 + \beta \left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial l_{t+1}} - w_{t+1} - \lambda_2^{t+1}(1 - \nu) w_{t+1} - \lambda_3^{t+1}(1 - \nu)\right] = 0 \\
\frac{\partial}{\partial k_{t+1}} :& \lambda_4^t + \beta \left[\frac{\partial y(k_{t+1}, l_{t+1}, z_{t+1})}{\partial k_{t+1}} - \lambda_4^{t+1}(1 - \delta)\right] = 0
\end{aligned}
$$

The equilibrium conditions are the same, although there are 4 Lagrange multipliers, where only λ_4 can be substituted. The other have co-dependencies, which is why we decided to present the other formulation in the main part of the paper.

	(1)	(2)	(3)
Labor share	-0.0314**	-0.0558***	-0.0308**
	(0.0108)	(0.00942)	(0.0101)
Investment share	0.122***		0.0875***
	(0.0148)		(0.0207)
Job openings	0.0304*	0.0409*	0.0151
	(0.0126)	(0.0159)	(0.0139)
L.Investment share		0.0750***	
		(0.0152)	
L.Profit share			0.247*
			(0.119)
Constant	2.265**	4.418***	2.252**
	(0.827)	(0.674)	(0.798)
N	80	79	79
Adj. R^2	0.815	0.745	0.841

Notes: The dependent variable is profit share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 13: Regressions 1951Q1-1970Q4

C Estimation Results by Period

We now present a more detailed of the empirical analysis in each sub-period, which we already described in the main text. Each regression table is structured in the following manner. In column (1) we estimate the regression model described in equation (4). We see as we expect that the coefficients of labor share and hiring cost share enter with a negative coefficient. The coefficient of investment share is significantly positive, which is surprising and this result remains when we include lagged investment in (2). We detect autocorrelation in the residuals using the Breusch Godfrey-Test and therefore include the first Lag of profitshare in (3). We see that the coefficient of investment share changes signs in the middle period, consistent with our model predictions. However, this change is reversed in the post-2000 period.

	(1)	(2)	(3)
Labor share	-0.394***	-0.461***	-0.0591*
	(0.0662)	(0.0604)	(0.0231)
Investment share	0.423***		-0.0390*
	(0.0376)		(0.0172)
Job openings	-0.481***	-0.465***	0.0366
	(0.0933)	(0.0906)	(0.0297)
L.Investment share		0.413***	
		(0.0360)	
L.Profit share			0.980***
			(0.0294)
constant	24.83***	29.30***	4.377**
	(4.409)	(3.940)	(1.495)
N	120	119	119
adj. R^2	0.638	0.639	0.970

Notes: See Table 13

Table 14: Regressions 1971Q1-2000Q4

	(1)	(2)	(3)
Labor share	-1.184***	-1.263***	-0.540***
	(0.0682)	(0.0744)	(0.149)
Investment share	0.469***		0.260*
	(0.113)		(0.116)
Job openings	0.710**	0.568	-0.0425
	(0.253)	(0.298)	(0.209)
L.Investment share		0.514***	
		(0.135)	
L.Profit share			0.596***
			(0.0955)
constant	71.62***	76.01***	32.43***
	(3.599)	(3.719)	(8.300)
N	64	63	63
adj. R^2	0.852	0.841	0.926

Notes: See Table 13

Table 15: Regressions 2001Q1-2016Q4

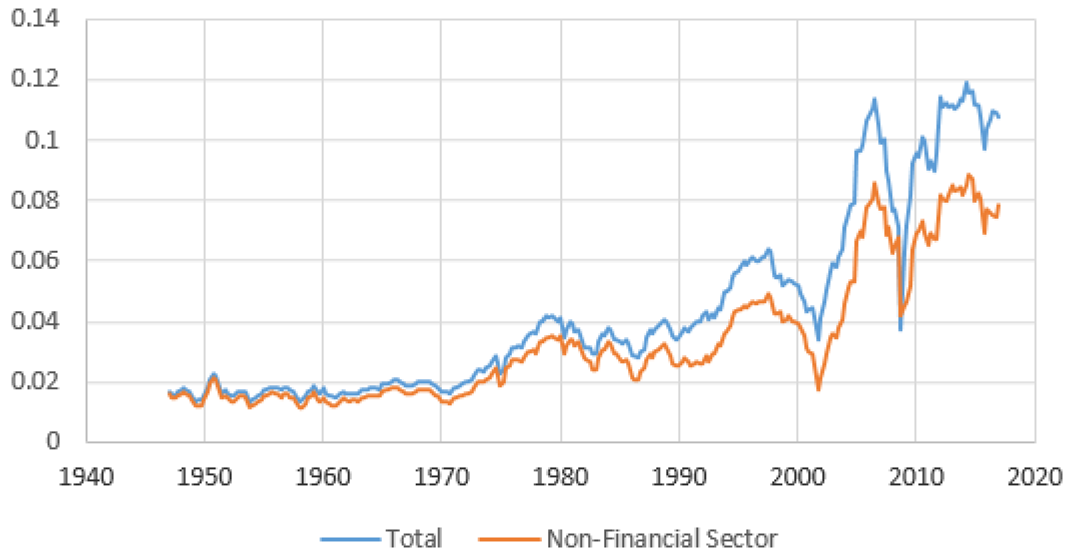

Figure 6: Profit Shares in the US

D Data Appendix

The data we use are of quarterly frequency. They relate to the United States and cover the period from 1951Q1 to 2016Q4. All data were downloaded from the FRED database unless noted otherwise.[22]

D.1 Output and Profit

We use the *Real Gross Domestic Product* in Billions of Dollar, which is seasonally adjusted and has 2009 as base year for chaining.

For profit, we take non-financial corporate profit, which is seasonally adjusted. The financial sector was excluded because we analyze a real model and therefore have no role for a financial sector. However, we also performed the empirical analysis with the entire corporate profit time series and the results are virtually unchanged. To illustrate this, we plot the resulting profit shares in figure 6.

The two series track each other quite closely but start to diverge around 1971. At this time the difference increases, meaning that the financial sector has become relatively more profitable. An interesting observation is the last quarter of 2008. In this quarter, the financial sector in total was making negative profits, thus the total

[22]FRED, Federal Reserve Bank of St. Louis; `https://fred.stlouisfed.org/series`

profit in the US was below the non-financial profit.

D.2 Investment, Price of Capital and Labor Share

As investment we use *Real Gross Private Domestic Investment*, which is seasonally adjusted and chained in 2009.

The price of capital which is displayed in Figure 2 is calculated by dividing the investment deflator by the consumption deflator. This is precisely the definition of the price of capital in our model and the rate at which output goods can be transformed into capital.

The labor share is constructed by normalizing the index of the non-financial corporate sector to its 2009 value of 60%.

D.3 Job Vacancies

For this time series we rely on the work by Barnichon (2010), who carefully combines the traditional Help-Wanted-Index taken from print version of newspapers with the Job Openings and Labor Turnover Survey (JOLTS), which is available from 2000 onwards. The author publishes updates on his website.[23] The data are available at a monthly frequency from 1951 to 2016. We aggregate them to a quarterly frequency using the mean. In this way, we obtain a time series which is consistent over a long time horizon.

D.4 Capital and Labor

When comparing capital intensities, we are restricted to using yearly data due to the availability of data on the US capital stock.

We use data on the capital stock at constant national prices. For employment we use two distinctive variables. One is the hours worked by full-time and part-time employees and the other one is the employees who are on a non-farm payroll.

When calculating the capital labor ratio, i.e. the capital intensity of production, we get two different series because we use different denominators. However both series are increasing in the period from 1950 to 2014, as can be seen in Figure 7. It depicts the ratios of capital to the number of workers, and the one to total hours worked, respectively. Both ratios are steadily increasing during the period of observation.

[23] `https://sites.google.com/site/regisbarnichon/data`

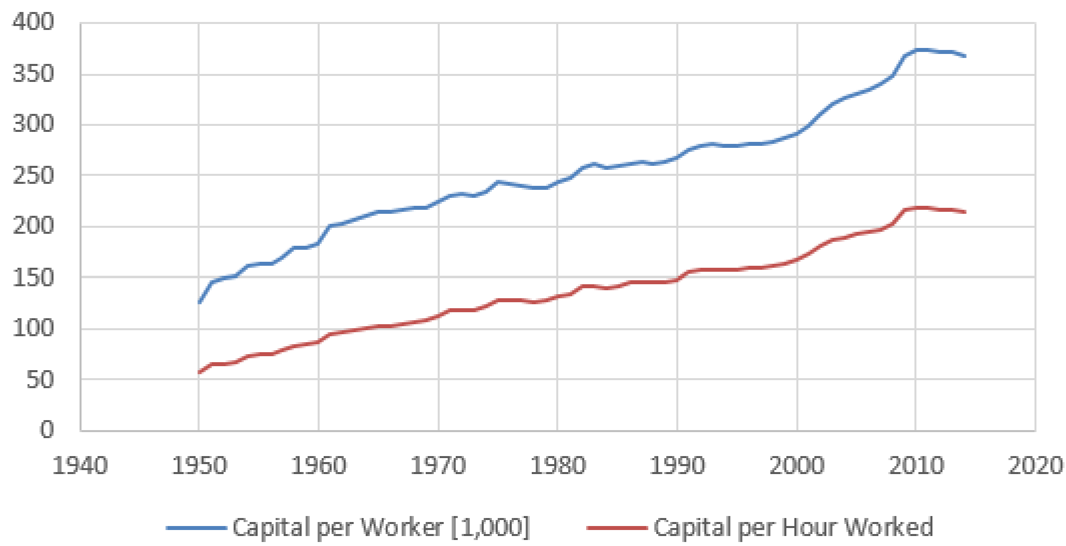

Figure 7: Capital Intensities in the US

E Using Dividends and Corporate Profits

As discussed in the main text, one important distinction between our definition of profits and the corporate profits we observe in the data is the treatment of investment. Investment reduces profits in our model but does not affect corporate profits which are defined according to legal accounting standards. We try to tackle this issue in two ways. First, we perform the regression analysis with dividend share as dependent variable instead of profit share. Second, we define a variable corporate profit in our theoretical model, which is the sum of profit and investment and compute its correlation with the other variables.

E.1 Regressions on Dividend Share

We construct the dividend share by using FRED data on dividends and divide it by GDP. We then run regressions for the full sample and per period, corresponding to the regressions in the text.

The main changes in the full sample regressions, presented in Table 16 are in the orders of magnitude. Now the number of job openings also enters with a significant negative sign, which arguably points to the fact that new hires are financed by current revenues, thus reducing profits.

	(1)	(2)	(3)
Labor share	-0.00404***	-0.00403***	-0.000253**
	(0.000277)	(0.000247)	(0.0000821)
Investment share	0.00353***		0.000329*
	(0.000222)		(0.000136)
Job openings	-0.00566***	-0.00574***	-0.000447**
	(0.000545)	(0.000517)	(0.000156)
L.Investment share		0.00370***	
		(0.000206)	
L.divshare			0.935***
			(0.0255)
Constant	0.247***	0.244***	0.0144*
	(0.0194)	(0.0171)	(0.00592)
Observations	264	263	263
Adjusted R^2	0.861	0.880	0.987

Notes: The dependent variable is dividend share. L. denotes the first lag of a variable. Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 16: Dividend Shares

	(1)	(2)	(3)
Labor share	0.00000207	-0.0000610	-0.00106***
	(0.0000128)	(0.0000419)	(0.000297)
Investment share	0.0000152	-0.000143**	0.000553
	(0.0000256)	(0.0000459)	(0.000523)
Job openings	-0.0000160	0.000233***	0.00163
	(0.0000242)	(0.0000601)	(0.00196)
L.dividend Share	0.980***	1.027***	0.770***
	(0.0288)	(0.00626)	(0.0949)
Constant	-0.000167	0.00507	0.0624**
	(0.00104)	(0.00304)	(0.0190)
Period	1951Q1-1970Q4	1971Q1-2000Q4	2001Q1-2016Q4
Observations	79	119	63
Adjusted R^2	0.966	0.999	0.873

Notes: See Table 16

Table 17: Regressions on Dividend Share by Sub-Period

E.1.1 Splitting up the Periods

This exercise corresponds to the one presented in Appendix C, where we divide our sample into 3 sub-periods, with dividend share as dependent variable. We will only report the model including one lag in the dividend share, due to autocorrelation in the other variants of the regression model.

For the period 1951Q1-1970Q4 we see that the only significant variable is lagged dividend share, which suggests that dividends in that time were not very volatile and are best explained by an AR-(1) process. In the middle period, the coefficient on labor share is not significant, but investment enters with a negative coefficient. Although this is in line with the predictions of our model, this result disappears again in the period from 2000 onwards, when the coefficient on labor share turns significantly negative. Overall, no clear picture emerges when looking at dividends as a proxy for economic rents, as they appear in our model. Our data series spans a long time period, and it is likely that corporate governance changes with respect to dividends have appeared over time.

	Profit	Labor	Hiring	Investment	Corp. Profit
Profit	1	0	0	0	0
Labor	-0.1288	1	0	0	0
Hiring	-0.8052	-0.4793	1	0	0
Investment	-0.9137	-0.2834	0.9748	1	0
Corp. Profit	0.3914	-0.9623	0.2239	0.0146	1

Notes: Approximated correlation of the model, including corporate profits.
All data are HP-filtered, with a smoothing factor of 1600.

Table 18: Correlations

E.2 Correlations of Corporate Profits

A different way to bridge the differences in the definition of profit between our model and the data is to define a variable *Corp. Profit*, which is revenue minus wage payments and hiring costs, and calculate its share. We present the obtained correlations in Table 18 The strong negative correlation between investment share and profit share in our model renders this variable almost acyclical to investment. Qualitatively, correlations now are the same as what we report in Table 8 for the US economy, as all signs are correct. Quantitatively, there are still discrepancies, due to our model abstracting from the financing decisions of firms.