A Model of Social Networks and Migration Decisions*

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Abstract

This paper is motivated by rich empirical evidence regarding the importance of social contacts abroad for the decision making of potential migrants. It aims at providing a microfoundation for the well-established hypothesis in the chain migration literature that attributes much of the impact of social contacts to their role as information providers about jobs in the foreign labor market. The proposed theoretical model has two countries, a home country and a destination country, and agents in both countries share information about job opportunities through an explicitly modelled network of social relations. The information about jobs affects employment prospects and hence, expected income of the agents in each country. This in turn determines the decisions of the home country residents on whether to migrate or to stay, which in the model is represented by an outcome of the migration decision game. I study the effects of the social network on equilibrium migration decisions and in particular, the effects of a small change in the number of social contacts in the destination country on the size of the migration flow. Using the results of analytical work and numerical simulations, I find that even a marginal increase in the number of links between countries may lead to a substantial increase in migration.

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1 Introduction

In this paper I aim to contribute to an important class of migration literature, the so-called chain migration literature. The central finding of the chain migration literature is the existence of a strong influence of previous migration from a certain geographic location on further migration from that location.\(^1\) According to one of the main hypotheses in the chain migration literature, much of the impact of previous migrants on migration decisions of new potential migrants can be attributed to their role as information providers about jobs in the foreign country. In this paper I offer a microfoundation for this hypothesis.

One remarkable phenomenon that motivates this and many previous studies of chain migration is the tendency of same-ethnicity immigrants to locate within the same geographically defined area(s) in the destination country. Such geographical concentration leads to the formation of so-called ethnic enclaves, ethnic neighborhoods in the host country with characteristic cultural identity and economic activity.\(^2\) Numerous examples include China towns in New York, the large overseas community of Poles in Chicago, Little Havana and Spanish barrios in different cities across the U.S., Little Odessa and Ukrainian Village in New York and Chicago, the Japanese community in the district of Liberdade in Sao Paulo, Turkish enclaves such as Marxloh in Duisburg and Kreuzberg in Berlin, and a range of smaller ethnic groups such as the Icelandic community in the town of Gimly in Canada, Greektown in Toronto, Little India in Bangkok, Little Finland in Thunder Bay, Ontario, Chinese villages on Caribbean islands and many others.

Even more suggestive is the case of 19th century Italian emigration, that for concreteness I will use as the main motivating example of the paper.\(^3\) Originally emigrants from Italy revealed strong preferences for countries of South America, especially Argentina and Brazil. As emigration from Italy grew, the prominence of the United States as a destination increased. Most of this shift occurred due to emigration from the southern part of Italy where emigration traditions to South America were weakest. However, despite the substantial wage gaps favoring North America over South America and despite the higher level of urbanization in North America, urban Northern Italians continued migrating to rural South America.\(^4\) Over time this resulted in an essential


\(^{2}\) See, for example, Abrahamson (1996), Portes and Jensen (1992), and MacDonald and MacDonald (1964).

\(^{3}\) This case is particularly remarkable since having taken place in the era of “free migration”, it is exempt from the influence of family reunification laws and policies, which to a certain extent account for contemporary migration movements.

\(^{4}\) Baily (1983) describes that "only 25 percent of those who migrated to the United States and 40 percent of those who migrated to Argentina were listed as skilled or white-collar workers" (p.295). Further, Italians from the north were "in general not only more skilled and literate than Italians from the south but also more familiar with organizations
difference in Italian regional emigration; the Southern Italians migrated in large numbers to the United States, while the Northern Italians predominantly moved to South America.\footnote{This apparent anomaly has attracted the attention of historians and migration economists. See, for example, Baily (1983), Klein (1983), Hatton and Williamson (1998).}

The existence of ethnic enclaves and the pattern of 19th century Italian emigration suggest the value of friends and family ties in promoting migration. Chain migration literature confirms this fact and proposes several hypotheses regarding the channels through which previous migrants influence choices of potential migrants. Among those it distinguishes job information provision and assistance as a factor of major importance.\footnote{In more detail the methodology, hypotheses and findings of the chain migration literature are discussed in section ??}. Empirical evidence supports this hypothesis with the findings of high rates of employment among new migrants via assistance of their relatives and friends. For example, Massey and Espinosa (1997) show in a survey of Mexican immigrants to the United States, that almost forty percent of migrants received jobs through informal networks of friends. Even larger proportion – about three-fourths of migrants – obtained jobs via relatives and friends according to Gilani et al. (1981), in his study of Pakistani migration to the Middle East.\footnote{Similar results are found for immigrants from Bangladesh and India to the Middle East (over 63 percent according to Mahmood (1991) and Nair (1991)), from Pakistan and Cyprus to Britain (Joly (1987) and Josephides and Rex (1987)), from Portugal to France (Hily and Poinard (1987)), and from Turkey to Germany (Wilpert (1988)).}

In this paper I aim to contribute to the chain migration literature by offering a microfoundation for the job information provision hypothesis. I seek to formally model how the job information from previous migrants transforms into migration decisions of would-be migrants, so as to examine the effect of (even marginal) changes in the information transmission on migration flows. Despite the stated importance of job information factor in shaping migration decisions and despite the substantial empirical evidence supporting that fact, this seems to be the first theoretical model that explicitly accounts for the role of previous migrants as job information providers in determining migration choices of their friends and relatives back home.

In earlier theoretical work, the paper that is closest in its motivation and results to the considered framework is Carrington et al. (1996). They study a competitive equilibrium model of labor migration and show that when moving costs decrease with the number of migrants already settled in the destination country, migration flows may increase even as wage differentials between countries narrow. Despite the fact that the content message of the two papers is similar, the key point of this paper is very different. While in Carrington et al. (1996) the cost of moving is assumed to be decreasing with the stock of previous migrants (and this is the central assumption of the model), in this paper I specifically focus on studying why and how it is decreasing, with the only presumption that this happens due to the job information provision by the previous migrants.
To study the role of individual social links, the structure of social connections between people is defined formally and explicitly. Residents of the home and destination country use social network to share information about employment opportunities and influence each others’ expected income and hence, migration choices of the home country residents (potential migrants) by either providing job information or competing for it. The resulting decisions to migrate or to stay are then determined as a Nash equilibrium outcome of the game between potential migrants. I study the dependence of this outcome on the specific structure of links between potential migrants and residents of the foreign country.

The advantages of modelling job information exchange via social network are twofold. First, in compliance with the main goal of the paper, the explicit design of the network allows me to study the impact on migration decisions of just a marginal change in the structure of information exchange. For example, a change in just one link between a potential migrant and his contact in the destination country, may have an impact not only on his own migration decisions but also indirectly – on migration decisions of his friends since their employment prospects at home and abroad are affected by the decision of that one potential migrant. Second, social network approach seems but natural as it merely establishes a “missing link” between two empirically and theoretically ascertained facts. The first fact is the relevance of social network structure for individual employment prospects. The second is the key influence of employment prospects on emigration rates.

Regarding the first fact, a range of studies for a variety of occupations, skill levels and socioeconomic backgrounds document that 30 to 60 percent of jobs are found through friends or relatives. Moreover, Burt (1992, 2001) and Coleman (1990) argue that not just the number of friends or the strength of friendship ties matter for employment and career prospects but also other network characteristics such as closure and presence of structural holes. Furthermore, recent theoretical works by Calvó-Armengol and Jackson (2004, 2007) and Calvó-Armengol (2004) establish that agents’ employment and wages over time depend on their position in the social network, how many social ties they have, and how well-employed those social ties are. Eventually this results in correlation patterns in wages and employment of connected agents, and these patterns depend on the network configuration. In a different setting, Calvó-Armengol and Zenou (2005) also show that the size of the social network is important for employment rates. Regarding the second fact, the strong influence of employment prospects, defined in terms of earnings and employment rates, on migra-

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8See Bewley (1999), Corcoran et al. (1980), Granovetter (1973, 1995) and Rees (1966)).
9Quantitatively these statements are tested in Burt (1992), Podolny and Baron (1997), Gabbay (1997) and others.
10They introduce an aggregate job matching function in the analysis and find that when the network size increases, on average, the unemployed agents hear about more vacancies but, at a certain point, multiple vacancies are likely to reach the same unemployed agent and the further expansion of the network decreases the job matching rate.
tion decisions constitutes the core statement of traditional migration literature. It is confirmed by both theoretical and empirical findings. The two ascertained facts together imply that indirectly social network structure must have an important effect on migration decisions. And in fact, this is supported by findings of the paper: even small changes in the structure of social connections may induce changes in migration flows.

The network of social connections between people is modelled as a set of nodes that represent agents and a set of links that indicate the existence of social relation between the linked agents. Links serve for transmission of information about jobs and the transmission of information is modelled as in Calvó-Armengol and Jackson (2004). As a result, links predetermine employment prospects and expected earnings of each agent. Agents are residents of either home or destination country/region.

The migration decisions of the home country residents are modelled as an outcome of a one-shot simultaneous move game. The home country residents (players) maximize their lifetime expected income (payoff) and in the beginning of the first time period chose one of two strategies: either migrate to the destination country or stay at home. This game is similar to the drop-out decision game described in Calvó-Armengol and Jackson (2004) where labor market participants decide whether to stay in the work force or drop out. However, the two important distinctions between the games are in order. First, in Calvó-Armengol and Jackson (2004) a value of the outside option (to staying) is normalized to 0, whereas in this paper, the outside option is a labor market of the other country. The value of this option is the expected stream of income in that country, which is firstly, not 0 and secondly, affected by the decisions of other potential migrants with whom a given player is directly or indirectly linked. Second, and more importantly, Calvó-Armengol and Jackson (2004) assume that the network structure of the labor market is not altered when an agent drops out; the dropout’s employment status is simply set to be zero forever but his social contacts continue to pass on information about jobs. Instead, in this model, migration actually alters the structure of the labor market in both countries. When someone migrates, he is de facto removed from the set of home country residents (added to the set of destination country residents) and cannot exchange job information with those who decided to stay. This results in a different dynamics of employment in two models, with the outcome that straightforward analytical solution of the migration decision game appears feasible only for the case of complete networks, while Calvó-Armengol and Jackson (2004) derive a generic result.

To study the equilibrium of the game, I first show that for any configuration of the social network, there exists a unique stationary distribution of agents’ employment states in every country.

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Moreover, the stationary probability of employment of an agent is increasing in the number of his social connections in the country. Therefore, at least in case of a complete network, where all agents are connected with each other, the long-run probability of employment in the home country is decreasing with migration, whereas the probability of employment in the destination country is increasing with migration. As a result, when agents assign substantial weight to their long-run expected earnings, their migration decisions are strategic complements. The theory of the games in strategic complements then suggests that there exists a unique maximal Nash equilibrium in pure strategies.\textsuperscript{12} In this paper, the result on strategic complementarity of players’ decisions and the existence of a maximal Nash equilibrium is proved analytically for the case of a complete network. Then the numerical work supplements the analysis by addressing other network topologies.

The main finding of the paper is that the marginal change in the number of social connections between the home and the destination country/region may cause large variation in agents’ migration decisions. So, the substantially higher migration to North America from the south of Italy than from the north of Italy could indeed have resulted from minor superiority in the initial number of North American connections of Italians from the south. Specifically, for the case when the network of social relations is complete and agents primarily care about their long-run expected income, I find that there exists a threshold value for the number of social links to the destination country (North America), such that no one migrates in the maximal equilibrium from a region or a social group in the home country where the actual number of these links is below the threshold (northern Italy), but everyone migrates from a region or a social group where the number of these links is above the threshold (southern Italy). This conclusion is also supported by the results of numerical simulations for the case of other network structures: the higher the number of agents’ contacts in the destination country, the higher the equilibrium migration flow, and even a marginal increase in the number of these contacts may lead to a substantial increase in migration.

The remainder of the paper is organized as follows. Section ?? discusses the main findings and hypotheses of chain migration literature. The model describing the economy, evolution of employment, and migration decision game is presented in section ?? . The solution of the game and its dependence on the structure of links in the network are demonstrated in section ?? . There I first describe the results of analytical work for the case when the network is complete and agents assign a high weight to their long-run expected earnings. Then I present the results of numerical simulations for arbitrary network topologies. Finally, section ?? concludes.

\textsuperscript{12}By definition in Calvó-Armengol and Jackson (2004), a maximal equilibrium is such that the set of players staying at home is maximal meaning that the set of players staying at home in any other equilibrium of the game is a subset of those staying in at the maximal equilibrium.
2 Chain Migration Literature: Findings and Hypotheses

Chain migration literature documents the strong effect of the previous migrations on continuation of migration flow. For example, Hatton and Williamson (1998) report that as many as 90 percent of immigrant arrivals to the US at the turn of the 19th century were travelling to meet a friend or relative who had previously emigrated. Also, their estimation results suggest that for each thousand of previous emigrants a further 20 were "pulled" abroad each year. Other chain migration studies include Munshi (2003), Drever and Hoffmeister (2008), Faist (2000), Gurak and Caces (1992), Dahya (1973), Dunlevy (1991), Jasso and Rosenzweig (1986), Massey and Garcia (1987) and Levy and Wadycki (1973).

To demonstrate the relevance of past migrations, chain migration literature mostly uses a *lagged emigration rate* or a *migrant stock* as an explanatory variable, the latter pertaining to the cumulative number of all previous migrants from a source region. The estimated positive effect on the probability of migration or the size of current migration flow is then attributed to various forms of assistance that previous migrants provide to newcomers.

The commonly discussed forms of assistance can be formulated as three hypotheses regarding the influence of past migrants on the decisions of new migrants. The first hypothesis is information provision. Previous migrants often provide their friends with information about jobs, housing, health care and social services, customs and practices of the foreign culture, etc. (Grossman (1989), Hansen (1940), Gottlieb (1987), Aguilera (2005), Drever and Hoffmeister (2008), Lin (1999)). The second hypothesis is financial assistance. Previous migrants can offer food, temporary lodging, credit and even money to finance moves (Scott (1920), Grossman (1989), Munshi (2003), Granovetter (1995)). Finally, the third hypothesis is social and psychological help, such as access to ethnic goods and recreation and emotional support (Massey et al.(1987), Marks (1989), Mazzucato (2009), Menjívar (2002)). All these ways of assisting new migrants lower the costs and risks of movement and increase the expected net returns to migration.

In this paper, I focus on the first, arguably most relevant, information provision hypothesis and develop a formal model to study how the job-search assistance of previous migrants affects migration decisions of their acquaintances at home.

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13For a survey of international migration theory and empirical findings on migrant networks see Massey et al.(1993) and Boyd (1989). More recent trends, topics and methods in migration literature are discussed in Gurgand et al. (2012).
3 The Model

3.1 Economy

There are two countries/regions: the home country/region (H) and the destination country/region (D). The economy in (H) and (D) is infinite horizon in discrete time, \( t \in \{1, 2, \ldots \} \).

In the beginning of period 1, some residents of (H) migrate to (D). An agent in (H) migrates to (D) as soon as his expected life-time income in (D), net of the sunk moving cost \( c \), is higher than his expected life-time income in (H). Having decided to migrate, the agent leaves (H) immediately, before any other event has taken place, and becomes initially unemployed resident of (D). The migration decision is made just once and no reentry is allowed for agents choosing to migrate.\(^{14}\) Two ideas are implicit in the latter condition. First, the majority of costs of staying in the labor market of a particular country (education, acquiring labor market specific skills and opportunity) are usually borne at an early stage of an agent’s career and are sunk, so that there is little incentive to change the migration decision. Secondly, costs of moving are also sunk and usually high, which prevents the return migration. Furthermore, all native residents of (D) stay in their country permanently; they do not make any migration decisions.\(^{15}\)

Let \( H \) be the set of all initial residents in (H), \( D \) be the set of all initial residents in (D) and \( \mathcal{M} \) be the set of migrants. Let \( a_H = |H|, a_D = |D|, \) and \( m = |\mathcal{M}| \). So, after migration the total number of residents in (H) is \( a_H - m \) and the total number of residents in (D) is \( a_D + m \).

For convenience of exposition, I assume that residents of both countries are assigned a number so that \( H = \{1, \ldots, a_H\} \) and \( D = \{a_H + 1, \ldots, a_H + a_D\} \). Besides, to address residents of (H) and (D) after migration I define two bijections

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\alpha : H \setminus \mathcal{M} \to \{1, \ldots, a_H - m\}, \\
\gamma : D \cup \mathcal{M} \to \{1, \ldots, a_D + m\},
\]

such that \( \alpha \) associates a certain number \( \alpha(i) \in \{1, \ldots, a_H - m\} \) with each agent \( i \) who is a permanent resident of (H) and \( \gamma \) associates a certain number \( \gamma(j) \in \{1, \ldots, a_D + m\} \) with each agent \( j \) who is either a migrant from (H) or an initial resident of (D).

Agents in (H) and (D) are connected by the network of social relations, \( G \). Network \( G \) is undirected and for any \( i \neq j \), \( G_{ij} = 1 \) indicates that agents \( i \) and \( j \) know each other, and \( G_{ij} = 0 \) indicates that they do not know each other. Also, by convention, \( G_{ii} = 0 \) for any agent \( i \). I assume that network \( G \) is given exogenously and its structure does not change over time.

\(^{14}\)Similar assumptions are made for the drop-out decision game in Calvó-Armengol and Jackson (2004).

\(^{15}\)The precise description of the migration process is presented in section ??.
All residents of (H) and (D) are labor market participants. Jobs within each country are identical with the wage rate $w_H$ per period in country (H) and the wage rate $w_D$ per period in country (D). Both wages are exogenous, constant over time and $w_D > w_H$. Unemployed agents in each country earn nothing and receive no unemployment benefits.

For any period $t \geq 1$, the end-of-period $t$ employment state of the labor market in (H) and (D) is characterized by vectors $h_t$ and $d_t$, respectively. If agent $i$ from (H) (or (D)) is employed at the end of period $t$, $h_t(\alpha(i)) = 1$ ($d_t(\gamma(i)) = 1$), otherwise $h_t(\alpha(i)) = 0$ ($d_t(\gamma(i)) = 0$). Thus, at any $t \geq 1$ $h_t$ is a 0-1 vector of length $a_H - m$, $h_t \in \{0,1\}^{a_H-m}$, and $d_t$ is a 0-1 vector of length $a_D + m$, $d_t \in \{0,1\}^{a_D+m}$.

Furthermore, let vectors $h_0$, $d_0$ characterize the initial employment state of the labor market in (H) and (D) or equivalently, the initial distribution of employment among agents in the two countries. Precisely, let $h_0$ be a vector of starting employment statuses in (H) of agents from the set $H$ of initial (H) residents and $d_0$ be a vector of starting employment statuses in (D) of agents from the set $D \cup H$ of initial (D) and (H) residents. For the purposes of further analysis, the initial employment statuses in (D) of agents from $H$ are set to unemployment, the status which these agents would have in the beginning of period 1 if they migrated to (D). Just like vectors $h_t$, $d_t$ for $t \geq 1$, vectors $h_0$, $d_0$ are composed of 0’s and 1’s, where 1 indicates initial employment and 0 indicates initial unemployment; $h_0 \in \{0,1\}^{a_H}$ and $d_0 \in \{0,1\}^{a_H+a_D}$.

In the following, I denote by $E_H$ all initially employed and by $U_H$ all initially unemployed in (H) agents from $H$. Similarly, $E_D$ denotes initially employed and $U_D$ – initially unemployed in (D) agents from $D$. Formally,

$$E_H = \{i \in H \text{ s.t. } h_0(i) = 1\}, \quad U_H = \{i \in H \text{ s.t. } h_0(i) = 0\},
$$

$$E_D = \{i \in D \text{ s.t. } d_0(i) = 1\}, \quad U_D = \{i \in D \text{ s.t. } d_0(i) = 0\}.$$


Furthermore, let $E_H^i$ and $U_H^i$ denote the sets of direct initially employed and unemployed in (H) contacts of agent $i$ from $H$. Likewise, $E_D^i$ and $U_D^i$ denote the sets of direct initially employed and unemployed in (D) contacts of agent $i$ from $D$.

$$E_H^i = \{j \in H \text{ s.t. } G_{ij} = 1, h_0(j) = 1\}, \quad U_H^i = \{j \in H \text{ s.t. } G_{ij} = 1, h_0(j) = 0\},
$$

$$E_D^i = \{j \in D \text{ s.t. } G_{ij} = 1, d_0(j) = 1\}, \quad U_D^i = \{j \in D \text{ s.t. } G_{ij} = 1, d_0(j) = 0\}.$$

Let $e_H^i = |E_H^i|$, $e_D^i = |E_D^i|$, $u_H^i = |U_H^i|$, and $u_D^i = |U_D^i|$.

Below I describe the transmission of job information and dynamics of employment in the network.

They are modelled as in Calvó-Armengol and Jackson (2004).
3.2 Timing of events

Each period $t \geq 1$ starts with some agents being employed and others unemployed as described by the employment states $h_{t-1}, d_{t-1}$ of the previous period. Then the timing of events in period $t$ is the following. First, residents of both, (H) and (D) obtain information about new job openings in their own country. Any agent in (H) and (D) hears of a new job opening directly with a probability $p_a \in (0, 1)$. This job arrival process is independent across agents. If the agent is unemployed, he accepts the job offer. If the agent is already employed, he passes the offer along to one of the direct unemployed contacts in the same country. This is where the network structure of relationships between people becomes important: it determines the job information exchange process and ultimately affects long-run employment prospects of any agent.

Finally, the last event that happens in a period is that some employed agents lose their jobs. This happens randomly with a breakup probability $p_b \in (0, 1)$ which is also independent across all agents in (H) and (D).

Admittedly, the condition that job arrival and breakup rates, $p_a$ and $p_b$, are constant and independent of the number of workers is rather restrictive. However, keeping all economic conditions – wages and rates $p_a, p_b$ – fixed allows me to (a) isolate the effects of network structure on migration decisions in otherwise simple setting and (b) focus on the supply side of the labor market, that is, workers and job information exchange between them, that constitute the main subject of interest in this paper.

3.3 Job information exchange

I consider the job information exchange between agents that satisfies four properties.

First, the job information is transmitted in at most one stage\footnote{The termology of Boorman (1975).}: if the job offer is neither taken by the agent who hears of the job directly nor by the direct contacts of this agent, the job is wasted rather than being passed on to some other needy party. Secondly, if several job offers arrive to one unemployed agent, he randomly selects only one of them and all other job offers remain unfilled. These first two properties rule out the study of chain effects in the transmission of job information. However, the one-stage information transmission is strongly supported by the empirical study in Granovetter (1974). The finding of the study is that information chains of length two or more account for only 15.6 percent of the sample. Thirdly, an employed agent passes the job information to one of his direct unemployed contacts with uniform randomization. This is the so-called equal neighbors treatment assumption from Calvó-Armengol and Jackson (2004). Finally, the information

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may only flow between agents within the same country, that is, those who decided to stay in (H) can only share job information with other agents in (H) and those who decided to migrate can only exchange information with other migrants and with initial residents in (D).

Notice that the last property does not mean that agents in (H) can never obtain job information from their contacts in (D). They can conditional on making a decision to migrate. In fact, as will be stated in section ??, residents of (H) take into account their future employment chances and earnings in each country but make decisions to migrate or to stay at the very beginning of period 1, before any information about new job openings arrived in either of the two countries. Given that and given the irreversibility of the decisions, the last property just means that whenever an agent has chosen to stay in (H) or go to (D), employment prospects in the other country become irrelevant for him, and also he himself becomes irrelevant for the employment conditions in the other country. Technically this implies that links between an agent in (H) and agents in (D) are only "active" if the agent from (H) migrates to (D). At the same time, links connecting a migrant with his friends in (H) "stop functioning".17

3.4 Transition between employment states

The evolution of employment of any agent \(i\) over time is predetermined by the sequence of stochastic events in each time period. For example, the transition from unemployment at the end of period \(t - 1\) to employment at the end of period \(t\) is possible if agent \(i\) either receives a job offer directly, at rate \(p_a\), or hears about a job offer from one of his direct employed contacts. The chance to hear about a job offer from an employed friend is increasing in probability \(p_a\) that this friend will receive an extra job offer but is decreasing in the number of other unemployed contacts of this friend. Moreover, having received a job, the agent can only keep it till the end of a period with probability \(1 - p_b\). On the other hand, the transition from employment at the end of period \(t - 1\) to unemployment at the end of period \(t\) is fully determined by the breakup rate, \(p_b\), and does not depend on the pattern of social connections of an agent. The other two transitions, when the employment status of an agent does not change over the period, are defined by the corresponding complementary events. Formally, for any agent \(i \in H \setminus M\) and for any \(t \geq 2\), the four probabilities of transition between employment

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17Such specification of the model suggests that although migration does not change the structure of the whole network \(G\), it actually alters the structure of the labor market in (H) and in (D). Reiterating the argument made in the introduction, this aspect of the model is of essential difference from the specification of the drop-out decision game in Calvó-Armengol and Jackson (2004), where the network structure of the labor market is not altered when an agent drops out.
states are given by:

\[ P(h_t(\alpha(i)) = 1|h_{t-1}(\alpha(i)) = 0) = \]
\[ = \begin{cases} (1 - p_b) \left[ 1 - (1 - p_a) \prod_{j \in E_{H \setminus M, t-1}^i} \left( 1 - \frac{p_a}{|U_{H \setminus M, t-1}^j|} \right) \right] & \text{if } E_{H \setminus M, t-1}^i \neq \emptyset \\
(1 - p_b)p_a & \text{if } E_{H \setminus M, t-1}^i = \emptyset \end{cases} \]
\[ = (1 - p_b) \left[ 1 - (1 - p_a) \prod_{j \in E_{H \setminus M, t-1}^i} \left( 1 - \frac{p_a}{|U_{H \setminus M, t-1}^j|} \right) \right] \] \quad (1)

\[ P(h_t(\alpha(i)) = 0|h_{t-1}(\alpha(i)) = 0) = 1 - P(h_t(\alpha(i)) = 1|h_{t-1}(\alpha(i)) = 0) \]
\[ P(h_t(\alpha(i)) = 0|h_{t-1}(\alpha(i)) = 1) = p_b \]
\[ P(h_t(\alpha(i)) = 1|h_{t-1}(\alpha(i)) = 1) = 1 - p_b \] \quad (2)

where \( E_{H \setminus M, t-1}^i \) is the set of direct contacts of agent \( i \) in \( H \setminus M \) who are employed in (H) at the end of period \( t-1 \), \( U_{H \setminus M, t-1}^j \) is the set of direct contacts of agent \( j \) (\( j \in E_{H \setminus M, t-1}^i \)) in \( H \setminus M \) who are unemployed in (H) at the end of period \( t-1 \), and \( 1_{E_{H \setminus M, t-1}^i} = 1 \) if \( E_{H \setminus M, t-1}^i \neq \emptyset \) but \( 1_{E_{H \setminus M, t-1}^i} = 0 \) otherwise.

Similarly, for any agent \( i \in D \cup M \) and any \( t \geq 2 \), the probabilities of transition between employment states are defined by four equations:

\[ P(d_t(\gamma(i)) = 1|d_{t-1}(\gamma(i)) = 0) = \]
\[ = \begin{cases} (1 - p_b) \left[ 1 - (1 - p_a) \prod_{j \in E_{D \cup M, t-1}^i} \left( 1 - \frac{p_a}{|U_{D \cup M, t-1}^j|} \right) \right] & \text{if } E_{D \cup M, t-1}^i \neq \emptyset \\
(1 - p_b)p_a & \text{if } E_{D \cup M, t-1}^i = \emptyset \end{cases} \]
\[ = (1 - p_b) \left[ 1 - (1 - p_a) \prod_{j \in E_{D \cup M, t-1}^i} \left( 1 - \frac{p_a}{|U_{D \cup M, t-1}^j|} \right) \right] \] \quad (5)

\[ P(d_t(\gamma(i)) = 0|d_{t-1}(\gamma(i)) = 0) = 1 - P(d_t(\gamma(i)) = 1|d_{t-1}(\gamma(i)) = 0) \]
\[ P(d_t(\gamma(i)) = 0|d_{t-1}(\gamma(i)) = 1) = p_b \]
\[ P(d_t(\gamma(i)) = 1|d_{t-1}(\gamma(i)) = 1) = 1 - p_b \] \quad (6)

where \( E_{D \cup M, t-1}^i \) is the set of direct contacts of agent \( i \) in \( D \cup M \) who are employed in (D) at the end of period \( t-1 \), \( U_{D \cup M, t-1}^j \) is the set of direct contacts of agent \( j \) (\( j \in E_{D \cup M, t-1}^i \)) in \( D \cup M \) who are unemployed in (D) at the end of period \( t-1 \), and \( 1_{E_{D \cup M, t-1}^i} = 1 \) if \( E_{D \cup M, t-1}^i \neq \emptyset \) but \( 1_{E_{D \cup M, t-1}^i} = 0 \) otherwise.

Notice that essentially the same equations determine the probabilities of transition from the initial employment status to the employment status at the end of period 1 in both countries. The
equations should only be corrected for the fact that the initial employment status of any agent \( i \in H \setminus M \) is given by \( h_0(i) \) and the initial employment status of \( i \in D \cup M \) is given by \( d_0(i) \).

**Remark** Time \( t \) employment status of any agent \( i \) in a given country is fully determined by the job arrival and breakup rates, \( p_a \) and \( p_b \), by the network structure and the time \( t-1 \) employment status of agents in a two-links-away neighborhood of agent \( i \) in that country. The probability of being employed at period \( t \) for agent \( i \) who is unemployed at the end of period \( t-1 \) is increasing in the number of direct employed contacts and decreasing in the number of two-links-away unemployed contacts who share at least one of their direct employed contacts with \( i \).

The Remark suggests that in a one-period-ahead perspective, direct employed contacts of unemployed agent \( i \) improve his prospects for hearing about a job offer while unemployed two-links-away contacts are agent \( i \)'s "competitors" for job information and therefore, decrease his chances of employment. More distant indirect relationships do not have an impact on one-period-ahead employment prospects of \( i \). However, in a longer time frame, the larger network and status of other agents affect employment status of agent \( i \) through the effect they have on the employment status of \( i \)'s contacts.

Given transition probabilities between employment states of any agent in (H) and in (D), transition probability between two employment states of the whole labor market in each country is simply a product of the transition probabilities between the corresponding employment states of agents. So, for any two employment states \( h, h' \) in (H) and \( d, d' \) in (D) and for any \( t \geq 2 \),

\[
P(h_t = h'|h_{t-1} = h) = \prod_{i \in H \setminus M} P(h_t(\alpha(i)) = h'(\alpha(i))|h_{t-1}(\alpha(i)) = h(\alpha(i)))
\]

\[
P(d_t = d'|d_{t-1} = d) = \prod_{i \in D \cup M} P(d_t(\gamma(i)) = d'(\gamma(i))|d_{t-1}(\gamma(i)) = d(\gamma(i)))
\]

The Remark, (??) and (??) imply that employment state \( h_t \) of the labor market in (H) and employment state \( d_t \) of the labor market in (D) follow two separate finite state Markov processes. I denote by \( M(G[H \setminus M], p_a, p_b) \) the Markov process for \( h_t \) and by \( M(G[D \cup M], p_a, p_b) \) the Markov process for \( d_t \).\(^{18}\)

Markov processes \( M(G[H \setminus M], p_a, p_b) \) and \( M(G[D \cup M], p_a, p_b) \) have several important characteristics. First, they are both *homogenous*. This follows from the fact that job arrival and breakup probabilities, \( p_a \) and \( p_b \), are constant and the structure of the network in (H) and (D) does not change after migration taking place in the very beginning of period 1. Secondly, Markov chains for \( h_t \) and \( d_t \) are *irreducible* and *aperiodic*, since all transition probabilities are strictly positive.

\(^{18}\)\(G[A]\) denotes the network induced by graph \( G \) on the set of agents \( A \).
These properties of Markov chains $M(G[H \setminus M], p_a, p_b)$ and $M(G[D \cup M], p_a, p_b)$ lead to the following statement:

**Proposition 3.1.** There exists a unique stationary distribution of employment states in (H) and a unique stationary distribution of employment states in (D).

The stationary distribution of employment in (H) and (D) defines a unique steady-state probability of employment of any agent in every country:

\[ p_{i}^{ss} = \sum_{h \text{ s.t. } h(\alpha(i)) = 1} \mu(h) \quad \forall \quad i \in H \setminus M \tag{11} \]

\[ q_{i}^{ss} = \sum_{d \text{ s.t. } d(\gamma(i)) = 1} \nu(d) \quad \forall \quad i \in D \cup M \tag{12} \]

where $\mu$ is the stationary distribution of employment states in (H) and $\nu$ is the stationary distribution of employment states in (D).

### 3.5 Migration decision game

**Description of the game** Migration decisions of residents in (H) are modelled as a one-shot simultaneous move game, $\Gamma$, where the structure of the game is common knowledge. The game takes place in the beginning of period 1 when no information about new job openings has yet arrived in either of the two countries. The players are all residents of (H). They simultaneously choose one of two actions: staying in (H), denoted by $s$, or migrating to (D), denoted by $m$, so as to maximize their expected life-time income. In the strategic form, $\Gamma = [H, \Sigma, \{\pi^i\}_{i \in H}]$, where $H$ is a set of players, $\Sigma = \{s, m\}$ is each player’s set of pure strategies, and $\pi^i$ is a payoff function of player $i$, the expected income of $i$ in (H) or (D).

**Strategies and payoff function** Let $\sigma = (\sigma(1), \ldots, \sigma(a_H))$ be a profile of pure strategies of all players in $H$ and $\sigma(-i)$ be a profile of pure strategies of player $i$’s opponents. Let $v_H^i(\sigma(-i))$ and $v_D^i(\sigma(-i))$ represent the expected life-time income of player $i$ in (H) and (D), provided that $i$’s opponents play $\sigma(-i)$. Then for each player $i \in H$ and any strategy profile $\sigma$, the payoff function $\pi^i$ is defined by:

\[ \pi^i(s, \sigma(-i)) = v_H^i(\sigma(-i)), \]

\[ \pi^i(m, \sigma(-i)) = v_D^i(\sigma(-i)). \]

To write functions $v_H^i$ and $v_D^i$ explicitly, I impose an important simplifying assumption. For the rest of the paper I assume that agents in both countries start in period 1, and then jump to the
steady state in the next "period". This gives a rough representation of the life-time optimization problem of players, but enough to see the effects of their interaction in the short and in the long run.

Under this assumption, the expected life-time income of player $i$ in either country is the sum of the expected income in period 1 and the present discounted value of the future expected income starting from period 2 onwards. The future expected income is represented by the infinite flow of the identical one-period expected earnings determined by the product of the steady-state probability of employment and constant wage rate. So, each player $i$ who stays in (H) receives a payoff

$$v^i_H(\sigma(-i)) = \left\{ \begin{array}{ll}
w_H + \beta w_H p^i_{ss}(\sigma(-i)) + \beta^2 w_H p^i_{ss}(\sigma(-i)) + \cdots 
& \text{if } h_0(i) = 1 \\
wh p^i(\sigma(-i)) + \beta^2 w_H p^i_{ss}(\sigma(-i)) + \cdots 
& \text{if } h_0(i) = 0
\end{array} \right. = \left\{ \begin{array}{ll}
wh + \frac{\beta}{1-\beta} w_H p^i_{ss}(\sigma(-i)) 
& \text{if } h_0(i) = 1 \\
wh p^i(\sigma(-i)) + \frac{\beta}{1-\beta} w_H p^i_{ss}(\sigma(-i)) 
& \text{if } h_0(i) = 0
\end{array} \right. \tag{13}$$

where $p^i$ and $p^i_{ss}$ are the probabilities of player $i$ to be employed in (H) at the beginning of period 1 (if $i$ is initially unemployed) and at the steady-state, and $0 < \beta < 1$ is a constant discount factor.

More compactly (??) can be written as:

$$v^i_H(\sigma(-i)) = w_H [(1 - h_0(i))p^i(\sigma(-i)) + h_0(i)] + \frac{\beta}{1-\beta} w_H p^i_{ss}(\sigma(-i)). \tag{14}$$

Likewise, if player $i$ moves to (D), he receives a payoff

$$v^i_D(\sigma(-i)) = w_D q^i(\sigma(-i)) - c + \beta w_D q^i_{ss}(\sigma(-i)) + \beta^2 w_D q^i_{ss}(\sigma(-i)) + \cdots = w_D q^i(\sigma(-i)) - c + \frac{\beta}{1-\beta} w_D q^i_{ss}(\sigma(-i)) \tag{15}$$

where $q^i$ and $q^i_{ss}$ are the probabilities of player $i$ to be employed in (D) at the beginning of period 1 and at the steady-state. The definition of $v^i_D$ takes into account that any migrant is initially unemployed in (D).

Notice that probabilities $p^i$ and $q^i$ in the above equations differ from the probabilities of transition between initial unemployment and end-of-period-1 employment as defined in section ???. In fact, $p^i$ and $q^i$ are higher than the corresponding transition probabilities, since they describe the chance of getting employed in the beginning of a period irrespective of the fact that the job can be lost in the end of the period:

$$p^i = \frac{P(h_1(\alpha(i)) = 1| h_0(i) = 0)}{1 - p_b}, \tag{16}$$

$$q^i = \frac{P(d_1(\gamma(i)) = 1| d_0(i) = 0)}{1 - p_b}. \tag{17}$$

---

19 The same simplification is made in Calvó-Armengol and Jackson (2004) for their study of the drop-out decision game.

20 Recall that the steady-state probabilities of employment of player $i$ in (H) and in (D), $p^i_{ss}$ and $q^i_{ss}$, were defined in (??) and (??).
The equilibrium of game $\Gamma$ is a standard Nash equilibrium in pure strategies. Below I study the properties of this equilibrium in order to infer how migration decisions of individuals are affected by the structure of their social relations and in particular, by the number of their contacts in the destination country.

### 4 Results

First, I consider the special case when network $G$ of social connections is complete (or represents a union of complete components), so that all individuals in the network are linked with each other. Furthermore, I assume that discount factor $\beta$ is sufficiently close to 1, that is, agents mainly care about their expected earnings in the long run. In fact, when $\beta$ is sufficiently "large", the weight $\frac{\beta}{1-\beta}$ assigned by agents to their expected earnings from period 2 onwards is substantially larger than the weight assigned to their expected income in period 1. As a result, migration decisions of home country residents are essentially driven by their long-run income considerations.

For this setting I solve the model analytically. I prove the existence of equilibrium and study the change in the equilibrium migration choices as the number of links to the destination country changes just marginally. After that I relax the restrictions on the network structure and the discount factor and examine the outcomes of the migration decision game in simple numerical examples.

#### 4.1 Steady-state probability of employment and maximal Nash equilibrium in the complete network

The long-run expected income in (H) and in (D), which determines migration choices of players when $\beta$ is large enough, hinges on the steady-state probability of employment. Below I consider the dependence of the steady-state probability of employment in the complete network on the total number of agents, or alternatively, on the number of direct social contacts of each agent.\(^\text{21}\)

The steady-state probability of employment can be derived using the subdivision of periods procedure proposed in Calvó-Armengol and Jackson (2004). The idea of this procedure is that as we divide job arrival rate, $p_a$, and job breakup rate, $p_b$, by some larger and larger factor, we are essentially looking at arbitrarily short time periods. In the limit, the resulting process for employment approaches a continuous time (Poisson) process, which is a natural situation where the short-run effects of agents’ interaction are inconsequential and employment prospects of any individual are determined by the long-run effects.\(^\text{22}\)

\(^{21}\)Clearly, since the network is complete, the steady-state probability of employment is fully defined by the total number of agents in the network.

\(^{22}\)Formally, by dividing $p_a$ and $p_b$ by some common factor $T$, we obtain an associated Markov process $M(g, \frac{p_a}{T}, \frac{p_b}{T})$ for the employment state on a complete network $g$. In terminology of Calvó-Armengol and Jackson (2004), this Markov process is called the $T$-period subdivision of Markov process $M(g, p_a, p_b)$. Lemma ?? addresses the situation when $T$
Lemma 4.1. Under fine enough subdivision of periods, the steady-state probability of employment in a complete network of size $n$ (formed by $n$ agents) is equal to

$$\theta(n) = \frac{1 + \sum_{l=1}^{n-1} \prod_{k=1}^{l} \frac{p_b}{p_a} (1 - \frac{k}{n})}{1 + \frac{p_b}{p_a} + \frac{p_b}{p_a} \sum_{l=1}^{n-1} \prod_{k=1}^{l} \frac{p_b}{p_a} (1 - \frac{k}{n})}.$$  \hfill (18)

It is strictly increasing in $n$.

The statement of Lemma 4.1 is illustrated with Figure 2. The strictly monotonic pattern of the steady-state probability is preserved for all combinations of parameters $p_a$, $p_b$; higher $p_a$ and lower $p_b$ result in better employment prospects in the long run.

![Figure 1: Steady-state probability of employment in the complete network of size $n$.](image)

Thus, despite the short-run competition for jobs between unemployed agents, in the long run, having more agents in the network is good news for everyone as it effectively improves future employment prospects of any agent. In particular, this implies that in the complete network, migration has a long-lasting positive effect on employment probability in (D) but is also detrimental for the long-run employment probability in (H).\textsuperscript{23} As a consequence, if the weight assigned to the long-run expected income is high enough, i.e. $\beta$ is sufficiently close to 1, the incentive of any agent to migrate is increasing as more of the other players migrate. Likewise, the greater the number of players who stay, the stronger the incentive for others to stay. This inference gives rise to Proposition 4.2.

\textsuperscript{23}This result supports the idea that migrants can promote the welfare of the destination country.

is sufficiently large, so that the true steady-state probability of employment under $M(g, p_a, p_b)$ and $M(g, \frac{p_a}{p_n}, \frac{p_b}{p_n})$ can be calculated using the approximate Markov process. Further details are provided in the Appendix.
Proposition 4.2. Given complete network \( G \) and fine enough subdivision of periods, there exists \( \bar{\beta} \) such that for all \( \beta \geq \bar{\beta} \):

1. Decisions of players in \( H \) are strategic complements, that is, game \( \Gamma \) is supermodular

2. There exists a unique maximal Nash equilibrium in pure strategies

The existence of a maximal Nash equilibrium follows from the theory of supermodular games. The maximal equilibrium is defined as in Calvó-Armengol and Jackson (2004). It is an equilibrium where the set of players staying in \( (H) \) is maximal, that is, the set of players staying in \( (H) \) at any other equilibrium of the game is a subset of those staying in \( (H) \) at the maximal equilibrium.

4.2 The effects of links between countries on equilibrium migration flow

Strategic complementarity of players’ decisions and interchangeability of players’ positions in the complete network imply that only four outcomes are possible at equilibrium: initially unemployed players either all migrate or all stay and initially employed players either all migrate or all stay. Moreover, since irrespectively of players’ initial employment status, expected earnings in period 1 are negligible compared to those from period 2 onwards (\( \beta \) is close to 1), not only the strategies of players with identical initial status are the same but the strategies of all players are the same. This leads to Proposition 4.3.

Proposition 4.3. Given complete network \( G \) and fine enough subdivision of periods, either no one or everyone migrates at the maximal Nash equilibrium. The situation when no one migrates is the maximal Nash equilibrium if and only if

\[
 w_H \theta(a_H) \geq w_D \theta(a_D + 1).
\]

Condition (19) states that for any agent, the steady-state expected income in \( (H) \) is at least as high as in \( (D) \), provided that all other players stay. Thus, the situation when no one migrates is sustainable at the Nash equilibrium. Furthermore, the situation when everyone migrates is always a Nash equilibrium, although not necessarily maximal, since for any \( w_D > w_H \),

\[
 w_H \theta(a_H) < w_D \theta(a_D + a_H)
\]

It is easy to see that the larger the number \( a_D \) of players’ contacts in \( D \), the more likely it is that in the maximal equilibrium, all players migrate. Formally, given \( a_H, w_H \) and \( w_D \), there exists a threshold value \( \bar{a}_D(a_H, w_H, w_D) \) of \( a_D \) such that as soon as \( a_D \) exceeds this threshold, the inequality \( w_H \theta(a_H) \geq w_D \theta(a_D + 1) \) does not hold and the outcome with all players staying in \( (H) \) is not an equilibrium. Therefore, as a corollary of Proposition 4.3, we obtain:
Proposition 4.4. The number of migrants in the maximal equilibrium of $\Gamma$ is weakly increasing in the number of players’ contacts in $D$. Moreover, for any $a_H$, $w_H$ and $w_D$, there exists a threshold value $\bar{a}_D(a_H, w_H, w_D)$ such that no one migrates in the maximal Nash equilibrium when the number of players’ contacts in $D$ is equal to $\bar{a}_D(a_H, w_H, w_D)$, but everyone migrates if this number of contacts exceeds $\bar{a}_D(a_H, w_H, w_D)$ at least by 1. The threshold $\bar{a}_D(a_H, w_H, w_D)$ is defined by the equation:

$$\frac{w_H}{w_D} \theta(a_H) = \theta(a_D + 1)$$

(20)

Thus, under certain conditions on the network structure and on the ratio of wages in (H) and in (D), just a minor increase in the number of social connections with (D) leads to a sharp increase in migration at the maximal Nash equilibrium.

The result of Proposition 4.4 is certainly very stylized. However, it still demonstrates the desired effects. In particular, for the case of 19th century Italian emigration, it suggests that the striking difference in migration flows to North America from the south and from the north of Italy could have resulted from a small initial difference in the number of social links to North America from each of the two regions.24

More formally, suppose that $H_N$ and $H_S$ are two social groups in the south and in the north of Italy respectively and that $D_N$ and $D_S$ are two groups in North America, such that $D_N$ has social connections with $H_N$ and $D_S$ has social connections with $H_S$. Suppose that communities in the south and in the north of Italy are "separated", so that they do not have an impact on the long-run expected earnings of each other. For example, let $G[H_N]$, $G[H_S]$ form separate components of the network in Italy and $G[D_N]$, $G[D_S]$ form separate components of the network in North America. Also, let both $G[H_N \cup D_N]$ and $G[H_S \cup D_S]$ be complete networks and $|H_N| = |H_S| = a_H$. Furthermore, assume that $|D_S| > \bar{a}_D(a_H, w_H, w_D) \geq |D_N|$, that is, the number of North American contacts of the Southern Italians is larger than the threshold $\bar{a}_D(a_H, w_H, w_D)$ defined above and the number of North American contacts of the Northern Italians is lower or equal to this threshold. Then according to Proposition 4.4, in the maximal Nash equilibrium of the migration decision game, everyone migrates from the group $H_S$ of Southern Italians but no one migrates from the group $H_N$ of the Northern Italians, provided that both, Northern and Southern Italians mainly care about their expected income in the long run.

For example, for the specific parameter values, $p_a = p_b = 0.05$, $a_H = 10$ and $\frac{w_H}{w_D} \in [0.6366, 0.7639)$, condition (4.4) suggests that in the maximal Nash equilibrium, no one migrates from $H_N$, if agents

24Moreover, the preceding analysis suggests that if Italian potential migrants were mainly concerned about their earnings in the long run, the gap in initial employment conditions favoring North America over Italy did not play a major role in determining the migration decisions.
in $H_N$ do not have any social connections in North America ($|D_N| = 0$), but everyone migrates from $H_S$, if agents in $H_S$ have just one acquaintance in North America ($|D_S| = 1$).  

### 4.3 The effects of links between countries on migration in non-complete networks: Numerical exercise

In this section I relax the restrictions that network $G$ is complete and discount factor $\beta$ is close to 1. I show that the basic insights from the previous analysis about the impact of social links between agents on the steady-state probability of employment and on the size of migration flow continue to hold.

First, just as in case of a complete network structure, I find that the steady-state probability of employment of any agent in a non-complete network is increasing in the number of the agent’s direct contacts. This result is demonstrated by examples in Table ??.

Table ?? displays the values of the steady-state probability of employment of agent 1 for different numbers of his direct connections. The upper part of Table ?? displays the change in the steady-state probability of employment of agent 1 when the number of his links with other agents increases but the size of the network remains fixed; the lower part of the table continues the exercise for the case when an increase in the number of links of agent 1 comes along with an increase in the size of the network. In either case, as the number of connections between agent 1 and other agents increases, the steady-state probability of employment grows. Moreover, everything else being equal, higher job information arrival rate, $p_a$, and lower job breakup rate, $p_b$, result in higher probability of employment in the long run.

As before, the finding of a positive impact of direct social contacts on each agent’s steady-state probability of employment suggests that as soon as discount factor $\beta$ is high enough, directly linked players tend to choose the same strategy. However, strategic complementarity of all players’ decisions and the existence of a (maximal) Nash equilibrium in the game are contingent on the particularity of the network structure and chosen parameter values.

If Nash equilibrium exists, it can be defined by simple conditions. To state these conditions formally for a generic network structure and any $0 < \beta < 1$, I introduce new notation. Let $C^i_H, C^j_D$ be the sets of those players in $H$ who are competitors or potential competitors of player $i$ for the

\[ w_i \theta(1) \leq w_H \theta(a_H) < w_i \theta(2) \]

where $\theta(1) \approx 0.5$, $\theta(2) \approx 0.6$, and $\theta(a_H) = \theta(10) \approx 0.7854$ according to Figure ?? (see the plot corresponding to $p_a = p_b = 0.05$).

---

\( ^{25}\)This result is implied by inequality $w_i \theta(1) \leq w_H \theta(a_H) < w_i \theta(2)$.
Table 1: Steady-state probability of employment of agent 1 for different values of $p_a$ and $p_b$

<table>
<thead>
<tr>
<th>network $G$</th>
<th>$p_a = 0.2$</th>
<th>$p_a = 0.1$</th>
<th>$p_a = 0.1$</th>
<th>$p_a = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_b = 0.015$</td>
<td>$p_b = 0.015$</td>
<td>$p_b = 0.03$</td>
<td>$p_b = 0.05$</td>
</tr>
<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9292</td>
<td>0.8678</td>
<td>0.7638</td>
<td>0.4872</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9583</td>
<td>0.9219</td>
<td>0.8463</td>
<td>0.5748</td>
</tr>
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<td></td>
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<td></td>
</tr>
<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9686</td>
<td>0.9426</td>
<td>0.8818</td>
<td>0.6269</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9770</td>
<td>0.9608</td>
<td>0.9163</td>
<td>0.6971</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>network $G$</th>
<th>$p_a = 0.2$</th>
<th>$p_a = 0.1$</th>
<th>$p_a = 0.1$</th>
<th>$p_a = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_b = 0.015$</td>
<td>$p_b = 0.015$</td>
<td>$p_b = 0.03$</td>
<td>$p_b = 0.05$</td>
</tr>
<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9584</td>
<td>0.9221</td>
<td>0.8469</td>
<td>0.5760</td>
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<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9686</td>
<td>0.9425</td>
<td>0.8818</td>
<td>0.6294</td>
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<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9740</td>
<td>0.9544</td>
<td>0.9052</td>
<td>0.6800</td>
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</tr>
<tr>
<td>$\begin{array}{c} \circ 1 \ \circ 2 \ \circ 3 \ \circ 4 \ \circ 5 \ \circ 6 \end{array}$</td>
<td>0.9772</td>
<td>0.9616</td>
<td>0.9196</td>
<td>0.7154</td>
</tr>
</tbody>
</table>

Remark: The probabilities are expressed in percentage points.
first-period job offers in (H) and in (D), respectively:

\[ \forall i \in U_H \ C^i_H = \bigcup_{j \in E^i_H} C^i_{H,\text{from } j}, \text{ where } C^i_{H,\text{from } j} = \{ l \in U_H \text{ s.t. } l \neq i, j \in E^i_H \}, \]

\[ \forall i \in H \ C^i_D = \bigcup_{j \in E^i_D} C^i_{D,\text{from } j}, \text{ where } C^i_{D,\text{from } j} = \{ l \in H \text{ s.t. } l \neq i, j \in E^i_D \}. \]

If player \( i \) is initially employed in (H), I assume that the set of his competitors in (H) is empty:

\[ \forall i \in E_H \ C^i_H = \emptyset \]

Using this notation, an equilibrium of game \( \Gamma \) can be defined as a strategy profile such that for any player \( i \) who migrates, \( i \in M^* \), the following inequality holds:

\[ w_H \left[ (1 - h_0(i))p^{i*} + h_0(i) \right] + \frac{\beta}{1 - \beta} w_H p^{i*}_{ss} \leq w_D q^{i*} - c + \frac{\beta}{1 - \beta} w_D q^{i*}_{ss} \]

where

\[ p^{i*} = 1 - (1 - p_a) \left[ \prod_{j \in E^i_H \setminus M^*} \left( 1 - \frac{p_a}{w_H - c^i_{H,\text{from } j}} \right) \right]^{1_{E^i_H \setminus M^*}}, \]

\[ q^{i*} = 1 - (1 - p_a) \left[ \prod_{j \in E^i_D} \left( 1 - \frac{p_a}{w_D + 1 + c^i_{D,\text{from } j}} \right) \right]^{1_{E^i_D}}, \]

\( c^i_{H,\text{from } j} = |C^i_{H,\text{from } j} \cap M^*| \), \( c^i_{D,\text{from } j} = |C^i_{D,\text{from } j} \cap M^*| \), and \( p^{i*}_{ss}, q^{i*}_{ss} \) are the steady-state probabilities of employment of player \( i \) on \( G[H \setminus M^*] \cup \{ i \} \) and \( G[D \cup M^*] \), respectively. At the same time, for any player \( k \) who does not migrate, \( k \in H \setminus M^* \), the opposite inequality or equality holds.

I use this definition of equilibrium in order to find equilibrium of the game numerically for a range of network structures and particular parameter values. I focus on a change in equilibrium strategies of players caused by a single link increase in the number of connections between (H) and (D).

Table ?? presents the results for four examples of different network structures. The values of parameters used to calculate these examples are \( w_H = 5 \), \( w_D = 7 \), \( c = 1 \), \( p_a = 0.1 \), \( p_b = 0.015 \), and \( \beta = 0.7 \). For these parameter values and for a large set of combinations of initial employment states in (H) and (D), I find that there exists a unique maximal Nash equilibrium of the migration decision game in each of the four examples. Moreover, for a subset of initial employment states in two countries, a minimal increase in the number of social links between (H) and (D) causes a substantial increase in the amount of agents who choose to migrate at the maximal equilibrium. For instance, in certain employment conditions, not only the agent whose own number of links increases changes his strategy in favor of migrating but also some other agents in (H) do so.\(^{27}\)

\( ^{26} \) The same values of \( p_a \) and \( p_b \) are used in the numerical examples of Calvó-Armengol and Jackson (2004). The authors argue that "if we think about these numbers from the perspective of a time period being a week, then an agent looses a job roughly on average once in 67 weeks, and hears (directly) about a job on average once in every 10 weeks" (p. 430).

\( ^{27} \) See examples 1, 3, and 4 of Table ??.
Table 2: Equilibrium strategy profiles of players in $H$. Changes caused by adding a link

**Example 1:** $H = \{1, 2, 3\}$, $D = \{4, 5, 6\}$, link $2 \rightarrow 4$ is added

<table>
<thead>
<tr>
<th>Initial employment states in $H$ and $D$</th>
<th>$h_0 = (0,1,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (0,1,1), d_{0,D} = (1,1,1)$</th>
<th>$h_0 = (0,1,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (0,1,1), d_{0,D} = (1,1,1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(m,s,s)$</td>
<td>$(m,s,s)$</td>
<td>$(s,s,m)$</td>
<td>$(s,s,s)$</td>
</tr>
<tr>
<td></td>
<td>$(m,m,s)$</td>
<td>$(m,m,s)$</td>
<td>$(m,m,s)$</td>
<td>$(m,m,s)$</td>
</tr>
</tbody>
</table>

**Example 2:** $H = \{1, 2, 3, 4\}$, $D = \{5, 6, 7\}$, link $2 \rightarrow 6$ is added

<table>
<thead>
<tr>
<th>Initial employment states in $H$ and $D$</th>
<th>$h_0 = (0,1,0,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (0,1,0,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (0,1,0,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (0,1,0,1), d_{0,D} = (1,1,0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(m,m,s)$</td>
<td>$(m,s,s,m)$</td>
<td>$(m,m,s)$</td>
<td>$(m,m,s)$</td>
</tr>
<tr>
<td></td>
<td>$(m,m,m)$</td>
<td>$(m,s,m,m)$</td>
<td>$(m,m,m)$</td>
<td>$(m,m,m)$</td>
</tr>
</tbody>
</table>

**Example 3:** $H = \{1, 2, 3, 4, 5\}$, $D = \{6, 7, 8\}$, link $1 \rightarrow 6$ is added

<table>
<thead>
<tr>
<th>Initial employment states in $H$ and $D$</th>
<th>$h_0 = (1,0,0,0,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (1,0,0,0,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (1,0,0,0,1), d_{0,D} = (1,1,0)$</th>
<th>$h_0 = (1,0,0,0,1), d_{0,D} = (1,1,0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(s,m,m,m,m)$</td>
<td>$(s,m,m,m,m)$</td>
<td>$(s,m,m,m,m)$</td>
<td>$(s,m,m,m,m)$</td>
</tr>
<tr>
<td></td>
<td>$(m,m,m,m,m)$</td>
<td>$(m,m,m,m,m)$</td>
<td>$(m,m,m,m,m)$</td>
<td>$(m,m,m,m,m)$</td>
</tr>
</tbody>
</table>

**Example 4:** $H = \{1, 2, 3, 4, 5, 6\}$, $D = \{7, 8, 9\}$, link $3 \rightarrow 8$ is added

<table>
<thead>
<tr>
<th>Initial employment states in $H$ and $D$</th>
<th>$h_0 = (0,1,1,0,0,1), d_{0,D} = (0,1,1)$</th>
<th>$h_0 = (0,1,1,0,0,1), d_{0,D} = (1,1,1)$</th>
<th>$h_0 = (0,1,1,0,0,1), d_{0,D} = (1,1,1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(m,s,m,m,m)$</td>
<td>$(m,s,m,m,m)$</td>
<td>$(m,s,s,m,m)$</td>
</tr>
<tr>
<td></td>
<td>$(m,m,m,m,m)$</td>
<td>$(m,s,m,m,m)$</td>
<td>$(m,m,m,m,m)$</td>
</tr>
</tbody>
</table>

22
Examples of Table ?? can be viewed as an illustration of Italian emigration case. Indeed, let $H$ represents a group of people in the north of Italy and $D$ represents a group in North America. Suppose that a group of people in the south of Italy has the same structure of social relations with each other and the same initial employment state as a group of people in the north of Italy, $H$. Then the examples of Table ?? suggests that for a certain combination of initial employment states of Italians and North Americans, a substantially higher proportion of Southern Italians migrates to North America as soon as the number of social connections between the Southern Italians and North Americans is just marginally higher than the number of connections between the Northern Italians and North Americans.

5 Conclusion

The aim of this paper is to provide a microfoundation for the hypothesis in the chain migration literature that job information provision by previous migrants to their social contacts at home plays an important role in determining migration decisions. In particular, I examine the effects of small changes in the structure of information exchange between past migrants and their friends at home on the size of the migration flow. This allows me to address the puzzling case of the 19th century Italian emigration and reason whether the lack of migration from the north of Italy to North America, despite the evident economic benefits of that move, and simultaneous large migration from the south of Italy could have resulted from the initial small difference in the number of social contacts in North America of the Northern and Southern Italians.

To study the role of interpersonal information exchange on migration decisions, I develop a model where the role of social networks to diffuse information about available jobs is explicitly taken into account. The model has two countries, a home and a destination country. Agents in both countries are labor market participants. They use their social links to exchange information about job opportunities in the local labor market. Employment of every agent in each time period is subject to the idiosyncratic risk. If an agent is unemployed, then having links to employed agents improves his prospects for hearing about a job offer. In contrast, having two-links-away unemployed competitors for job information worsens these prospects. In the beginning of period 1, residents of the home country compare their employment prospects at home and in the destination country and take once-and-for-all migration decisions. These migration decisions are modelled as a pure-strategy Nash equilibrium of a one-shot simultaneous move game. I study the properties of this equilibrium.

The analytical work in the paper is restricted to the case when the network of social connections is complete and discount factor $\beta$ is sufficiently close to 1. I find that in such a setting, the steady-
state probability of employment of any agent is increasing in the size of the network. This implies that migration improves the long-run employment prospects in the destination country but worsens those in the home country. As a result, migration decisions of the home country residents turn out to be strategic complements which guarantees the existence of a unique maximal Nash equilibrium in the game.

The main finding of the paper is that even a small difference in the number of links between countries can change migration patterns substantially. In particular, in the stylized setting of the model, when the network of social connections is complete and $\beta$ is close to 1, the equilibrium of the migration game displays a threshold property, i.e. there exists a threshold for a number of links to the destination country, such that as soon as the actual number of links is below (above) this threshold, nobody (everybody) migrates. More generally, numerical simulations for various network structures show that higher number of social connections between countries leads to higher migration flow and even a minor increase in the number of these connections may produce a large increase in migration.

The findings of the paper demonstrate the importance of the structure of social relations in determining migration decisions. With respect to the case of Italian emigration they suggests that substantially higher migration to North America from the south than from the north of Italy could indeed have resulted from an originally minor superiority in the number of social links with North America of the Southern Italians over those of the Northern Italians.
6 Appendix

To calculate the steady-state probability of employment, I analyze the subdivided dynamics of employment introduced in Calvó-Armengol and Jackson (2004). Consider T-period subdivision \( M^T(g, \frac{p_a}{T}, \frac{p_b}{T}) \) of the Markov process \( M(g, p_a, p_b) \) for the arbitrary network structure \( g \). Let \( P^T \) denote the matrix of transitions between different employment states under \( M^T \). That is, \( P^T_{ss'} \) is the probability that \( s_t = s' \) conditional on \( s_{t-1} = s \). It is easy to see that for large \( T \),

\[
P^T_{ss'} = \begin{cases} 
\frac{p_a + o\left(\frac{1}{T}\right)}{\frac{p_a}{T}} + o\left(\frac{1}{T}\right) & \text{if } s \text{ and } s' \text{ are adjacent and } s(i) > s'(i) \text{ for some } i \\
\frac{p_a}{T} & \text{if } s \text{ and } s' \text{ are adjacent and } s(i) < s'(i) \text{ for some } i \\
1 - \#s \frac{p_a}{T} - \sum_{i \text{ s.t. } s(i) = 0} \frac{p_b(s)}{T} + o\left(\frac{1}{T}\right) & \text{if } s = s' \text{ and } s \text{ and } s' \text{ are non-adjacent and } s \neq s'
\end{cases}
\]

where \( \#s \) denotes the number of agents who are employed in state \( s \), \( \#s = \sum_k s(k) \), and \( p_i(s) \) is the probability that agent \( i \) who is unemployed in state \( s \) hears about a new job offer and at most ones:

\[
p_i(s) = p_a + p_a \cdot \sum_{j \text{ s.t. } s(j) = 1, g_{ij} = 1} \sum_{k \text{ s.t. } s(k) = 0} \frac{1}{g_{jk}},
\]

The definition of \( P^T_{ss} \) suggests that Markov process \( M^T(g, \frac{p_a}{T}, \frac{p_b}{T}) \) is a Poisson process. In particular, when \( T \) is high enough, the probability of even a single shock to employment state, \( s \), at every subperiod is very low (of order \( \frac{1}{T} \)). The probability of two or more shocks is even lower (of order \( \frac{1}{T^2} \) or lower). Therefore, instead of \( M^T(g, \frac{p_a}{T}, \frac{p_b}{T}) \), I consider an approximate Markov process \( \hat{M}^T(g, \frac{p_a}{T}, \frac{p_b}{T}) \) where only one-shock transitions are retained while the transitions involving two or more shocks are disregarded. Matrix \( \hat{P}^T \) of transitions under \( \hat{M}^T \) is defined as

\[
\hat{P}^T_{ss'} = \begin{cases} 
\frac{p_a}{T} & \text{if } s \text{ and } s' \text{ are adjacent and } s(i) > s'(i) \text{ for some } i \\
\frac{p_b(s)}{T} & \text{if } s \text{ and } s' \text{ are adjacent and } s(i) < s'(i) \text{ for some } i \\
1 - \#s \frac{p_a}{T} - \sum_{i \text{ s.t. } s(i) = 0} \frac{p_b(s)}{T} & \text{if } s = s' \text{ and } s \text{ and } s' \text{ are non-adjacent and } s \neq s'
\end{cases}
\]

In the following, I calculate the stationary probability distribution of employment using the approximate Markov process, \( \hat{M}^T \), instead of Markov process \( M^T \). This substitution is justified since for large enough values of \( T \), transition probabilities of the approximate Markov process, \( \hat{M}^T \), are close to those of the true Markov process, \( M^T \). As a result, the stationary probability distributions under \( M^T \) and under \( \hat{M}^T \) are also close, which is proved formally in Calvó-Armengol and Jackson (2004):

\[
\lim_{T \to \infty} \mu^T = \lim_{T \to \infty} \hat{\mu}^T
\]

where \( \mu^T \) is the steady-state distribution of employment under \( M^T \) and \( \hat{\mu}^T \) is the steady-state distribution of employment under \( \hat{M}^T \).
Proof of Lemma ??

To simplify the notation, below I use $a$ for $\frac{p_a}{T}$ and $b$ for $\frac{p_b}{T}$.

The steady-state probability of employment of any agent $i$ in the complete network of size $n$ can be defined as:

$$\theta(n) = \sum_{s \in \{0,1\}^n \text{ s.t. } s(i)=1} \sum_{k=1}^{n} \binom{n-1}{k-1} \hat{\mu}^T(s : \#s = k), \quad (22)$$

where $\hat{\mu}^T(s : \#s = k)$ is the steady-state probability of employment state $s$, such that exactly $k$ agents are employed. To derive $\hat{\mu}^T(s : \#s = k)$ for any $k \in 0 : n$ I use the definition of transition probabilities, $\hat{P}^T_{ss'}$, in (??) applied to the case of a complete network: 28

$$\hat{P}^T_{ss'} = \begin{cases} 
    b(1 + \frac{\#s - \#s'}{n-\#s'}) & \text{if } s \text{ and } s' \text{ are adjacent and } s(i) > s'(i) \text{ for some } i \\
    a(1 + \frac{k}{n-k+1}) \hat{\mu}^T(s : s(-i) = s'(-i) \text{ and } s(i) < s'(i))1_{\{k\geq1\}} + b(n-k)\hat{\mu}^T(s : s(-i) = s'(-i) \text{ and } s(i) > s'(i))1_{\{k\leq n-1\}} + (1 - bk - an1_{\{k\leq n-1\}})\hat{\mu}^T(s' : \#s' = k), & \text{if } s = s' \\
    0 & \text{if } s \text{ and } s' \text{ are non-adjacent and } s \neq s'
\end{cases} \quad (23)$$

(??) enables representation of $\mu^T$ in the following form:

$$\hat{\mu}^T(s' : \#s' = k) = ak \left(1 + \frac{k-1}{n-k+1}\right) \hat{\mu}^T(s : s(-i) = s'(-i) \text{ and } s(i) < s'(i))1_{\{k\geq1\}} + b(n-k)\hat{\mu}^T(s : s(-i) = s'(-i) \text{ and } s(i) > s'(i))1_{\{k\leq n-1\}} + (1 - bk - an1_{\{k\leq n-1\}})\hat{\mu}^T(s' : \#s' = k), \quad k \in 0 : n \quad (24)$$

The first term in the sum on the right-hand side of (??) corresponds to a change in the employment status of agent $i$ from unemployment in $s$, $s(i) = 0$, to employment in $s'$, $s'(i) = 1$. There are $k$ such agents and each of them may find a job either directly, with probability $a$, or using the information about a job offer from one of his $k - 1$ employed friends, with probability $a\frac{k-1}{n-k+1}$, since every friend chooses $i$ as a job offer recipient randomly among $n - k + 1$ unemployed candidates.

The second term on the right-hand side of (??) corresponds to a change in the employment status of agent $i$ from employment in $s$ to unemployment in $s'$. There are $n - k$ such agents and each of them loses a job at exogenous rate $b$.

Finally, the third term in the sum corresponds to the case when none of the agents changes his employment status.

Cancelling the term $\hat{\mu}^T(s' : \#s = k)$ on the left- and on the right-hand side of (??) results in

28The derivation of the expression for $\hat{\mu}^T$ in this proof is based on the unpublished manuscript by A. Calvó-Armengol (2007).
the system of equations (22):

\[(a1_{k \leq n-1})n + bk\hat{\mu}^T(s') : #s' = k) = \]
\[= \frac{ak}{n} \left(1 + \frac{k-1}{n-k+1}\right)\hat{\mu}^T(s : s(-i) = s'(-i) \text{ and } s(i) < s'(i)) + b(n-k)\hat{\mu}^T(s : s(-i) = s'(-i) \text{ and } s(i) > s'(i)), \quad k \in 0 : n \]

(25)

To solve this system, I first rewrite it in terms of probabilities \(\mu_k, k \in 0 : n\), that \(k\) out of \(n\) agents are employed at the steady state. Due to interchangeability of nodes in the complete network,

\[\mu_k = \binom{n}{k} \hat{\mu}^T(s : #s = k), \quad k \in 0 : n \]

(26)

Then (22) reduces to a system of \(n + 1\) equations with \(n + 1\) unknowns, \(\mu_0, \ldots, \mu_n\):

\[(an1_{k \leq n-1}) + bk)\mu_k = 1_{k \geq 1}an\mu_{k-1} + 1_{k \leq n-1}b(k + 1)\mu_{k+1}, \quad k \in 0 : n \]

The solution can be found recursively:

\[
\begin{align*}
\mu_{n-1} &= \frac{b}{a}\mu_n \\
\mu_{n-2} &= \frac{b}{a} \frac{n-1}{n} \mu_{n-1} \\
\mu_{n-3} &= \frac{b}{a} \frac{n-2}{n} \mu_{n-2} \\
&\vdots \\
\mu_1 &= \frac{b}{a} \frac{2}{n} \mu_2 \\
\mu_0 &= \frac{b}{a} \frac{1}{n} \mu_1
\end{align*}
\]

Hence,

\[\mu_{n-l} = \left[\frac{b}{a}\right]^l \frac{(l-1)!}{n^{l-1}(n-l)!} \mu_n = \left[\frac{b}{a}\right]^l \frac{(n-1)!}{n^{l-1}(n-l)!} \mu_n, \quad 1 \leq l \leq n \]

(27)

Using (27), probability \(\mu_n\) can be found from the normalization condition, \(\sum_{k=0}^n \mu_k = 1\). We have:

\[
1 = \sum_{k=0}^n \mu_k = \sum_{l=0}^n \mu_{n-l} = \sum_{l=0}^n \left[\frac{b}{a}\right]^l \frac{(n-1)!}{n^{l-1}(n-l)!} \mu_n = \mu_n \left[1 + \sum_{l=1}^n \left[\frac{b}{a}\right]^l \frac{(n-1)!}{n^{l-1}(n-l)!}\right] = \\
= \mu_n \left[1 + \frac{b}{a} + \sum_{l=2}^n \left[\frac{b}{a}\right]^l (1 - \frac{1}{n}) \cdots (1 - \frac{p-1}{n})\right] = \\
= \mu_n \left[1 + \frac{b}{a} + \sum_{l=2}^n \left[\frac{b}{a}\right]^l \left(\frac{1}{n}\right) \cdots \left(\frac{1}{n}\right)\right] = \\
= \mu_n \left[1 + \frac{b}{a} + \sum_{l=2}^n \left[\frac{b}{a}\right]^l \left(\frac{1}{n}\right) \cdots \left(\frac{1}{n}\right)\right] = \\
= \mu_n \left[1 + \frac{b}{a} + \frac{n - 1}{a} \sum_{l=2}^n \left[\frac{b}{a}\right]^l \left(\frac{1}{n}\right) \cdots \left(\frac{1}{n}\right)\right] = \\
= \mu_n \left[1 + \frac{b}{a} + \frac{n - 1}{a} \sum_{l=2}^n \left[\frac{b}{a}\right]^l \left(\frac{1}{n}\right) \cdots \left(\frac{1}{n}\right)\right]
\]

27
So,
\[
\mu_n = \frac{1}{1 + \frac{b}{a} + \frac{k}{a} \sum_{l=1}^{n-1} \prod_{k=1}^{l} \left( \frac{b}{a} \right) (1 - \frac{k}{n})}, \quad n \geq 1
\]

Now, the closed-form expressions for the remaining \( n \) probabilities, \( \mu_0, \ldots, \mu_{n-1} \) follow from (??).

At last, plugging \( \mu_k, k \in 0 : n \), into (??) and using the obtained sequence of \( \hat{\mu}^T(s : \#s = k) \), \( k \in 0 : n \), in (??), leads to the following expression for the steady-state probability of employment:

\[
\theta(n) = \sum_{k=1}^{n} \frac{k}{n} \mu_k = \sum_{l=0}^{n-1} \frac{n-l}{n} \mu_{n-l} = \sum_{l=0}^{n-1} \left[ \frac{b}{a} \right]_l \frac{(n-1)!}{n! (n-l-1)!} \mu_n = \\
\mu_n \left[ 1 + \sum_{l=1}^{n-1} \left( \frac{b}{a} \right)_l \frac{(n-1) \cdots (n-l)}{n^l} \right] = \mu_n \left[ 1 + \sum_{l=1}^{n-1} \left( \frac{b}{a} \right)_l \left( 1 - \frac{1}{n} \right) \cdots \left( 1 - \frac{l}{n} \right) \right] = \\
\mu_n \left[ 1 + \sum_{l=1}^{n-1} \prod_{k=1}^{l} \left( 1 - \frac{k}{n} \right) \right] = \frac{1 + \sum_{l=1}^{n-1} \prod_{k=1}^{l} b_k \left( \frac{b}{a} \right)_l \left( 1 - \frac{k}{n} \right)}{1 + \frac{b}{a} + \frac{k}{a} \sum_{l=1}^{n-1} \prod_{k=1}^{l} b_k \left( \frac{b}{a} \right)_l \left( 1 - \frac{k}{n} \right)}, \quad n \geq 1
\]

Given \( \theta(n) \), it is now straightforward to show that \( \theta(n) \) is strictly increasing in \( n \) for any \( n \geq 1 \).

Reducing \( \theta(n) \) and \( \theta(n+1) \) to a common denominator and subtracting \( \theta(n) \) from \( \theta(n+1) \), we obtain a ratio where denominator is unambiguously positive and the nominator is equal to

\[
\left( 1 + \sum_{l=1}^{n} \prod_{k=1}^{l} p_k \left( 1 - \frac{k}{n+1} \right) \right) \left( 1 + \frac{p_b}{p_a} + \frac{p_b}{p_a} \sum_{l=1}^{n-1} \prod_{k=1}^{l} p_k \left( 1 - \frac{k}{n} \right) \right) - \\
\left( 1 + \sum_{l=1}^{n-1} \prod_{k=1}^{l} p_k \left( 1 - \frac{k}{n} \right) \right) \left( 1 + \frac{p_b}{p_a} + \frac{p_b}{p_a} \sum_{l=1}^{n-1} \prod_{k=1}^{l} p_k \left( 1 - \frac{k}{n+1} \right) \right)
\]

Simple algebra transforms this expression to

\[
\prod_{k=1}^{n} \frac{p_b}{p_a} \left( 1 - \frac{k}{n+1} \right) + \sum_{l=1}^{n-1} \left[ \frac{p_b}{p_a} \right]_l \left[ \prod_{k=1}^{l} \left( 1 - \frac{k}{n+1} \right) - \prod_{k=1}^{l} \left( 1 - \frac{k}{n} \right) \right] > 0
\]

Hence, \( \theta(n+1) - \theta(n) > 0 \) for any \( n \geq 1 \). In particular, as the size of the network, \( n \), becomes arbitrarily large, the steady-state probability of employment approaches 1.
References


