Starting Point

Varian starts off his model by the observation of price dispersion, that brings him to the question of why this can occur. Normally one would expect customers to act rationally and therefore choosing to buy at the store with the lowest price. That however would lead to all stores charging the lowest possible price at which profits are not already negative. Considering stores with identical products and purchasing costs, the price charged should be the same, namely that price that is equal to the costs and therefore would lead to zero profits. (Varian p. 651)

Varian identifies two different types of price dispersion that are taken into account in the literature: spatial and temporal price dispersion. Spatial price dispersion is referring to different stores charging different prices for the same good at the same point in time, while temporal price dispersion means that all stores vary their prices over time (then it clearly can also occur that the same good is sold at different prices in different stores at a given point in time). (Varian p. 651)

As a starting point Varian refers to two older models of price dispersion. The model of “bargains and ripoffs” by Salop and Stiglitz, where they act on the assumption of two different types of customers, the informed ones, who know the whole price distribution, and the uninformed ones, who know nothing about the prices. Varian however considers this implausible as due to the persistence of the price dispersion, the uninformed customers should learn (at least something) about the price distribution over time. The second model was invented by Shilony, who models an economy where consumers can buy in a neighbourhood store, not inflicting any additional costs on them, or buy in stores further away, where prices could possibly be lower but search costs would be imposed. (Varian p. 651-652)
The Model by Varian

Varian combines the two models described above in order to allow for different types of customers (informed and uninformed ones) as well as for randomized pricing strategies by firms at the same time. In his model he makes the simplifying assumption, that the only sales strategy used is that of price discrimination between the two different types of customers. (Varian p. 652)

In his model each customer has a reservation price (denoted by $r$). Informed customers (their number is denoted by $I$) always buy in the store with the lowest price. Uninformed customers (there are $M$ of them) choose a store at random and buy there as long as the price of the store is not above their reservation price. All stores in the economy (their number is given by $n$) have a density function $f(p)$ that denotes the probability of charging the price (denoted by $p$) for each possible $p$. (Varian p. 652)

Each week stores decide on their price via the density function $f(p)$. (The period after which prices change seems to be chosen more or less arbitrarily; but it should not become to large, as then one would again expect learning should take place, so that in the end there should not be uninformed customers any more.) The store with the lowest price gets $I+U$ customers (where $U = M/n$ is the share of uninformed customers); while all other stores only get $U$ customers. By assumption all stores have the same strictly declining average cost curves (denoted by $c(q)$). (Varian p. 652)

$p^* = \frac{c(I+U)}{(I+U)}$ denotes the average cost at the highest possible number of customers $I+U$ a store can achieve (by charging the lowest price). No store will charge any price above the reservation price $r$ or below $p^*$. Stores will not go beyond the reservation price, as there demand and therefore profits will be zero. Going below $p^*$ too will also not occur, as there profits would be negative. Therefore it should hold that:

\begin{equation}
(1) \quad f(p)=0 \text{ for } p>r \text{ or } p<p^*
\end{equation}

(Varian p. 653)
There exists no symmetric equilibrium where the price is the same for all stores. Would all stores be charging the same price only a slight cut in price by one store would be sufficient to get all informed customers and thereby making a positive profit. (Varian p. 653)

No single point in the pricing strategy has a positive probability. If there were such points with a positive probability, the probability of a tie at that point would also be positive, therefore a small shift of this point with positive probability arbitrarily little to the left will imply a gain in profits. Therefore none of the points in the equilibrium density function will have positive probability. By those findings Varian comes to the conclusion, that the cumulative distribution function (denoted by $F(p)$) will be a continuous function. (Varian p. 653-654)

Varian than calculated the expected profit by considering the two possible cases occurring for a store. The first is that of having picked the lowest price, which happens with probability $(1-F(p))^{n-1}$. The second case is that of not charging the lowest price which happens with the converse probability $1-(1-F(p))^{n-1}$. The store with the lowest price (that therefore gets $U+I$ customers) makes the profit $\pi_s(p)=p(U+I)-c(U+I)$, stores that charge a higher price (and therefore only have $U$ customers) get $\pi_f(p)=pU-c(U)$, therefore the expected profit is:

\[
E(\pi) = \int p \cdot \{\pi_s(p)((1-F(p))^{n-1})+\pi_f(p)(1-(1-F(p))^{n-1})\} f(p) \, dp
\]

(Varian p. 654)

The firms face a maximization problem, where they have to choose the density function $f(p)$ by which to determine their prices, in order to maximize the expected profit. The expected profit for all prices must be equal, as otherwise it would pay off to increase the probability of charging one of the prices with a higher expected profit. As Varian considers the case where free entry to the market is given (therefore as long as positive profits occur firms enter the market), this profit must be equal to zero:

\[
\pi_s(p)((1-F(p))^{n-1})+\pi_f(p)(1-(1-F(p))^{n-1}) = 0
\]

(Varian p. 654)
The cumulative distribution function of Varians model therefore follows directly from equation (3) and looks as follows:

\[ (4) \quad 1-F(p) = \left[ \frac{\pi_f(p)}{\pi_f(p) - \pi_s(p)} \right]^{1/(n-1)} \]

\( \pi_f(p) - \pi_s(p) \) is definitely negative because \( p(U+I) - c(U+I) > pU - c(U) \). Therefore \( \pi_f(p) \) must also be negative in order not to get a negative probability. \( F(p) \) should be an increasing function in \( p \), as \( \pi_f(p)/(\pi_f(p) - \pi_s(p)) \) decreases in \( p \). This can be shown by taking the derivative, which yields that \( (\pi_f(p) - \pi_s(p))U - \pi_f(p)(-I) \) has to be negative. By substituting for \( \pi_f(p), \pi_s(p) \) and rearranging the equation one gets the inequality \( c(I+U)/(I+U) < c(U)/U \), which has to be true as \( c(p) \) has been defined to be strictly decreasing in \( p \). (Varian p 654-655)

As those equations only hold as long as \( f(p) > 0 \) Varian continues his paper by showing that \( f(p) \) is positive in all three relevant cases, i.e. when approaching the lower and upper limit as well as between the limits.

- At the lower limit it should hold that \( F(p^* + \varepsilon) > 0 \) for any \( \varepsilon > 0 \). This is trivially true, as if \( F(p^* + \varepsilon) \) would be 0 one store could charge a lower price, e.g. \( p^* + \varepsilon/2 \) and would thereby get all informed customers and making a positive profit as its price would be higher than \( p^* \) (by \( \varepsilon/2 \)).

- At the upper limit it should hold that \( F(r - \varepsilon) < 1 \) for any \( \varepsilon > 0 \). Defining \( p_{max} < r \) the highest price ever charged, then a firm charging \( p_{max} \) will only get the uninformed customers. As the expected profit for every possible price has to be 0 it follows that \( p_{max} U - c(U) = 0 \), as \( r > p_{max} \) it follows that \( rU > pU \) and therefore \( rU - c(U) > 0 \), therefore charging \( r \) would lead to a positive profit.

- As a last step Varian shows, that there is no gap in between any two prices \( p_1, p_2 \) where \( f(p) = 0 \) Suppose there were such a gap and denote a price in this gap by \( p' \) such that \( p_1 < p' < p_2 \) Where \( p' \) succeeds in being the lowest price under the same circumstances as \( p_1 \), that is when all other prices are higher than \( p_2 \), if \( p_1 \) is not the lowest price, \( p' \) is neither. Then in both cases \( p' \) would yield a higher profit than \( p_1 \) as \( p' > p_1 \), from which it follows that charging price \( p' \) will lead to positive profits .

(Varian p. 655)
It should be clear that a store that charges the reservation price $r$ gets only its share of the uninformed customers so that $\pi_f(r)=0$, on the other side, a store charging $p^*$ will get all informed customers and $\pi_s(p^*)=0$. Using these equations one can also determine $n$ and $p^*$.

(Varian p. 655)

To get some insights into the model, Varian calculates the equilibrium density for the case where $c(p)=k>0$. Then one gets:

- $\pi_s(p)=p(U+I)-c(U+I) \iff \pi_s(p)=p(U+I)$
- $\pi_f(p)=pU-c(U) \iff \pi_f(p)=pU-k$
- $\pi_f(r)=rU-k \iff 0=rM/n-k \iff k=rM/n \iff n=rM/k$
- $U=M/n \iff U=k/r$
- $\pi_s(p^*)=p^*(U+I)-k \iff 0=p^*(k/r+I)-k \iff k=p^*(k/r+I)$
- $\pi_f(p)=F(p)=F'(p)$

(Varian p. 656)

Varian's model leads to a U-shaped density function, that is prices closer to $r$ and $p^*$ are charged with a higher probability than intermediate ones. This seems natural as shops would like to charge informed customers the price $p^*$ and the uninformed price $r$, as they are required to sell at the same price to all consumers they tend to switch between the two extremes. In practice the density function is dependent on the proportions of informed and uninformed customers. This becomes immediately clear if one considers extreme cases: If there would only be one uninformed customer and a large number of informed ones, high prices would be charged with smaller probability as the expected number of customers for all stores not charging the lowest price is only $1/n$. If there would only be one informed customers and a large number of others who are uninformed, small prices would be charged with a smaller probability, as being the store with the lowest price only gives the store one additional customer, while the profit made by selling to the uninformed customers decreases with the decrease in price. (Varian p. 656)
The next matter of interest to Varian is the question of the benefit of being informed. Therefore one wants to know the average prices paid by the informed respectively uninformed customers.

- For the uninformed customers:  
  \[ \bar{p} = \int_{r}^{p^{*}} p f(p) dp \]  
  \[ \Rightarrow \bar{p} = r - \int_{r}^{p^{*}} F(p) dp \]  
  \[ \Rightarrow r - \int_{r}^{p^{*}} \left[ 1 - \kappa / (1/p - 1/r) \right]^{1/(n-1)} dp \]  
  \[ \Rightarrow p^{*} + \kappa / (1/p - 1/r) \left[ 1/(n-1) \right] \]  

- For the informed customers \((p_{min})\):  
  \[ f_{min}(p) = (1-F(p))^{n-1} f(p) \]  
  \[ \Rightarrow f_{min}(p) = (1-(1-(k/l(1/p-1/r)))^{n-1}) f(p) \]  
  \[ \Rightarrow f_{min}(p) = 1 - (1-k/l(1/p-1/r)) f(p) \]  
  \[ \Rightarrow f_{min}(p) = k/l(1/p-1/r) f(p) \]  
  \[ \Rightarrow \bar{p}_{min} = k/l \int_{p^{i}}^{p} p (1/p-1/r) f(p) dp \]  
  \[ \Rightarrow k/l(1-p/r) \]  

(Varian p. 657)

Varian then calculates comparative statics for the case of fixed costs. His results are shown in the following table:

<table>
<thead>
<tr>
<th></th>
<th>k</th>
<th>l</th>
<th>M</th>
<th>r</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^{*} )</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( n )</td>
<td>-</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( F(p) )</td>
<td>?</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>( \bar{p} )</td>
<td>?</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>( \bar{p}_{min} )</td>
<td>?</td>
<td>?</td>
<td>-</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

\( \lambda \) denotes the fraction of informed customers:  
\[ \lambda = l/(l+M) \]  
(Varian p. 657)

Two features of the comparative statics seem interesting to Varian. The first is, that is can be shown that the average price paid by the uninformed consumers rises in the number of uninformed consumers. This means the more uninformed consumers, the higher the average price they pay. The second interesting feature is, that on the other hand informed customers tend to pay lower prices on average when the number of uninformed customers gets bigger. Varian explains that by the observation, that with a higher number of uninformed customers more shops enter the market \((n \text{ rises})\), and as informed customers in this model always only pay the lowest price a larger number of stores tends to lower the average price of the informed customers. (Varian p. 657)
Interestingly empirically we can observe, that Germany, where the number of Supermarket chains is higher then in Austria, where the market is more or less split between two companies prices tend to be lower.

In this model Varian takes the informed an uninformed customers as exogenously given, but he points out that the decision of whether to be informed or not can be made endogenous quite easily. He applies the Salop and Stiglitz example and considers the case, where customers can decide to become fully informed about the price distribution by paying a fixed amount $c$. Furthermore he supposes that there are two types of consumers, that have different search costs $c_1$ and $c_2$, where $c_2 \geq c_1$. Therefore the average prices of uninformed customers stay the same, while the average price of informed customers now is $\bar{p}_{\text{min}} - c_i$. (Varian p. 657-658)

For an equilibrium to be a full equilibrium none of the groups of customers should see any advantage in changing their behaviour of being informed or uninformed respectively. Varian considers the case of $c_2 > c_1 = 0$ (actually one can always assume $c_1$ to be zero, as only the difference between $c_1$ and $c_2$ is of importance). This would be a situation where the group with $c_1$ search costs will always be informed. For an equilibrium to exist (for which one also needs uninformed customers) consumers with search costs of $c_2$ will not want to be informed. This is the case as long as $p < \bar{p}_{\text{min}} + c_2$. (Varian p. 658)

Varian concludes his article by a summary in which he recurs on his findings, that in his model stores trying to discriminate in prices between uninformed and informed customers will come to play mixed strategies with a U-shaped density function. This seems intuitively plausible and he also finds some evidence in real live retailing to support his finding of intermediate prices being less probable in America at his time. (Varian p. 658)

Even though Varian wrote his article over 30 years ago, one can still find a price dispersion of the kind Varian observed nowadays even on another continent. In Austria supermarkets tend to sell some of there products at a discounted price. Those discounts normally are about 25 % and last for a week or a month. (For example: Spar Monatssparer, Friends of Merkur Angebot der Woche, Billa Tiefpreislatte)
Advantages of the Model

From my perspective, the main advantages of Varian's model are:

- The model is very simple but nevertheless explains some of the price dispersion we observe in reality.

- Varian's model accounts for both, the differences between informed and uninformed customers and randomized pricing strategies. This adds plausibility to his model in comparison to the models he refers to as a starting point, as due to the time dimension in price dispersion customers cannot predict prices by learning over time.

- He also shows, that one can also easily model a situation where the decision of being informed or not is perfectly rational within his model.

Shortcomings

Besides these advantages there are also some shortcomings, most of them already identified by Varian himself. It should not be forgotten that this was one of the early works in the field of price dispersion and therefore a model not only allowing for, but most presumably designed for, further elaboration.

The following points could be starting points for further improvement:

- Reasons for sales behaviour other than price discrimination between informed and uninformed customers like “inventory costs, cyclical fluctuations in costs or demand, loss leader behaviour, advertising behaviour” (Varian p. 652) are not taken into account. (Varian p. 652)

- The model does only consider completely informed and completely uninformed customers. The possibility of different degrees of being informed (e.g. knowing only the prices of some stores) is ruled out in the model but certainly existent in reality. (Varian p. 652)
• He only considers the case of a symmetric equilibrium meaning all firms have the same pricing strategy. In reality we observe some stores operation on a lower price basis than others. *(Varian p. 652)* (However, the question whether these stores compete for the same costumers or not would need further investigation).

• It may also be of interest to consider pricing strategies of combining high prices of one product by low prices of another, as shops are often observed selling some of there goods at discounted prices but seldom all their products.

• In addition this may also add another dimension to the information of customers. Maybe some customers are informed about some products while uninformed about others?

**Literature:**