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The Edinburgh Building
Shaftesbury Road
Cambridge CB2 2RU, UK

TELEPHONE 01223 312393
FAX 01223 315052
http://www.cup.cam.ac.uk

Hofbauer, Josef; Sigmund, Karl * 90287
Evolutionary games and population dynamics.
Cambridge: Cambridge University Press, (ISBN 0-521-62570-X/pbk; 0-521-62365-0/hbk). xxvii, 323 p. £ 16.95; $ 27.95/pbk; £ 50.00; $ 69.95/hbk (1998).

The monograph under review displays a mathematical analysis of population dynamics in ecosystems. The models and equations which are examined reflect a game theoretic approach, that is, each species acts as if it participates in a multi-agent game, and the species which we observe are those that survive. The mathematics portrays much more than survival, as the possible dynamics resulting out of the behavioral strategies are outcome of the models.

Before delving into the details of the models, it should be noted that the authors did an excellent job in the presentation. Enlightening comments relate the mathematical models and derivations to the reality they depict. Comments on the history of the subject accompany the text, and extensive references to the literature enable to complete the picture. New results are displayed along with the classical analysis of the subject. Exercises provide the newcomers the possibility to test their understanding. The text is addressed to both mathematicians and biologists (and social scientists interested in the sociology of species). The conclusions concerning ecology are given through mathematical eyeglasses, and thus proficiency in the mathematical vocabulary is required. For most of the text the mathematical prerequisites are advanced calculus, linear algebra and basic differential equations. At some points higher level is of help, for instance when concepts like Hopf bifurcation and homotopy are introduced and employed.

The first part of the text examines dynamical systems on a finite-dimensional space, where each coordinate represents a species. Thus, the chapter offers glance at differential equations, concentrating on equations which arise in population dynamics. The text points out the relations between the formulation of the equations and the reflected behaviour of the different species. The interpretation of characteristics of the resulting solutions is then examined. The equations and concepts analyzed in this category include the logistic equation, Lotka-Volterra systems, cooperative versus competitive systems, ω-limit sets, Poincaré-Bendixson theorem, cyclic phenomena, chaotic dynamics.

Part two introduces game-theoretic considerations. It starts with basic notions of games in normal form, and then shifts to the more involved framework of repeating and evolutionary games. The prime game theoretic concept in this framework is the Nash equilibrium, which is quite transparent when a game is finitely repeated, but its reflection on dynamic games is far from being agreed upon. The chapter offers little about classical game theory, and concentrates on the dynamical aspects. The text introduces and analyses notions of stable equilibria. Strategies leading to such equilibria are examined. Replicator dynamics, the relatively new development offered in the text, is a behavioral law within which the species try, roughly, to repeat evolutionary success. The text examines the effect of such dynamics in several situations. Other behavioral laws such the best response, adjustment rules and adaptive dynamics are examined. The latter concept is dealt with in detail, including prisoner dilemma like games in various situations.

The third part concentrates on stability, and permanence. Stability is taken in the Lyapunov sense, namely that perturbations from, say, an equilibrium, that occur spontaneously, are corrected by the dynamics. Permanence reflects the possibility the trajectory may not lead to an equilibrium, yet it wonders in a region that allows for all the participating species to survive. These notions are examined in various ecological frameworks. The mathematics involves Lyapunov functions considerations, and linear algebra tools, like the Perron-Frobenius theorem.

The last part displays issues specific to populations genetics. The mathematics here are more of a discrete type, introducing selection rules, mutation-selection, recombination arrangements, etc. Here the game theoretic aspects play a lesser role, although the dynamics in interpreted as resulting from applying mixed strategies.

Z. Artstein (Rehovot).