This review appeared in

INTERNATIONAL MATHEMATICAL NEWS 179

Dated DEC 1998

CAMBRIDGE UNIVERSITY PRESS

with compliments

The Edinburgh Building
Shaftesbury Road
Cambridge CB2 2RU, UK

TELEPHONE 01223 312393
FAX 01223 315052
http://www.cup.cam.ac.uk

HOFBAUER J. — SIGMUND K.: *Evolutionary Games and Population Dynamics.* Cambridge University Press, 1998, XXVII+323 S. ISBN 0-521-62570-X P/b £ 16,95, ISBN 0-521-62365-0 H/b £ 50,–.

Ten years ago, the authors published their book „Theory of Evolution and Dynamical Systems", which was a seminal work in Evolutionary Game Theory. The confessed aim of the present book is to replace that one. Evolutionary Game Theory is by now a well-established subject, with applications both in Economics and in Biology. Thus, the authors concentrate now on the mathematical theory underlying one part of this theory, namely deterministic models in continuous time.

The book is divided into four parts of markedly different character. The first three parts form a mathematical textbook on replicator dynamics and Lotka-Volterra equations, while the fourth focuses on population genetics.

The first part is an introduction to dynamical systems, focused on Lotka-Volterra equations. These equations describe the evolution of population numbers in ecosystems. The basic concepts of dynamical systems are reviewed, from Lyapunov functions to the Poincaré-Bendixson theorem and Hopf bifurcations. It is noteworthy how the authors have managed to give such a complete view of the basics of dynamical systems in only 50 pages, when their aim was to concentrate in Lotka-Volterra equations.

Once the basics are established, the second part focuses on the Replicator Dynamics, which arises when a population of agents evolves according to the results of an underlying symmetric game. This dynamics describes the evolution of the frequencies with which the strategies of the game are played in a large population framework. The basic results for replicator dynamics and evolutionary stability are reviewed here. Next, the authors briefly mention some alternative game dynamics and devote a chapter to a discussion of adaptive dynamics, seen as a generalization of the Replicator Dynamics. This part is completed with a discussion on dynamics arising from bimatrix games. All together, the topics in this part cover what might be a first basic course in Evolutionary Game Theory.

The third part is devoted to stability, permanence, and persistence in replicator and Lotka-Volterra equations. This is a much deeper study of both of them. The main point of the authors is that these two models are mathematically equivalent.

The fourth part leaves the realm of pure mathematics and discusses models of population genetics. After an initial chapter on discrete dynamical systems (which stands in contrast to the rest of the book, devoted to continuous-time dynamics), this part focuses on the applications of replicator equations to the biological sciences.

The book is a must for any mathematician, economist, or biologist working in Evolutionary Game Theory. The only criticism could come from economists, since the book is definitely biased towards biological applications. It is worth to note that it is suitable for use as a textbook, including an enormous amount of exercises.

C. Alos-Ferrer (Wien)