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Hofbauer, J. & **Sigmund**, K. 1998: Evolutionary Games and Population Dynamics. Cambridge University Press, Cambridge. 323 pp., figs, Hb £50.00, US$69.95, ISBN 0-521-62365-0; Pb £16.95, US$27.95, ISBN 0-521-62570-X.

Reviewed by Jürgen Tomiuk, University of Tübingen.

Species' competition, predator–prey relations and behavior of individuals or groups of individuals have been always fascinating topics in biology. In the 1920s Lotka & Volterra first modelled the dynamics of host-parasite and predator-prey populations. Shortly after this the Russian ecologist Gause (1935) pointed to the significance of the Lotka–Volterra equation for species' coexistence with his well-known experiments in which he studied competition among different *Paramecium* species. Gause's exclusion principle states that species using identical niches, i.e. individuals being affected by the same intensity in space and time through all biotic and abiotic factors, cannot coexist. At the beginning of the 19th century mathematics was obviously already accepted in ecological research, but nevertheless studies in behavior ecology have been almost exclusively descriptive for a long time. It was at the beginning of the 1970s when game theory was applied to animal behavior (e.g. Maynard Smith 1974). Payoff, cost and benefit of behavioral strategies have been analysed in order to understand their evolutionary significance. Interactions among individuals have been considered in an evolutionary genetic context and today ESS, the evolutionary stable strategy, is a well-known concept in population biology. Meanwhile evolutionary game theory evolved into an interesting topic for both biologists and mathematicians.

Mathematical models simplify reality to the expense of proving the existence and stability of nontrivial solutions that describe the behavior of a biological system (e.g. a trivial genetic state is given in a population whose individuals are genetically identical at a locus). Biological modelling also provides insight into the dynamics of biological systems: The state of systems can be static or oscillating. Sets of parameters determining biological mechanisms can be identified for which the state of a system can drastically be changed, or systems might pass from one state into another and this process can be (ir-)reversible. Finally, simple models can be the basis for our understanding of more complex biological systems.

This book focuses on the mathematical treatment of models in population dynamics, evolution and population genetics. First Lotka–Volterra equations, e.g. predator–prey relations and competiton of two and more species, are considered in detail which turns out to be useful in the succeeding chapters about evolutionary games. However, biologists should not let themselves be deceived by the excellent and nonmathematical introduction: later readers will have to deal with pure mathematics. Indeed the authors clearly point out in the preface that the new issue of their former book *Theory of Evolution and Dynamical Systems* is no longer an interdisciplinary approach but a mathematical textbook in which the biological chapters are removed. There are definitions, theorems, proofs and the focus of attention is the existence and stability of solutions of mathematical models.

The book comprises four sections with 22 chapters in total: Part I, *Dynamical Systems and Lotka–Volterra Equations*, has five chapters considering logistic growth, Lotka–Volterra equations for predator–prey systems, for competing species, and ecological equations for two species and more than two species. Part II, *Game Dynamics and Replicator Equations*,consists of six chapters. Part III, *Permanence and Stability*, and Part IV, *Population Genetics and Game Dynamics*, have six and five chapters, respectively. Short notes about the intention of the respective parts introduce each new section of the book which, in addition, close with an interesting short description of the history and names of scientists dealing with the problems under consideration. A lot of exercises are embedded in the theoretical framework which enables readers to learn and to test their new mathematical knowledge.

This book can highly be recommended to mathematicians interested in applications in social sciences, biology, and population genetics. As a textbook, students of mathematics especially will profit if they want to go into biological modelling. Biologists, however, will benefit from this book only if they have a good mathematical background in algebra, analysis, function theory and ordinary differential equations.

References

Gause, G. F. 1935: Experimental demonstration of Volterra's periodic oscillation in the numbers of animals. J. Exp. Biol. **12,** 44—48.

Maynard Smith, J. 1974: The theory of games and the evolution of animal conflicts. J. Theor. Biol. **47,** 209—221.