Proefexamen Capita Selecta

Naam : _

Nummer : _

- Zijn toegelaten: Cursusnota's. Zijn niet toegelaten: Boeken, rekenmachines, GSM's enz.
- U hebt 2 uur tijd om de vragen op te lossen. De maximumscore is 20.
- Veel Succes !

Probleem	**1**	**2**	**3**	**4**
Punten	**4**	**6**	**7**	**3**
Score				

$$\boxed{\sum = \quad}$$

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Capita Selecta

— Proefexamen —

Summer Term 2004

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1. Split exact sequences. Let $n \geq 1$ be an integer. Give group homomorphisms α and β such that

$$0 \to \mathbb{Z}/n\mathbb{Z} \xrightarrow{\alpha} \mathbb{Z}/n^2\mathbb{Z} \xrightarrow{\beta} \mathbb{Z}/n\mathbb{Z} \to 0$$

becomes a short exact sequence of abelian groups (written additively). Is this sequence always split ?

(4 Points)

2. Group extensions. Let $0 \to A \xrightarrow{\alpha} E \xrightarrow{\beta} G \to 1$ be a group extension of A by G and assume that A is abelian.

(a) For $x \in G$ choose an $e \in E$ such that $\beta(e) = x$. Then set $x \cdot a = eae^{-1}$ for $a \in A$. Show that this is a well-defined action of G on A so that A becomes a G-module.
(b) Show that equivalent extensions of A by G give rise to the same action.

(6 Points)

3. Cohomology groups.

(a) Let $G = \mathcal{A}_5$ be the alternating group on five letters and $A = \mathbb{Z}/7\mathbb{Z}$ be a trivial $\mathcal{A}_5$-module. Compute $H^n(G, A)$ for $n = 0, 1, 2$.
(b) Let G be a finite group and $\mathbb{Q}$ be a G-module. Show that $H^n(G, \mathbb{Q}) = 0$ for all $n \geq 1$.

(7 Points)

4. Profinite groups. Let G be a profinite group and H be an open subgroup of G. Show that H is of finite index in G.

Honors problem: Give an example of an infinite nonabelian profinite group.

(3 Points)

Bespreking: Dinsdag, 18 Mei 2004