Optimal demand management policies with probability weighting

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Abstract

We review the optimality of partial insurance contracts in the presence of moral hazard when consumers are affected by probability weighting. In valuing risky prospects individuals commonly overweight extreme outcomes at the expense of intermediate outcomes; these deviations systematically bias the ex-ante value of insurance. We characterize optimal contracts for the case of perfectly competitive insurance markets taking this bias into account and find that full insurance is preferred more often for illness that has low probability, while partial insurance or even no insurance are preferred for illness that has higher probability. The latter result justifies the existence of a universal compulsory minimum level of health insurance.

1 Introduction

Demand management policies are a main element in the design of health insurance contracts. They typically take the form of deductibles or coinsurance rates that insurance holders must bear when they make use of the services being insured. Received theory of insurance suggests that this form of risk sharing allows to strike the right balance between the consumer's desire to insure and the incentives problem created by insurance; since insurance holders have access to the demand for health

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services without having to face its full financial cost, insurance results in overexpenditure on those services if the state of health is not fully contractible. This problem of moral hazard in health care and the desirability of partial insurance were originally pointed out by Pauly (1968) and Zeckhauser (1970). While there is general agreement regarding the estimates for the price-elasticity of demand for health services that determine the extent of moral hazard (Zweifel and Manning 2000), there are still wide disparities in the estimates of the optimal insurance policies in the presence of moral hazard, that have been proposed by different papers in the literature (Cutler and Zeckhauser 2000, p. 587). These authors suggest that different degrees of risk aversion and moral hazard might underlie such disparities.

Optimal insurance policies are usually calculated to maximize ex-ante consumer's welfare measured by expected utility, taking into account how the fraction of cost born by consumers affects demand after the purchase of insurance. However, the theoretical literature on decision under risk is ripe with experimental and empirical observations of systematic violations of the axioms underlying expected utility. Promising alternative theories that fare better, at least in the description of how choices under risk are actually taken, are rank-dependent theories like the theory of anticipated utility (Quiggin 1982) and cumulative prospect theory (Tversky and Kahneman 1992). These theories distinguish between attitudes towards risk and attitudes towards probability. The former refer to the features of utility functions which measure sensitivity towards outcomes. The latter substitute objective probabilities with so called decision weights in calculating the value of a prospect. The properties of weighting functions have been tested empirically giving extensive support to inverted S-shaped functions that overweight outcomes with extreme ranks and underweight intermediate outcomes.[1] Different methods for studying the qualitative properties of attitudes towards probabilities independently from outcome sensitivity have been proposed by Wakker (2001) and Abdellaoui (2002).

The present paper reviews the optimality of partial insurance contracts when individuals are affected by probability weighting. We consider a simple model of health insurance where illness comes in two different levels of severity; a fixed-cost treatment that fully eliminates the consequences of illness should be applied in the severe state only.[2] Ideally, individuals affected by illness would like to be be fully compensated for the financial losses incurred due to illness. These, however, depend on the level of severity which cannot be observed by the insurance company. If the difference between the cost of treatment and the losses due to the low severity is large enough, it is optimal from an ex-ante perspective to purchase partial insur-

[1] See Wakker (2001, Appendix A) for an account of this empirical evidence. An early paper that points to the relevance of prospect theory considerations in the choice of health plans is Ellis (1989).

[2] Our model is a simplified version of the one used in Ma and Riordan (2002).

ance, that induces demand for the service in the severe state only, provided that consumers behave according to expected utility theory. The introduction of decision weights changes the value of insurance ex-ante. As a result, the optimality of partial insurance depends on the probability of illness. We show that full insurance is optimal for a larger range of severities if the probability of illness is low, whereas partial insurance turns out to be preferred in a larger range of severities if the probability of illness is high enough. In general, the purchase of insurance may change the rank of consequences. To simplify the analysis, we restrict our attention to insurance policies that would not alter this rank, excluding the possibility of overinsuring some contingencies. Our result basically follows from overweighting the state of perfect health (best consequence) when its probability is low and underweighting it when its probability is high, basic features that are implied by inverted S-shaped weighting functions.

Although the normative implications of behavioral theories are arguable, two implications seem to follow readily from our preliminary results. First, if demand management policies are aimed at restoring efficiency to the extend they can, fine-tuning is required in the design of policies, taking into account how individuals perceive risk. Moreover, in extreme cases,it might be necessary to impose a minimum compulsory level of insurance for important losses that are typically underweighted.

Our paper is closely related to Ryan and Vaithianathan (2003), who also study health insurance with rank-dependent utility. In a model where strictly positive coinsurance rates are always optimal, provided there is no probability distortion, they show that full insurance (zero coinsurance rate) may be optimal when decision weights are introduced. Whereas analysis there is restricted to local optimality, our simple model allows to explicitly compute global optima, and the role that the different magnitudes of losses and their probabilities play in the optimality of full and partial insurance. They conclude that the introduction of decision weights explains the prevalence of zero coinsurance rate in privately contracted supplemental insurance. Our analysis also sheds light on the reasons for the opposite phenomenon, namely lack of insurance.

Finally, the paper also points to the limits of demand management policies and provides and additional rationale for the use of supply-side cost containment measures in aiming at efficiency in health care markets. Although we assume perfect competition in insurance markets and use fair insurance premia, probability distortions, and in particular excess pessimism, open the door to unfair premia in the design of contracts and imperfect competition in insurance markets, if insurance companies can better understand and estimate the incidence of illness than consumers do. The implications of probability weighting for imperfectly competitive markets are an important topic left for future research.

The paper is organized as follows. Section 2 introduces the model and obtains optimal insurance policies for the case of linear weights. Section 3 then characterizes optimal contracts for the problem introduced in the presence of probability weighting. Section 4 concludes. Proofs are deferred to the Appendix.

2 A simple moral hazard problem

In the present section we review a simple benchmark model of insurance where individuals are affected by moral hazard. In this simple setting, we identify conditions under which partial (instead of full) insurance is optimal.

Consider a population of *ex ante* identical individuals, each of them facing the risk of illness, who consider the purchase of health insurance. Each individual can end up in one of three possible states of health: state h which corresponds to *perfect health*, state m which corresponds to *mild illness*, and state s which corresponds to *severe illness*. We denote $S = \{s, m, h\}$ the set of states and $0 < p_i < 1$ the probability of state $i \in S$, with $\sum_i p_i = 1$.

The consequences of illness for any individual can be measured in terms of a financial loss; we denote by ℓ_i the loss associated to state i and assume that $\ell_h = 0$ and $\ell_s > \ell_m > 0$. Illness can be treated at a fixed cost C, independent of severity, and the application of treatment results in full recovery of any loss. We assume that $\ell_m < C < \ell_s$.

Consumers have preferences over states of wealth represented by a utility function $u : \mathbb{R} \to \mathbb{R}$ defined over wealth levels $\omega \in \mathbb{R}$. We assume that $u(\omega)$ is a twice continuously differentiable function satisfying $u'(\omega) > 0$ and $u''(\omega) < 0$ for all $\omega \in \mathbb{R}$.

No insurance

In the absence of insurance and given that $\ell_m < C < \ell_s$, individuals would prefer to suffer loss ℓ_m rather than going for treatment at cost C in the case of mild illness, while they would prefer to go for treatment rather than suffering loss ℓ_s in the case of severe illness. Therefore, the expected utility of an uninsured individual would be

$$U^0 \equiv p_s u(\omega - C) + p_m u(\omega - L) + p_h u(\omega)$$

where $L = \ell_m$ henceforth and ω denotes initial wealth.

Ideal insurance

We now introduce the possibility that individuals insure against illness. First suppose that illness is *contractible*; that is, the insurance company is able to condition the level of indemnity on the state of health. An insurance contract in that case specifies a premium P, the price of the insurance the individual must pay, and a collection of indemnities (I_s, I_m) that the individual gets from the insurance company in each possible state of illness. We assume that the insurance market is perfectly competitive, so in equilibrium firms earn zero expected profit; that is, insurance policies always satisfy the break-even constraint

$$P - \sum_{i \in S} p_i I_i = 0. \tag{1}$$

The expected utility of purchasing insurance contract (P, I_s, I_m) is then given by

$$U(P, I_s, I_m) = p_s u(\omega - P - C + I_s) + p_m u(\omega - P - L + I_m) + p_h u(\omega - P). \tag{2}$$

The ideal insurance (P^*, I_s^*, I_m^*) would maximize U as given in (2) subject to the break-even constraint given in (1). It is well-known that the optimal insurance in such a setup corresponds to full insurance provided individuals are risk averse. Indeed, it is easy to see in our model that the optimal contract is such that a individual's marginal utility is equalized over all three states.[3] This implies that the ideal contract should equalize income over all states and, thus, it should be such that severely ill individuals get paid for treatment with $I_s^* = C$, while mildly ill individuals do not get treatment but are fully reimbursed for their financial loss with $I_m^* = L$. The corresponding break-even premium is $P^* = p_m L + p_s C$. The expected utility of individuals with ideal insurance is

$$U^* \equiv u(\omega - p_m L - p_s C).$$

The ideal contract provides efficient risk spreading, fully eliminating individuals' financial risk associated with illness.

Non-contractible illness

A problem of *moral hazard* arises, if the insurance contract cannot condition the level of indemnity on the realized event; this is the case for example if the insurance company cannot observe the state of health and payment must be based on the individual's level of expenditure in health services. Such contracts cannot provide efficient risk spreading because they create a problem of incentives: if the cost of treatment is always fully reimbursed, even mildly ill individuals would opt for it after they have paid the insurance premium, thus inducing medical overexpenditure. For the case of health insurance this problem was first formally treated by Zeckhauser

[3] Concavity of u guarantees that individuals purchase strictly positive amounts of insurance against both illness severities and that they do not want to overinsure any of the severities; i. e. at an optimum $0 < I_m^* \leq L$ and $0 < I_s^* \leq C$.

(1970) who suggested that demand management policies, where individuals must bear part of the treatment costs may strike the right balance between risk spreading and appropriate incentives.

To see this in the context of our model suppose now that only insurance contracts of the form (P, I) can be offered, where the insured pays premium P and gets indemnity I in case of treatment. If $I = C$ the contract provides *full insurance* and *ex post* all insurance holders prefer to be treated independent of severity. To break even, the insurance company sets $P = (1 - p_h)C$. The expected utility of full insurance is $\widehat{U} \equiv u(\omega - (1 - p_h)C)$ which is strictly smaller than U^* since $L < C$. *Ex-ante* individuals may prefer a *partial insurance* contract with $I < C$ where they must bear a strictly positive *deductible* payment $C - I$ in case of treatment.

Assume that individuals refrain from treatment when they are exactly indifferent; i. e. when the deductible is exactly equal to the financial loss caused by illness. To justify this assumption we may think that individuals perceive some small disutility of treatment which is not modeled (e. g. discomfort or uncertainty about success of treatment). After the purchase of insurance at price P, demand for treatment depends on the level of indemnity I. If $I \leq C - L$, insurance holders opt for treatment in the severe state only. In that case the value of insurance is given by

$$U(P, I) = p_s u(\omega - P - C + I) + p_m u(\omega - P - L) + p_h u(\omega - P). \tag{3}$$

If $I > C - L$, then insurance holders would opt for treatment in both states of illness. The value of insurance is then

$$U(P, I) = (1 - p_h)u(\omega - P - C + I) + p_h u(\omega - P). \tag{4}$$

In the former case, individuals are in fact not insured against the minor loss; the corresponding break-even premium is then $P = p_s I$. In the latter case, individuals receive at least partial reimbursement in both illness states; the premium is then $P = (1 - p_h)I$. Obviously, given I the premium is higher when all insurance holders are expected to opt for treatment.

Substituting the corresponding values of P in expressions (3) and (4), expected utility can be expressed as a function of indemnity alone. Figure 1 illustrates the shape of expected utility if L is small relative to C. As long as individuals are not fully insured expected utility increases as income is equalized among states with an increasing indemnity. For I slightly higher than $C - L$, however, mildly ill individuals start to get treatment; although the level of insurance is almost the same, the premium suddenly increases and expected utility drops. In an optimum individuals will either buy full insurance with $I = C$ or partial insurance with $I = C - L$, just enough to refrain from treatment in the mildly severe case. Proposition 1 below

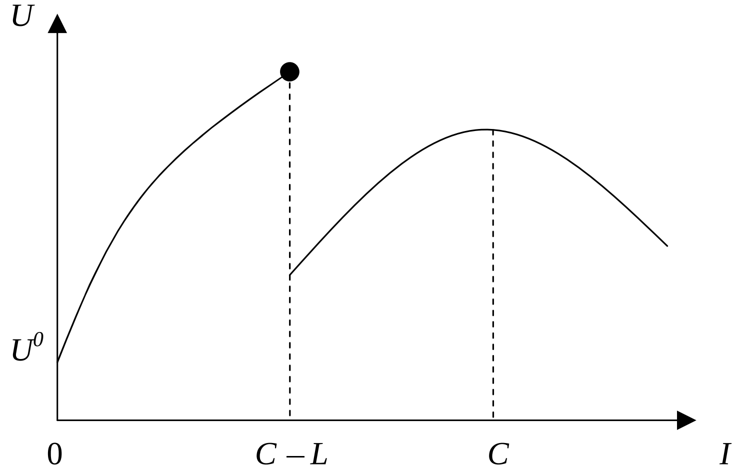

Figure 1: Expected utility with a fair premium when L is small relative to C.

shows that the expected utility of partial insurance is higher than the utility of full insurance if L is small relative to C. For high values of L, however, full insurance is still optimal.

Proposition 1. Given C, there exists $\widehat{L}(C) \in \left(\frac{p_m}{p_h+p_m}\, C, C\right)$ such that the *partial insurance* contract with $I = C - L$ and $P = p_s(C - L)$ is optimal for all $L \leq \widehat{L}(C)$. *Full insurance* with $I = C$ and $P = (1 - p_h)C$ is optimal for all $L > \widehat{L}(C)$.

The demand for treatment when individuals hold the partial insurance contract derived in Proposition 1 coincides with the demand when they hold ideal insurance. Yet with partial insurance mildly ill individuals are not compensated for their loss and severely ill individuals pay a deductible equal to L for treatment. Risk aversion then implies that the expected utility of partial insurance is always lower than the utility of ideal insurance. Indeed, by concavity of u we have

$$\widetilde{U} \equiv (1 - p_h)u(\omega - p_s(C - L) - L) + p_h u(\omega - p_s(C - L)) < U^*$$

The optimality of partial insurance depends mainly on the comparison between the efficiency gains derived from risk sharing and the losses due to medical overexpenditure. For $L \leq \widehat{L}(C)$ the losses due to overexpenditure when individuals are paid for treatment in both illness states would more than offset the gains of risk sharing derived from full insurance.

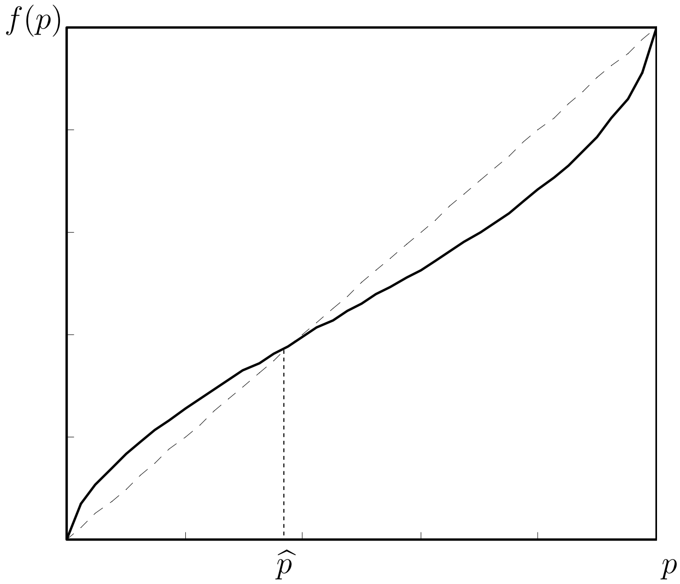

Figure 2: Weighting function.

3 The effect of probability weighting

Here the optimality of partial insurance obtained in the previous section is re-examined for the case of individuals who are affected by the kind of probability weighting proposed by *cumulative prospect theory* (henceforth CPT) developed by Tversky and Kahneman (1992). We maintain the assumptions on the utility function u, strictly increasing and strictly concave; therefore any change in individuals' behavior will come exclusively from the different treatment of probabilities.

Denote $f : [0,1] \to [0,1]$ the weighting function, assumed continuous, non-decreasing, and such that $f(0) = 0$ and $f(1) = 1$. Let $\widehat{p} \in (0,1)$ be such that

$$f(p) \gtreqless p \quad \text{if} \quad p \lesseqgtr \widehat{p}\ ;$$

with $f(p)$ being strictly concave for all $p < \widehat{p}$ and strictly convex for all $p > \widehat{p}$. Figure 2 illustrates the shape of the weighting function. Wu and Gonzalez (1996) estimate the value of $\widehat{p} \approx 0.40$.

Consumers attach decision weights to each consequence which depend on the probability distribution over states and on the rank of consequences. In particular, if $\{1, 2, \ldots, n\}$ is the set of states resulting in consequences $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ ordered from worst to best and $\mathbf{p}$ is the corresponding vector of probabilities, the weight attached to consequence $i \in \{1, 2, \ldots, n\}$ denoted $\pi_i(\mathbf{p})$ is such that $\pi_1(\mathbf{p}) = f(p_1)$ and for all $i \geq 2$

$$\pi_i(\mathbf{p}) = f\left(\sum_{j=1}^{i} p_j\right) - f\left(\sum_{j=1}^{i-1} p_j\right)$$

The value of a prospect $(\mathbf{x}, \mathbf{p})$ is then given by

$$V(\mathbf{x}, \mathbf{p}) = \sum_{i=1}^{n} \pi_i(\mathbf{p}) u(x_i)$$

CPT proposes a different treatment of gains and losses; in particular, it allows to construct weights for gains using a different weighting function as that for losses. Our analysis restricts to prospects with negative consequences corresponding to the losses due to illness. In that case, CPT with weighting function f coincides with *rank-dependent utility* (henceforth RDU; see Quiggin (1982)) using the dual weighting function given by $\widehat{f}(p) = 1 - f(1-p)$, which is also S-shaped and has a fixed point at $1 - \widehat{p}$. That is, constructing weights from the worst to the best consequence using f, as CPT does, is equivalent to doing so from best to worst using $\widehat{f}$, as RDU would prescribe (on this see Tversky and Wakker (1995)).[4]

No insurance

Let us now go back to the simple problem introduced in Section 2. In the absence of insurance, individuals face the prospect with consequences $(\omega - C, \omega - L, \omega)$ and probabilities (p_s, p_m, p_h). Decision weights are given by

$$\pi_s = f(p_s) \qquad \pi_m = f(p_s + p_m) - f(p_s) \qquad \pi_h = 1 - f(p_s + p_m).$$

Without insurance, individuals *perceive* utility equal to the value of this prospect

$$V^0 \equiv \pi_s u(\omega - C) + \pi_m u(\omega - L) + \pi_h u(\omega)$$

Obviously, decision weights change the perception of risk. Suppose for example that illness comes with low probability ($p_s + p_m < \widehat{p}$); the assumptions on f imply that the weight individuals place on illness in general and on severe illness in particular is higher than their respective objective probabilities and, thus, the weight they place on perfect health is smaller than its objective probability. To see this note that $1 - \pi_h = f(p_s + p_m) > p_s + p_m = 1 - p_h$ and that $\pi_s = f(p_s) > p_s$ since $p_s < \widehat{p}$. In this case, individuals are pessimistic towards the risk of illness; comparing V^0 with U^0 we see that they are more disturbed by the consequences of bad risks. Intuitively, this perception should make them behave as if they were more risk averse; in particular, if illness is not contractible, pessimism should make individuals want to insure fully against the risk of illness more often than without probability distortion. On the other hand, if illness comes with high probability ($p_s + p_m > \widehat{p}$), perfect health will be overweighted; less pessimistic individuals of

[4] This is relevant for the comparison of our results to those in Ryan and Vaithianathan (2003). In particular, their π^* corresponds to $1 - \widehat{p}$ in our case.

this kind should intuitively favor partial insurance more often. In fact, individuals may become truly optimistic if the probability of severe illness is very high ($p_s > \widehat{p}$) in which case the severe state will be underweighted and perfect health overweighted. We would not expect that overexpenditure in health services should be a problem for these individuals; more likely they will tend to underinsure. These intuition is confirmed by the analysis in the next section.

Moral hazard with decision weights

We restrict the analysis to insurance contracts (P, I) such that $0 \leq I \leq C$ so that the rank of consequences is not altered. Again, two separate cases must be distinguished. For low levels of indemnity, in particular if $0 \leq I \leq C - L$, only severely ill individuals opt for treatment and a fair premium is of the form $P = p_s I$. After the purchase of insurance individuals would face the prospect with consequences $(\omega - P - C + I, \omega - P - L, \omega - P)$ and probabilities (p_s, p_m, p_h). This type of contract provides the incentives to avoid inefficient treatment and it has value

$$V_1(P, I) = \pi_s u(\omega - P - C + I) + \pi_m u(\omega - P - L) + \pi_h u(\omega - P). \tag{5}$$

On the other hand, all individuals opt for treatment if $C - L < I \leq C$; a fair premium is of the form $P = (1 - p_h)C$. After the purchase of this type of contract, individuals would face consequences $(\omega - P - C + I, \omega - P - C + I, \omega - P)$ with the same probabilities as before. The value is then

$$V_2(P, I) = (1 - \pi_h)u(\omega - P - C + I) + \pi_h u(\omega - P). \tag{6}$$

Substituting the corresponding value of P in expression (5), the expected value of insurance for $0 \leq I \leq C - L$ expressed as a function of indemnity alone is

$$V_1(I) = \pi_s u(\omega - p_s I - C + I) + \pi_m u(\omega - p_s I - L) + \pi_h u(\omega - p_s I). \tag{7}$$

Taking derivatives and rearranging we obtain

$$V_1'(I) = \pi_s u'(\omega_s) - p_s \sum_{j \in S} \pi_j u'(\omega_j) \tag{8}$$

where ω_j denotes wealth in state $j \in S$. Since $\omega_s \leq \omega_m < \omega_h$, it follows from $u'' < 0$ that $u'(\omega_s) > \sum_j \pi_j u'(\omega_j)$. Comparing with the behavior of expected utility with respect to I depicted in Figure 1, we immediately see from expression (8) how weights may change the value of insurance. We still obtain that the value of insurance is increasing with indemnity as long as $\pi_s \geq p_s$ corresponding to the case of $p_s \leq \widehat{p}$. This case is analyzed in the next two propositions. As long as the probability of the severe state is small enough, it will be overweighted and individuals will want to insure at least partially. Remark 1 below refers to the opposite case in which consumers underweight the likely consequences of severe illness.

Proposition 2. If $p_s + p_m \leq \widehat{p}$, there exists $\widetilde{L}(C, \pi_h) \in \left(\frac{p_m}{p_m+p_h}C, C\right)$ such that the *partial insurance* contract (P, I) with $I = C - L$ and $P = p_s(C - L)$ is optimal for all $L \leq \widetilde{L}(C, \pi_h)$ while *full insurance* is optimal for all $L > \widetilde{L}(C, \pi_h)$. Moreover, $\widetilde{L}(C, \pi_h) < \widehat{L}(C)$.

Proposition 2 shows that the case of pessimistic individuals is analogous to the case of linear probabilities. Consumers optimally choose partial insurance for low values of L and they optimally choose full insurance for high values of L. Given C and π_h, the threshold value for L is $\widetilde{L}(C, \pi_h)$. The difference with Proposition 1 is that the range of small losses L for which individuals prefer to insure fully is larger than before. This is implied by the fact that $\widetilde{L}(C, \pi_h) < \widehat{L}(C)$.

Important for the result obtained in Proposition 2 is that both the severe state and the state of perfect health are overweighted. This follows from the assumption that the probability of illness is small in general, $p_s + p_m \leq \widehat{p}$. We now substitute $P = (p_s+p_m)I$ in expression (6) and obtain the value of insurance for $C-L < I \leq C$.

$$V_2(I) = (1 - \pi_h)u(\omega - (p_s + p_m)I - C + I) + \pi_h u(\omega - (p_s + p_m)I). \tag{9}$$

Taking derivatives and rearranging we obtain

$$V_2'(I) = (1 - \pi_h)u'(\omega_{-h}) - (1 - p_h)\{(1 - \pi_h)u'(\omega_{-h}) + \pi_h u'(\omega_h)\} \tag{10}$$

where now $\omega_{-h} = \omega - (p_s + p_m)I - C + I$ and $\omega_h = \omega - (p_s + p_m)I$. Evaluating the derivative at $I = C$ we have

$$V_2'(C) = (p_h - \pi_h)u'(\omega - (1 - p_h)C) < 0,$$

which shows that full insurance can never be optimal in this case. Proposition 3 below treats this case in more detail.

Proposition 3. If $p_s + p_m > \widehat{p}$ and $p_s \leq \widehat{p}$, then full insurance is never optimal; there exists $\overline{L}(C, \pi_h) \in \left(\frac{p_m}{p_m+p_h}C, C\right)$ such that the *partial insurance* contract (P, I) with $I = C-L$ and $P = p_s(C-L)$ is optimal for all $L \leq \overline{L}(C, \pi_h)$; for $L > \overline{L}(C, \pi_h)$ optimal insurance is given by a contract with $I = I(C, \pi_h) \in (C - L, C)$ and $P = (1 - p_h)I$. Moreover, $\overline{L}(C, \pi_h) > \widehat{L}(C)$.

Proposition 3 shows that with higher probability of illness consumers actually become less pessimistic. Voluntarily, they would never insure fully in that case. Consumers still optimally choose partial insurance for low values of L. For high values of L they may opt for a more generous partial insurance contract. Now given

C and π_h, the threshold value for L is $\overline{L}(C, \pi_h)$. The difference with Propositions 1 and 2 is also that the range of small losses L for which individuals prefer to insure partially is larger than before. This is implied by the fact that $\overline{L}(C, \pi_h) > \widehat{L}(C)$.

Remark 1. If $p_s > \widehat{p}$ the analysis is similar to case ii) but more cumbersome. In particular, full insurance is never optimal; further partial insurance might correspond to an interior optimum with $I < C - L$. In extreme cases individuals might prefer to purchase no insurance.

4 Final comments

The use of a simple model with three possible states, perfect health, severe illness, and mild illness, where the consequences of illness can be eliminated at a fixed cost, allows us to easily compute the optimal partial insurance that would exactly balance risk sharing and overexpenditure. We then review the optimality of this contract after substituting objective probabilities with decision weights of the form proposed by rank-dependent theories of decision under risk. We conclude that individuals will prefer to insure fully more often if the probability of illness is small. If the probability of illness is higher, however, they will never choose to fully insure. The range of losses for which partial insurance is optimal is larger than in the case of linear probabilities.

Probability weighting changes the ex-ante value of insurance and thus the optimality properties of partial insurance contracts from the consumer's perspective. This casts doubt on the normative value of optimal contracts obtained neglecting systematic bias in the perception of illness probabilities.

Appendix

Proof of Proposition 1. The purchase of insurance changes wealth in each state. To simplify exposition we will often refer to ω_j as the wealth in state $j \in S$. No confusion should arise by the omission of its dependence on the level of insurance I.

Suppose first $0 \leq I \leq C - L$; only severely ill individuals opt for treatment and $P = p_s I$. The expected utility of contract $(p_s I, I)$ is given by

$$U(I) = p_s u(\omega - p_s I - C + I) + p_m u(\omega - p_s I - L) + p_h u(\omega - p_s I).$$

Taking derivatives and rearranging we obtain

$$U'(I) = p_s \left(u'(\omega_s) - \sum_{j \in S} p_j u'(\omega_j) \right) > 0$$

where ω_j denotes wealth in state $j \in S$. Expected utility is strictly increasing in the level of indemnity, since $\omega_s \leq \omega_m < \omega_h$ for $0 \leq I \leq C - L$ and $u'' < 0$, and it attains a maximum at $I = C - L$ with value

$$U_1(C, L) \equiv (1 - p_h)u(\omega - p_s(C - L) - L) + p_h u(\omega - p_s(C - L)) \tag{11}$$

Suppose now that $I > C - L$; all insurance holders opt for treatment in case of illness independent of severity and $P = (1 - p_h)I$. Consumers are reimbursed in both states of illness with $\omega_s = \omega_m \equiv \omega_{-h} = \omega - (1 - p_h)I - C + I$. Expected utility is given by

$$U(I) = (1 - p_h)u(\omega - (1 - p_h)I - C + I) + p_h u(\omega - (1 - p_h)I).$$

It follows from $u'' < 0$ that

$$U'(I) = p_h(1 - p_h)\left(u'(\omega_{-h}) - u'(\omega_h)\right) \gtreqless 0 \qquad \text{if} \qquad I \lesseqgtr C$$

It is easy to check that $U'' < 0$. Thus, U is strictly concave with a maximum at $I = C$ where it takes the value

$$U_2(C) \equiv u(\omega - (1 - p_h)C) \tag{12}$$

Define $\Phi(C, L) = U_1(C, L) - U_2(C)$; partial insurance (resp. full insurance) is optimal if $\Phi(C, L) \geq 0$ (resp. if $\Phi(C, L) < 0$). From expressions (11) and (12)

$$\begin{aligned} \Phi(C, L) \quad = \quad & (1 - p_h)\left[u\left(\omega - p_s(C - L) - L\right) - u\left(\omega - (1 - p_h)C\right)\right] + \\ & + p_h\left[u\left(\omega - p_s(C - L)\right) - u\left(\omega - (1 - p_h)C\right)\right] \end{aligned} \tag{13}$$

Note that the second term in the right-hand side of expression (13), which is the utility gain of a smaller premium in the healthy state, is always strictly positive. Moreover, if $L \leq \frac{p_m}{p_h + p_m}\, C$ then the first term is also positive. Therefore, for any C partial insurance is optimal for values of L small enough. On the other hand, it follows from $u'' < 0$ that

$$U_1(C, C) = (1 - p_h)u(\omega - C) + p_h u(\omega) < u(\omega - (1 - p_h)C) = U_2(C)$$

Thus, $\Phi(C, C) < 0$; i. e. for $L = C$ full insurance is optimal. To complete the proof note that $u' > 0$ and $u'' < 0$ imply

$$\begin{aligned} \frac{\partial \Phi}{\partial L} = p_h p_s u'(\omega - p_s(C - L)) - (1 - p_h)(1 - p_s)u'(\omega - p_s(C - L) - L) < \\ -p_m u'(\omega - p_s(C - L)) < 0 \end{aligned}$$

It follows that for each C, there exists $\widehat{L}(C) \in \left(\frac{p_m}{p_h + p_m}\, C, C\right)$ such that $\Phi(C, \widehat{L}(C)) = 0$. Partial insurance (full insurance) is then optimal for all $L \leq \widehat{L}(C)$ $(L > \widehat{L}(C))$. $\square$

Proof of Proposition 2. We denote by ω_j the wealth level in state $j \in S$, omitting its dependence on I.

Suppose $0 \leq I \leq C - L$ and $P = p_s I$. Recall from expression (8) that

$$V_1'(I) = \pi_s u'(\omega_s) - p_s \sum_{j \in S} \pi_j u'(\omega_j) \tag{14}$$

and $u'(\omega_s) > \sum_j \pi_j u'(\omega_j)$.

Suppose now that $I > C - L$ and $P = (1 - p_h)I$. This contract equalizes income in both illness states; i. e. $\omega_s = \omega_m \equiv \omega_{-h} = \omega - (1 - p_h)I - C + I$. Recall from expression (10) that

$$V_2'(I) = (1 - \pi_h)u'(\omega_{-h}) - (1 - p_h)\{(1 - \pi_h)u'(\omega_{-h}) + \pi_h u'(\omega_h)\} \tag{15}$$

Note that $u'(\omega_{-h}) > u'(\omega_h)$ for $I < C$ and $u'(\omega_{-h}) = u'(\omega_h)$ for $I = C$. It is easy to check that $V_2'' < 0$.

Suppose now $p_s + p_m \leq \widehat{p}$; the properties of f imply $\pi_s = f(p_s) > p_s$ and $\pi_h = 1 - f(p_s + p_m) \leq 1 - (p_s + p_m) = p_h$. Expression (14) is strictly positive in this case implying that the best contract such that mildly ill individuals do not get treatment is $I = C - L$ and $P = p_s(C - L)$. From expression (15) and $u' > 0$ we have that for $C - L < I < C$ [5]

$$V_2'(I) > (p_h - \pi_h)u'(\omega_{-h}) \geq 0.$$

Therefore the best contract that would cover both illness severities would correspond to full insurance with $I = C$ and $P = (1 - p_h)C$.

Analogously to (13), let $\widetilde{V}_1(C, L)$ be the maximum of V_1 on $0 \leq I \leq C - L$ and define $\Phi(C, L, \pi_h) = \widetilde{V}_1(C, L) - V_2(C)$, writing explicitly the dependence of π_h for convenience. This yields

$$\begin{aligned} \Phi(C, L, \pi_h) \quad &= \quad (1 - \pi_h)\left[u\left(\omega - p_s(C - L) - L\right) - u\left(\omega - (1 - p_h)C\right)\right] + \\ &\quad +\pi_h\left[u\left(\omega - p_s(C - L)\right) - u\left(\omega - (1 - p_h)C\right)\right]. \end{aligned} \tag{16}$$

Partial insurance is an optimum if $\Phi \geq 0$. Given C and π_h, we have $\Phi(C, L, \pi_h) > 0$ if $L \leq \frac{p_m}{p_h + p_m} C$. It follows from $u'' < 0$ and $\pi_h \leq p_h$ that for $L = C$

$$\widetilde{V}_1(C, C) = (1 - \pi_h)u\left(\omega - C\right) + \pi_h u(\omega) < u\left(\omega - (1 - \pi_h)C\right)) \leq u\left(\omega - (1 - p_h)C\right)) = V_2(C).$$

Thus, $\Phi(C, C, \pi_h) < 0$. Moreover,

$$\begin{aligned} \frac{\partial \Phi}{\partial L} = \pi_h p_s u'(\omega - p_s(C - L)) - (1 - \pi_h)(1 - p_s)u'(\omega - p_s(C - L) - L) < \\ (p_s + \pi_h - 1)u'(\omega - p_s(C - L)) < 0 \end{aligned} \tag{17}$$

[5] The order of consequences would change if $I > C$ and therefore also the weights used to construct V.

since $f' > 0$ implies $p_s + \pi_h - 1 = p_s - f(p_s + p_m) < p_s - f(p_s) < 0$. Therefore, for each C and π_h there exists $\widetilde{L}(C, \pi_h) > \frac{p_m}{p_h + p_m} C$, which is implicitly defined by $\Phi(C, \widetilde{L}, \pi_h) = 0$.[6] Partial insurance (full insurance) is then optimal for all $L \leq \widetilde{L}(C, \pi_h)$ ($L > \widetilde{L}(C, \pi_h)$).

Finally, note that

$$\frac{\partial \Phi}{\partial \pi_h} = u(\omega - p_s(C - L)) - u(\omega - p_s(C - L) - L) > 0.$$

We can now make use of the implicit function theorem to conclude

$$\frac{\partial \widetilde{L}}{\partial \pi_h} = -\frac{\partial \Phi / \partial \pi_h}{\partial \Phi / \partial L} > 0$$

For $\pi_h = p_h$ we have that $\widetilde{L} = \widehat{L}$ from Proposition 1; for $\pi_h < p_h$ it follows that $\widetilde{L}(C, \pi_h) < \widehat{L}(C)$. □

Proof of Proposition 3. The proof proceeds analogously to the proof of Proposition 2. In particular, we still make use of expressions (14) and (15). Now suppose $p_s + p_m > \widehat{p}$ and $p_s \leq \widehat{p}$. In this case, we have $\pi_s \geq p_s$ and $\pi_h > p_h$. Expression (14) is still strictly positive and the best contract such that mildly ill individuals do not get treatment is again $I = C - L$ and $P = p_s(C - L)$. Again, we denote by $\widetilde{V}_1(C, L)$ the maximum value of V_1 in $0 \leq I \leq C - L$.

Evaluating expression (15) at $I = C$ we get

$$V_2'(C) = (p_h - \pi_h) u'(\omega - (1 - p_h)C) < 0.$$

Thus full insurance cannot be optimal. Formally, V_2 is well-defined and continuous on $[0, C]$. Let I_2 be its maximum, that is $V_2(I_2) \geq V_2(I)$ for all $0 \leq I \leq C$. Since $V_2'(C) < 0$ and $V_2'' < 0$, we have that either $I_2 = 0$ or $I_2 = I(C, \pi_h) > 0$ with $V_2'(I(C, \pi_h)) = 0$ from expression (15).

Suppose $I_2 = 0$, implying that V_2 is decreasing on $[0, C]$. We have that

$$\widetilde{V}_1(C, L) > V_1(0) > (1 - \pi_h) u(\omega - C) + \pi_h u(\omega) = V_2(0)$$

and thus partial insurance is optimal for all values of $L < C$. In this case, we have that $\overline{L}(C, \pi_h) = C$ and the proof is completed. Hence we can assume $I_2 > 0$ from now on.

Suppose $I_2 = I(C, \pi_h) \leq C - L$. Since $V_2'' < 0$, we have that for all $I \geq C - L$

$$\widetilde{V}_1(C, L) = V_1(C - L) > V_2(C - L) \geq V_2(I).$$

[6] The function $\widetilde{L}(C, \pi_h)$ is well-defined because Φ is strictly decreasing in L. Further, continuity of u' implies continuity of $\widetilde{L}$.

Hence partial insurance is optimal for all $L \leq C - I_2$.
Suppose $I_2 = I(C, \pi_h) > C - L$. Define

$$\Phi(C, L, \pi_h) = \widetilde{V}_1(C, L) - V_2(I_2) =$$
$$(1 - \pi_h) \left[u\left(\omega - p_s(C - L) - L\right) - u\left(\omega - (1 - p_h)I(C, \pi_h) - C + I(C, \pi_h)\right)\right] +$$
$$+ \pi_h \left[u\left(\omega - p_s(C - L)\right) - u\left(\omega - (1 - p_h)I(C, \pi_h)\right)\right].$$

Partial insurance is an optimum if $\Phi \geq 0$. Given C and π_h, we have $\Phi(C, L, \pi_h) > 0$ if $L < C - \frac{p_h}{p_h + p_m} I(C, \pi_h)$. Given C this implies that partial insurance is optimum for L small enough. Now for $L \to C$ we have

$$\lim_{L \to C} V_1(C - L) = (1 - \pi_h)u(\omega - C) + \pi_h u(\omega) < V_2(I(C, \pi_h))$$

which implies that full insurance is optimum for L close to C. Moreover, $\partial\Phi/\partial L$ is again given by expression (17) and hence Φ is strictly decreasing with L. Therefore, there exists $\overline{L}(C, \pi_h) < C$ such that $\Phi(C, \overline{L}, \pi_h) = 0$. Partial insurance (full insurance) is then optimal for all $L \leq \overline{L}(C, \pi_h)$ ($L > \overline{L}(C, \pi_h)$). Note that $\overline{L}(C, \pi_h) > C - \frac{p_h}{p_h + p_m} I(C, \pi_h) > \frac{p_m}{p_h + p_m} C$.

Finally, using the fact that $V_2'(I(C, \pi_h) = 0$ we obtain

$$\begin{aligned}
\frac{\partial\Phi}{\partial\pi_h} &= u(\omega - p_s(C - L)) - u(\omega - p_s(C - L) - L) - \\
&\quad \{u(\omega - (1 - p_h)I(C, \pi_h)) - u(\omega - (1 - p_h)I(C, \pi_h) - C + I(C, \pi_h))\} > 0.
\end{aligned}$$

The last inequality follows from the fact that $\overline{L}(C, \pi_h) > C - I(C, \pi_h)$ whenever $\pi_h > p_h$.

We can now make use of the implicit function theorem to conclude

$$\frac{\partial\overline{L}(C, \pi_h)}{\partial\pi_h} = -\frac{\partial\Phi\left(C, \overline{L}(C, \pi_h), \pi_h\right)/\partial\pi_h}{\partial\Phi\left(C, \overline{L}(C, \pi_h), \pi_h\right)/\partial L} > 0$$

For $\pi_h \to p_h$ we have that $I(C, \pi_h) \to C$ and hence $\overline{L} \to \widehat{L}$ from Proposition 1; it follows that $\overline{L}(C, \pi_h) > \widehat{L}(C)$ for $\pi_h > p_h$. □

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