

## Final exam

1. A single indivisible object is sold to one of two potential buyers. Buyer 1's value,  $X_1$ , is uniformly distributed in the interval  $[0, 1]$ , whereas buyer 2's value,  $X_2$ , is uniformly distributed in the interval  $[0, 2]$ . The two values are independently distributed.
  - (a) If the seller uses a second-price auction with reserve price  $r$ , what is the optimal  $r$  for a seller who attaches value  $x_0 = 0$  to the good?
  - (b) What is the optimal selling mechanism for this problem?
  - (c) How would the VCG mechanism allocate the object and what would be the payments in that case?
  - (d) Which of the selling mechanisms above are efficient? Argue why or give a counterexample.
  - (e) Rank the three mechanisms above in terms of revenues to the seller, without computing expected revenues.
  
2. A good with common value  $V$  is auctioned among  $N = 3$  bidders.  $V$  is unknown at the time of the auction, but assumed to be uniformly distributed over  $[0, 1]$ . Bidders get private signals  $\mathbf{X} = (X_1, X_2, X_3)$  on the value of the good. Given  $V = v$ , signals are independently and uniformly distributed on  $[0, 2v]$ .
  - (a) Derive the joint density of  $\mathbf{X}$ . What is the density of  $V$  conditional on  $\mathbf{X}$ ?
  - (b) Compute the symmetric equilibrium bidding strategy for a SPSB auction.
  - (c) Compute the symmetric equilibrium bidding strategy for a FPSB auction.
  - (d) As a seller, which one of the last two auctions would you prefer and why?