

Final exam

Question 1 is optional for students who want to improve their midterm grade.

1. Consider the following two versions of an all-pay auction with two bidders and private values uniformly distributed in the interval $[0, 1]$.
 - (First-price) all-pay auction: Highest bidder wins and both pay their bids.
 - (Second-price) war of attrition: Highest bidder wins and both pay 2nd-highest bid.
 - (a) Explain why both of these auctions are standard? State the revenue equivalence theorem.
 - (b) Compute the optimal equilibrium bidding strategy for the first-price version, $\beta^{\text{AP}}(x)$, using the revenue equivalence theorem and the fact that the expected equilibrium payment of a bidder with value x in this auction is $m^{\text{AP}}(x) = \beta^{\text{AP}}(x)$.
 - (c) Again using revenue equivalence, compute now the optimal equilibrium bidding strategy for the war of attrition, $\beta^{\text{WA}}(x)$.
2. Consider a first-price auction with two bidders where bidder 1's value, X_1 , is uniformly distributed on $[0, 2]$ and bidder 2's value, X_2 , is uniformly distributed on $[\frac{3}{2}, \frac{5}{2}]$.
 - (a) Consider a first-price auction with two bidders and reserve price $r = 1$. Verify that $\beta_1(x) = \frac{x}{2} + \frac{1}{2}$ and $\beta_2(x) = \frac{x}{2} + \frac{1}{4}$ constitute a pair of equilibrium bidding strategies.
 - (b) Argue carefully whether the auction is efficient. If not, give a counterexample.
 - (c) What is the optimal selling mechanism for this problem? Is this mechanism efficient? If not, give a counterexample.
 - (d) What would be the VCG mechanism applied to this case? What is the expected revenue to the designer of the VCG in this case?
3. Consider an auction with interdependent values where each bidder $i = 1, 2$ gets a signal $X_i \in [0, 1]$ on the value of the item offered. The valuation for bidder i is given by $V_i = X_i + \frac{1}{2}X_{-i}$. Signals have joint density function $f(x_1, x_2) = \frac{1+4x_1x_2}{2}$. Compute the equilibrium bidding strategy in the second-price auction as well as the expected revenue to the seller.