Complexity-Sensitive Decision Procedures for Abstract Argumentation

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Formal Models of Argumentation are a vivid field within KR and AI.

Underlying model: Dung’s abstract argumentation frameworks.

Simple, yet powerful formalism:

- Some important reasoning problems are hard even for the second level of the polynomial hierarchy.
- However, certain fragments show milder complexity.
This calls for complexity sensitive procedures that handle concrete problem instances with “appropriate effort”: 
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- If an instance is from a tractable fragment, the procedure should terminate in polynomial time.
- If an instance of a second-level problem is from an NP-fragment, the procedure should terminate after a single call (or small number of calls) to an NP-oracle (e.g. a SAT solver).
1. Motivation

Introduction (ctd.)

- This calls for complexity sensitive procedures that handle concrete problem instances with “appropriate effort”:
  - If an instance is from a tractable fragment, the procedure should terminate in polynomial time.
  - If an instance of a second-level problem is from an NP-fragment, the procedure should terminate after a single call (or small number of calls) to an NP-oracle (e.g. a SAT solver).

- Taking a SAT-solver as underlying engine
  - gives access to the sophisticated SAT solver technology
  - might even pay off for general instances (where an exponential number of calls is required).
Main Contributions

In this work we concentrate on complexity sensitive procedures for argumentation reasoning problems at the second level of the polynomial hierarchy (i.e. for preferred, semi-stable and stage semantics):

- We identify new NP/coNP fragments
- We augment NP fragments by certain parameterizations (syntactic distance and semantic parameters)
- We provide a novel schema for complexity sensitive procedures (in the talk: focus on preferred semantics).
Dung’s Abstract Argumentation Frameworks

Definition
An argumentation framework (AF) is a pair \((A, R)\) where
- \(A\) is a set of arguments
- \(R \subseteq A \times A\) is a relation representing the conflicts (“attacks”)

Example

![Graph](image-url)
Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

Example

$cf(F) = \{\{a, c, e\}, \{b, e\}, \{a, c\}, \{a, e\}, \{c, e\}, \{a\}, \{b\}, \{c\}, \{e\}, \emptyset\}$
### Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

### Example

![Diagram](attachment:image.png)

$\text{adm}(F) = \{\{a, c, e\}, \{b, e\}, \{a, c\}, \{a, e\}, \{c, e\}, \{a\}, \{b\}, \{c\}, \{e\}, \emptyset\}$
2. Background

Semantics

Complete Extension [Dung, 1995]

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

- \(S\) is admissible in \(F\)
- each \(a \in A\) defended by \(S\) in \(F\) is contained in \(S\)
  - \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).

Example

\[
\text{com}(F) = \{\{a, c, e\}, \{b, e\}, \{a, c\}, \{c, e\}, \{b\}, \{c\}, \{e\}, \emptyset\}
\]
2. Background

Semantics (ctd.)

Preferred Extensions [Dung, 1995]
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of $F$, if
- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S \not\subset T$

Example

$$\text{prf}(F) = \{\{a, c, e\}, \{b, e\}, \{a, c\}, \{c, e\}, \{b\}, \{c\}, \{e\}, \emptyset\}$$
### Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a **stable extension** of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

---

Semantics (ctd.)
Semantics (ctd.)

**Stable Extensions [Dung, 1995]**

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a **stable extension** of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

**Semi-Stable/Stage Extensions [Verheij 1996, Caminada 2006]**

Based on admissible/conflict-free sets but maximizing the “range”.
Main Reasoning Problems

<table>
<thead>
<tr>
<th>Credulous Acceptance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Cred_{\sigma}$: Given $AF \ F = (A, R)$ and $a \in A$; is $a$ contained in at least one $\sigma$-extension of $F$?</td>
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## Computational Complexity

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$Cred_\sigma$</th>
<th>$Skept_\sigma$</th>
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</thead>
<tbody>
<tr>
<td>$stb$</td>
<td>NP-c</td>
<td>coNP-c</td>
</tr>
<tr>
<td>$com$</td>
<td>NP-c</td>
<td>P-c</td>
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<tr>
<td>$prf$</td>
<td>NP-c</td>
<td>$\Pi^P_2$-c</td>
</tr>
<tr>
<td>$sem$</td>
<td>$\Sigma^P_2$-c</td>
<td>$\Pi^P_2$-c</td>
</tr>
<tr>
<td>$stg$</td>
<td>$\Sigma^P_2$-c</td>
<td>$\Pi^P_2$-c</td>
</tr>
</tbody>
</table>

Implications:
- For AFs with unique preferred extension, Skept$_{prf}$ becomes easier;
- For coherent AFs ($stb = prf$), Skept$_{prf}$ becomes easier;
- For stablecons AFs ($stb \neq \emptyset$), sem and stg become easier.
2. Background

Computational Complexity

<table>
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<tr>
<td>$\text{prf}$</td>
<td>NP-c</td>
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<tr>
<td>$\text{sem}$</td>
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Implications:
- for AFs with unique preferred extension, $\operatorname{Skept}_{\text{prf}}$ becomes easier;
- for coherent AFs ($\text{stb} = \text{prf}$), $\operatorname{Skept}_{\text{prf}}$ becomes easier;
- for stablecons AFs ($\text{stb} \neq \emptyset$), $\text{sem}$ and $\text{stg}$ become easier.
3. New Complexity Results

NP / coNP Fragments

Goal: identify classes of instances where reasoning tasks fall into first level of polynomial hierarchy.
NP / coNP Fragments

**Goal:** identify classes of *instances* where reasoning tasks **fall into first level** of polynomial hierarchy.

**Example:** For *AFs without odd-length cycles* all reasoning tasks are in NP or coNP.

The table below shows the complexity when the AF belongs to a sub-class.

<table>
<thead>
<tr>
<th>Sub-class</th>
<th>Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skept</td>
<td>P-c</td>
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<tr>
<td>perf</td>
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</tr>
<tr>
<td>Cred</td>
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<tr>
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<td>stg</td>
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<tr>
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<td>P-c</td>
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<tr>
<td>cof</td>
<td>coNP-c</td>
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<tr>
<td>w cyc</td>
<td>Σ₂⁻c</td>
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<tr>
<td>uniqp ref</td>
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<tr>
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<td>Σ₂⁻c</td>
</tr>
<tr>
<td>stablecons</td>
<td>Π₂⁻c</td>
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<tr>
<td></td>
<td>coNP-c</td>
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<td></td>
<td>NP-c</td>
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NP / coNP Fragments

Goal: identify classes of instances where reasoning tasks fall into first level of polynomial hierarchy.

Example: For AFs without odd-length cycles all reasoning tasks are in NP or coNP.

Table: Complexity when the AF belongs to a sub-class $G$.

<table>
<thead>
<tr>
<th>$G$</th>
<th>$Skept_{prf}$</th>
<th>$Cred_{sem}$</th>
<th>$Skept_{sem}$</th>
<th>$Cred_{stg}$</th>
<th>$Skept_{stg}$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$P$-c</td>
<td>$P$-c</td>
<td>$P$-c</td>
<td>$P$-c</td>
<td>$P$-c</td>
</tr>
<tr>
<td>ocf</td>
<td>coNP-c</td>
<td>NP-c</td>
<td>coNP-c</td>
<td>NP-c</td>
<td>coNP-c</td>
</tr>
<tr>
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<td>coNP-c</td>
<td>$\Sigma^P_2$-c</td>
<td>$\Pi^P_2$-c</td>
<td>$\Sigma^P_2$-c</td>
<td>$\Pi^P_2$-c</td>
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<tr>
<td>uniqpref</td>
<td>in NP</td>
<td>in NP</td>
<td>in NP</td>
<td>$\Sigma^P_2$-c</td>
<td>$\Pi^P_2$-c</td>
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<tr>
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</tr>
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Syntactic Distance

Is there a way to extend these fragments?
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Idea of backdoors [Ordyniak & Szeider, 2011]: The distance $\text{dist}_G(F)$ of an AF $F$ to a sub-class $G$ is defined as the minimal number of arguments one has to delete from $F$ such that $F$ falls into $G$. 
Syntactic Distance

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Table: Complexity when parameterized by the distance to a sub-class $G$

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Semantical Parameterization

**Idea:** Take number of extensions into account, e.g.
- \( \text{sol}^k\sigma \) as the class of AFs with at most \( k \sigma \)-extensions;
- coherent\(^k\) as the class of AFs where \( |\text{prf}(F) \setminus \text{stb}(F)| \leq k \);
- stablecons\(^k\sigma\) as the class of AFs where the range of extensions misses at most \( k \) arguments.
### Semantical Parameterization

**Idea:** Take number of extensions into account, e.g.

- $\text{sol}_\sigma^k$ as the class of AFs with at most $k$ $\sigma$-extensions;
- $\text{coherent}^k$ as the class of AFs where $|\text{prf}(F) \setminus \text{stb}(F)| \leq k$;
- $\text{stablecons}_{\sigma}^k$ as the class of AFs where the range of extensions misses at most $k$ arguments.

**Table:** Complexity when the AFs belong to a sub-class $G$ (for fixed $k$).

<table>
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<tr>
<th>$G$</th>
<th>$\text{Skept}_{\text{prf}}$</th>
<th>$\text{Cred}_{\text{sem}}$</th>
<th>$\text{Skept}_{\text{sem}}$</th>
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<tbody>
<tr>
<td>$\text{sol}_\sigma^k$</td>
<td>in $\text{P}^{\text{NP}}$</td>
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<td>$\text{coherent}^k$</td>
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<td>in $\text{P}^{\text{NP}}$</td>
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<tr>
<td>$\text{stablecons}_{\sigma}^k$</td>
<td>–</td>
<td>in $\text{P}^{\text{NP}}$</td>
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CEGARTIX for Skeptical Acceptance in Preferred Semantics

Input: AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{\text{com}}(\mathcal{F}) \land \neg x_\alpha$
2. while ($\varphi$ is satisfiable)
   1. find model $I$ of $\varphi$
   2. while (there is model $I'$ of $\varphi_{\text{com}}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha$)
      $I \leftarrow I'$
   3. if ($\varphi_{\text{com}}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) then reject
     else $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$
3. accept
Input: AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

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      $I \leftarrow I'$
   3. if ($\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) then reject
   4. else $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$

3. accept

Skeptical acceptance of argument e:
4. The Algorithm

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   4. else $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$
3. accept

Skeptical acceptance of argument e:
Step 1: initialize $\varphi$: $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg x_e$
CEGARTIX for Skeptical Acceptance in Preferred Semantics

**Input:** AF \( \mathcal{F} = (A, R) \), argument \( \alpha \in A 

1. \( \varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg x_\alpha \)

2. while (\( \varphi \) is satisfiable)
   1. find model \( I \) of \( \varphi \)
   2. while (there is model \( I' \) of \( \varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha \))
       \( I \leftarrow I' \)
   3. if (\( \varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \) is unsatisfiable) then reject
   4. else \( \varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x) \)

3. accept

Skeptical acceptance of argument \( e \):

**Step 2:** \( \varphi \) is satisfiable, e.g. \( \emptyset \) is complete and \( e \notin \emptyset \).
4. The Algorithm

CEGARTIX for Skeptical Acceptance in Preferred Semantics

Input: AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg x_\alpha$
2. while ($\varphi$ is satisfiable)
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   2. while (there is model $I'$ of $\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha$)
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   4. else $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$
3. accept

Skeptical acceptance of argument $e$:

Step 2.1: We keep the model representing $\emptyset$. 
CEGARTIX for Skeptical Acceptance in Preferred Semantics

Input: AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg x_\alpha$
2. while ($\varphi$ is satisfiable)
   1. find model $I$ of $\varphi$
   2. while (there is model $I'$ of $\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha$)
      $I \leftarrow I'$
   3. if ($\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) then reject
   4. else $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$
3. accept

Skeptical acceptance of argument e:

**Step 2.2**: maximizing the complete set without adding e.
We can add $b$ and thus we obtain the extension $\{b\}$. 
CEGARTIX for Skeptical Acceptance in Preferred Semantics

**Input:** AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg x_\alpha$
2. **while** ($\varphi$ is satisfiable)
   1. find model $I$ of $\varphi$
   2. **while** (there is model $I'$ of $\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha$) $I \leftarrow I'$
   3. **if** ($\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in \cap X} x \land (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) **then** reject
   4. **else** $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$
3. **accept**

Skeptical acceptance of argument $e$:

**Step 2.2:** $\{b\}$ is maximal (given $\neg x_e$)
CEGARTIX for Skeptical Acceptance in Preferred Semantics

Input: \( AF \mathcal{F} = (A, R) \), argument \( \alpha \in A \)

1. \( \varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg x_\alpha \)
2. \textbf{while} (\( \varphi \) is satisfiable)
   1. find model \( I \) of \( \varphi \)
   2. \textbf{while} (there is model \( I' \) of \( \varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha ) \)
      \( I \leftarrow I' \)
   3. \textbf{if} (\( \varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \) is unsatisfiable) \textbf{then} reject
   4. \textbf{else} \( \varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x) \)
3. \textbf{accept}

Skeptical acceptance of argument \( e \):

Step 2.3: \{b, e\} is a superset of \{b\} and thus \{b\} is not a counter-example for \( e \) being skeptically accepted.
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**Input:** AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg x_\alpha$
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   1. find model $I$ of $\varphi$
   2. **while** (there is model $I'$ of $\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha$)
      $I \leftarrow I'$
   3. **if** ($\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) **then** reject
   4. **else** $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$
3. *accept*

Skeptical acceptance of argument e:

**Step 2.4:** We exclude \{b\} from further investigation by adding the clause $x_a \lor x_c \lor x_d \lor x_e$. 
CEGARTIX for Skeptical Acceptance in Preferred Semantics

**Input:** $\mathcal{AF} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{com}(\mathcal{F}) \land \neg \neg \alpha$

2. **while** ($\varphi$ is satisfiable)
   1. find model $I$ of $\varphi$
   2. **while** (there is model $I'$ of $\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg \alpha$)
      1. $I \leftarrow I'$
   3. **if** ($\varphi_{com}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) **then** reject
   4. **else** $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$

3. accept

Skeptical acceptance of argument $e$:

**Step 2:** $\varphi$ is unsatisfiable, as there is no complete extension neither containing $b$ or $e$. 
CEGARTIX for Skeptical Acceptance in Preferred Semantics

**Input:** AF $\mathcal{F} = (A, R)$, argument $\alpha \in A$

1. $\varphi \leftarrow \varphi_{\text{com}}(\mathcal{F}) \land \neg x_\alpha$

2. **while** ($\varphi$ is satisfiable)
   1. find model $I$ of $\varphi$
   2. **while** (there is model $I'$ of $\varphi_{\text{com}}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x) \land \neg x_\alpha$)
     
     $I \leftarrow I'$
   3. **if** ($\varphi_{\text{com}}(\mathcal{F}) \land \bigwedge_{x \in I \cap X} x \land (\bigvee_{x \in X \setminus I} x)$ is unsatisfiable) **then** reject
   4. **else** $\varphi \leftarrow \varphi \land (\bigvee_{x \in X \setminus I} x)$

3. accept

Skeptical acceptance of argument $e$:

**Step 3:** Skeptically accept $e$
Experimental Evaluation (ctd).

**Figure:** Comparison of CEGARTIX (using Minisat) and ASPARTIX (using claspD); cumulative running times over the random instances (left) and grid instances (right).
Conclusion

- Investigation of complexity of hard argumentation problems when frameworks are from certain fragments
  - focus on fragments which lower the complexity to NP/coNP;
  - only semantical parameterizations allow to extend these fragments.

- Novel algorithm which is complexity-sensitive for such a parameterization
  - in the paper: further shortcuts; also semi-stable and stage semantics.

- First experiments are promising, but further analyses required.

- Next Step: investigation of further fragments and parameters.

- System available under www.dbai.tuwien.ac.at/research/project/argumentation/cegartix/
Experimental Evaluation - Setting

Questions:
- Dedicated vs. SAT-based approach
- Does incremental SAT-solving help.

Machine: OpenSUSE with Intel Xeon processors (2.33 GHz) and 49 GB memory

Systems:
- ASPARTIX: gringo v3.0.3, claspD v1.1.1
- ASPARTIX (based on metasp): gringo v3.0.3, claspD v1.1.1
- CEGARTIX incremental: MINISAT v2.2.0
- CEGARTIX: MINISAT v2.2.0
- CEGARTIX: clasp v2.0.5

Benchmark instances:
- Fully Random Graphs varying the edge density
- Random graphs based on a grid structure
Experimental Evaluation - SAT-solvers

Figure: Comparison of different variants of CEGARTIX (non-incremental and incremental applications of Minisat, non-incremental application of Clasp): cumulative running times over the random instances (left) and grid instances (right).