

From ultimatum to Nash bargaining: Theory and experimental evidence

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Abstract We consider a sequential two-party bargaining game with uncertain information transmission. When the first mover states her demand she does only know the probability with which the second mover will be informed about it. The informed second mover can either accept or reject the offer and payoffs are determined as in the ultimatum game. Otherwise the uninformed second mover states his own demand and payoffs are determined as in the Nash demand game. In the experiment we vary the commonly known probability of information transmission. Our main finding is that first movers' and uninformed second movers' demands adjust to this probability as qualitatively predicted, that is, first movers' (uninformed second movers') demands are lower (higher) the lower the probability of information transmission.

Keywords Commitment · Imperfect observability · Ultimatum bargaining · Nash bargaining · Experiments

JEL Classification C72 · C78 · C92

1. Introduction

In many situations one can profit from the ability to make a binding commitment. This has been well known for a long time: In 1066 William the Conqueror, so the Battle Abbey Chronicle reports, ordered his soldiers to burn their ships after landing in England. He thus unmistakably demonstrated that he and his men were determined to fight until they won the day or to die.

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That commitment confers a strategic advantage has influenced many areas of economic theory like macroeconomics, international trade and industrial organization. If there is a first mover advantage, the necessary conditions for a commitment are its irreversibility and that it can be reliably communicated.¹ Both assumptions are granted in most leader-follower models in economic literature. Recently, the robustness of preemptive commitments has been reviewed in the presence of imperfect observability or error-prone information transmission.²

In an experimental analysis of a 2×2 game with full-support noise Huck and Müller (2000) find that followers ignore small levels of noise and play a best-response against the observed leader's action even though with some probability this might be the "wrong" action. Leaders quickly learn to exploit this tendency and play the Stackelberg leader's quantity. Only with high levels of noise, play converges to the Cournot equilibrium. Güth et al. (2001) experimentally examine the strategic behavior of leaders and followers in sequential duopoly markets assuming, like Chakravorti and Spiegel (1993), that followers either observe quantities of the leaders or nothing at all. Consistent with theory, leaders enjoy a greater first-mover advantage when followers observe their actions with higher probability.

In this paper we theoretically and experimentally investigate uncertain communication in the ultimatum game. More precisely, the rules of the game are as follows: First, the proposer X (or first mover) states her demand x about which the responder (or second mover) then receives an "all-or-nothing" signal. That is, with a commonly known probability w the second mover Y learns the first mover's demand and with complementary probability the responder receives no information at all.³ The *informed* second mover chooses between accepting or rejecting the implicit offer. Accepting yields x to X and the difference between the total available 'pie,' p , and x , i.e. $(p - x)$ to Y . Rejecting the offer yields zero payoff for both. The *uninformed* second mover states his own demand y and payoffs are then determined as in the usual demand game (Nash, 1950). That is, if the sum of the two demands does not exceed the pie ($x + y \leq p$), both players receive their demands; otherwise ($x + y > p$) both receive nothing. The reason for linking the ultimatum game and the demand game is that both rely on single choices by both parties. This allows for an easy transition from one game to the other and an easy description of the intermediate games.

The solution of this game—based on the notion of risk dominance (Harsanyi and Selten, 1988)—prescribes a continuous transition from the ultimatum bargaining to the Nash demand game: As the probability w decreases from 1 to 0, the risk dominant first mover demand decreases continuously from $x = p$ to $x = p/2$, with the (un)informed second mover always (demanding) accepting the residual $(p - x)$.

In our view, in the field imperfect observability of earlier actions is rather the rule than the exception. It is therefore, both important and desirable to explore theoretically and experimentally the implications of imperfect observability in a variety of settings. In this respect our bargaining model can enhance our understanding of first-mover advantages when actions are imperfectly observable. This can clarify the robustness of results that were derived in the extensive theoretical literature that studied the role of commitment in sequential games and indicate how appropriate these bench-mark solutions are in the presence of imperfect communication

¹ To come back briefly to the above example: To burn the ships is clearly an irreversible action and one can assume that both the English and William's men did observe this action and understood its significance.

² Van Damme and Hurkens (1997) distinguish between errors in perception and errors in communication. Bagwell (1995) investigates a model with errors in perception. Chakravorti and Spiegel (1993), on the other hand, investigate a model with errors in communication.

³ Note that this signal technology is widely used in the literature, e.g., Rubinstein (1989), Fershtman et al. (1991), Laffont and Tirole (1993), and Chakravorti and Spiegel (1993).

channels. In spite of many experimental studies of bargaining and negotiations⁴ we are not aware of studies investigating uncertain information transmission in a bargaining situation.

Our experiment employs a within-subject design regarding the probability w with which the *second mover* is informed. In each of the 5 sessions subjects either acted in the role of the first or second mover and were repeatedly and randomly re-matched to play the game. In each of the 60 rounds the probability w with which the *second mover* received a signal was randomly and independently drawn for each pair of players from the set $\{0.1, 0.3, 0.7, 0.9\}$.

Our main finding is that first movers' (uninformed second movers') average demands weakly increase (decrease) with the probability w of the ultimatum mode. However, demands do not significantly differ for the two small levels of w . In all treatment conditions, the mode of behavior is the equal-split demand. In general, first movers' (uninformed second movers') demands decrease (increase) as sessions progressed. Finally, there are no direct effects of the probability w on rejection behavior of informed second movers.

The paper is organized as follows: In Section 2 we precisely state the model and derive the game-theoretic predictions. In Section 3 we describe our experimental design and the procedures used. The results of the experiments are then presented in Section 4. Finally, Section 5 offers a discussion of our results and some concluding remarks.

2. Theory

2.1. The model

There are two parties: a first mover X and a second mover Y who can divide a positive amount of money $p (> 0)$ among themselves. The timing of decisions is as follows: First X states her demand x with $0 \leq x \leq p$. This demand x is revealed to Y with probability $w \in [0, 1]$. With complementary probability $1 - w$ the decision x is not revealed to Y (who in this case only knows that X has already stated her demand x). The probability w is commonly known. When the second mover Y is informed about player X 's demand x , he can choose between "accepting" or "rejecting" X 's demand and the implicit offer $p - x$. If Y accepts, player X earns x while player Y earns $y = p - x$. In case Y is not informed about player X 's demand, he must state his own demand y with $0 \leq y \leq p$. If the outcome is feasible, i.e. $x + y \leq p$, both get what they demanded (i.e. player X earns x and Y earns y). If $x + y > p$, both earn nothing.

Thus, if the second mover is informed about the first mover demand, the rules are those of the ultimatum game. Therefore, we will refer to this case as the U-mode. If the second mover is not informed about the first mover demand, the rules resemble those of the symmetric Nash demand game with a positional order protocol.⁵ Therefore, we will refer to this case as the N-mode. For any $w \in (0, 1)$ we refer to the game with this parameter w as the w -game.

2.2. Solution

We assume players to be selfish profit maximizers and this is commonly known. For the U-mode any weakly undominated response function has to accept all positive offers. Since for continuous offers $y(x) = p - x$ there exists no smallest positive offer, we assume that the demand $x = p$ is also accepted. The decision problem of Y in case of a given and known first mover demand x can be substituted by its solution outcome (x, y) with $y = p - x$. This

⁴ For reviews see for example Güth (1995) or Roth (1995).

⁵ The positional order protocol relies on sequential decisions without revealing earlier moves to second movers (see Rapoport, 1997).

substitution yields a game which is called the N-truncation. The rules of the N-truncation are that with probability w the demand x by X is automatically accepted leading to payoff x for X and $y = p - x$ for Y . With complementary probability $1 - w$ the demands x by X and y by Y lead to the payoff vector (x, y) only when $x + y \leq p$. In conflict ($x + y > p$) it leads to a payoff of 0 for both.

In the N-mode any strict equilibrium $x^*(w)$ with $0 < x^*(w) < p$ requires the best response $y^*(w) = p - x^*(w)$. As a consequence, all demand vectors (x, y) with $x + y = p$ are equilibria of the N-truncation which are even strict (one loses by deviating unilaterally) in case of $x, y > 0$. When solving N-truncations we therefore rely on equilibrium selection.

More specifically, we discriminate among solution candidates (the strict equilibria) by risk dominance.⁶ Risk dominance (Harsanyi and Selten, 1988) is usually an intransitive relation. When solving N-truncations, this problem does not arise, however, since N-truncations have a unique strict equilibrium that risk-dominates all other strict equilibria. In our view, this renders risk dominance more intuitive and possibly behaviorally attractive.⁷ Relying on risk dominance, we can prove (see Appendix A):

Proposition 1. *The risk dominant solution of the N-truncations with parameter $w \in [0, 1]$ is $(x^*(w), y^*(w))$, where $y^*(w) = p - x^*(w)$ and*

$$x^*(w) = \frac{p}{(2 - w)}.$$

The solution of w -games is the solution of the N-truncations (given in Proposition 1) together with the best-response function (i.e., universal acceptance of all offers in the U-mode) underlying the definition of N-truncations. That is, given probability w , the first mover will demand $x^*(w)$, the uninformed second mover will demand $y^*(w) = p - x^*(w)$ while the informed second mover would accept all offers.

Since

$$x^*(w) = \frac{p}{(2 - w)} \rightarrow \begin{cases} \frac{p}{2} & \text{for } w \rightarrow 0 \text{ (N-mode for certain)} \\ p & \text{for } w \rightarrow 1 \text{ (U-mode for certain),} \end{cases}$$

the outcome of the w -games moves monotonically from the outcome of the Nash demand game ($x^*(0) = p/2$) to the one of ultimatum bargaining ($x^*(1) = p$). We thus have naturally linked the two prominent bargaining models by w -games.

3. Experimental design

The computerized experiments were conducted at Humboldt University Berlin using the software tool kit *z-Tree* (Fischbacher, 1999). We ran 5 sessions with 12 subjects each. With a few exceptions, subjects were students of economics or business administration at Humboldt University. They were either randomly recruited from a pool of potential participants or

⁶ Payoff dominance does not allow for discrimination among strict equilibria in our game (see also footnote 7).

⁷ Theoretical support for risk dominance is provided, e.g., in Harsanyi (1995), Kandori, Mailath and Rob (1993) and Young (1993). Evidence for the behavioral relevance of risk dominance is documented, e.g., in Van Huyck, et al. (1990, 1991), Straub (1995), Charness (2000) and Cabrales et al. (2000).

invited by leaflets distributed around the university campus. Sessions lasted about 45 minutes. The average earnings were EUR 12.88.⁸

Upon arrival in the laboratory, each subject was seated in front of a computer screen where she received written instructions in German.⁹ After reading the instructions, subjects were allowed to ask clarifying questions which were answered privately. Instructions informed subjects that there were two roles (role X and role Y) and that in each session 6 subjects were randomly assigned to the role X and 6 subjects to the role Y . Roles were kept fixed during the entire session. We implemented the w -game described above with four different probabilities w in a within-subject design.¹⁰ The payoffs were denoted in ECU (Experimental Currency Unit) and subjects were informed that 200 ECU would pay 1 EUR at the end of the session. The available amount p was equal to 100 ECU in each round. In each of the 60 rounds one X was randomly matched with one Y and the computer randomly selected one of the four probabilities $w \in \{0.1, 0.3, 0.7, 0.9\}$ independently for each of the six X/Y -pairs.¹¹ Each pair was informed about the selected value of w .¹² The computer also selected the mode (U- or N-mode) according to the chosen probability w independently for each pair. Then the subject acting in role X , knowing only probability w but not the selected mode, stated her integer demand x with $0 \leq x \leq 100$. The round then continued depending on the selected mode.

In case of the U-mode (“mode 1”), participant Y learned the actual demand x of X and then decided between “accepting” or “rejecting” it. In case of the N-mode (“mode 2”), participant Y was not informed about x before stating his own integer demand y with $0 \leq y \leq 100$. Y only knew that “ X had just stated his demand”. Payoffs were then determined according to the respective payoff rule, described in Section 2.1. At the end of each round, each X/Y pair was informed about the random draws made by the computer and the individual decisions made in this round. Furthermore, every participant was informed about his own individual payoff in that round.

The relatively high number of 60 rounds gave subjects ample opportunity for learning and provided many (although not independent) observations for (un)informed second movers in all treatment conditions. The random matching scheme was employed to weaken possible repeated game effects.

4. Experimental results

4.1. Descriptive data analysis

In this subsection we first present some relevant summary statistics. Later in Sections 4.2 to 4.4 we analyze first and second mover behavior in more detail using regression techniques.

⁸ Average earnings of first movers were EUR 13.56 with standard deviation 1.07 and average earnings of second movers were EUR 12.22 with standard deviation 1.08.

⁹ An English translation of the instructions can be downloaded at: http://center.uvt.nl/staff/muller/Instructions_From_Ulimateum_to_Nash.pdf.

¹⁰ A within-subject design allows to explore the sensitivity to w -changes on an individual level rather than via a comparison of different groups of participants whose composition might vary in some rather uncontrolled way. (See also Morgan and Vardy (2004) who use a very similar design.)

¹¹ This definitely discourages to develop habits for a given w -value but should induce participants to pay attention to the specific realization of w .

¹² The “extreme” probabilities ($w = 0.1$ and $w = 0.9$) were used in order to see whether behavior approximates the usual findings for the boundary cases $w = 0$, respectively $w = 1$. The intermediate probabilities ($w = 0.3$ and $w = 0.7$) allow to test the sensitivity of behavior to w -changes, both in the lower and the upper w -range.

Table 1 Risk dominant solution demands and observed average demands over all rounds

Probability	Second movers					
	First movers		N-mode			U-mode
	$x^*(w)$	Mean $x(w)$	$y^*(w)$	Mean $y(w)$	Conflict rate	Rejection rate
$w = 0.9$	90.91	58.61	9.09	44.90	42.85%	22.11%
($N = 456$)	–	(3.12)	–	(4.38)	(21/49)	(90/407)
$w = 0.7$	76.92	55.66	23.08	47.84	38.06%	11.31%
($N = 470$)	–	(2.86)	–	(2.35)	(51/134)	(38/336)
$w = 0.3$	58.82	50.50	41.18	48.99	5.37%	0.009%
($N = 466$)	–	(0.65)	–	(0.88)	(19/354)	(1/112)
$w = 0.1$	52.63	50.32	47.37	49.53	6.78%	0%
($N = 408$)	–	(0.53)	–	(0.55)	(25/369)	(0/39)
Total N		1800			906	894

Note. Standard deviations based on session averages appear in parentheses.

With 5 sessions of 60 rounds each, and 6 leaders and 6 followers in each session, we have a total of 1800 decisions for each role. Second movers were informed about x (U-mode) 894 times and remained uninformed (N-mode) in 906 encounters.

Table 1 reports summary statistics separately for the four different probabilities $w \in \{0.1, 0.3, 0.7, 0.9\}$. For first movers it reports predicted demands $x^*(w)$, average observed demands $x(w)$ along with standard deviations (in parentheses). For second movers Table 1 distinguishes between the Nash demand (N) and the ultimatum (U) mode. In case of the N-mode it reports the predicted demand $y^*(w)$, average observed demands $y(w)$ along with standard deviations and the conflict rates, i.e. the relative frequency of cases with $x(w) + y(w) > 100$. For the U-mode it only reports rejection rates. As there is no case in which a first mover demanded the whole pie, the theoretical rejection rate is zero in all cases. The information given in Table 1 is complemented by figures 1 and 2 which illustrate the frequency distributions of first movers' and uninformed second movers' demands.

What are the main effects? Consider first-mover behavior and refer to Table 1. Average demands of first movers vary monotonically with w but much less than predicted (for a more detailed graphical illustration see figures 1 and 2). There is only a negligible difference between average demands for $w = 0.3$ and $w = 0.1$ (50.50 vs. 50.32). Average demands between $w = 0.9$ and $w = 0.7$ vary considerably more (58.61 vs. 55.66). The largest difference can be observed between $w = 0.7$ and $w = 0.3$ (55.66 vs. 50.50). Overall, first movers do not sharply differentiate when the level of w is small ($w \leq 0.3$) and mostly offer to split the pie equally in these cases what explains the very low conflict rates (5.37%, respectively 6.78% in case of the N-mode and 0.009%, respectively 0% in case of the U-mode). Furthermore, it is interesting to note that behavior is less dispersed the lower the probability w of the ultimatum mode as shown by the standard deviations of first and second mover demands.

Also with regard to uninformed second-mover demands (N-mode), we observe that the comparative statics properties regarding w are reflected in the data (although, again, not as pronounced as predicted). Average demands increase from 44.90 in case of $w = 0.9$ to 49.53 in case of $w = 0.1$. However, again average demands differ only slightly between $w = 0.1$ and $w = 0.3$ (49.53 vs. 48.99).

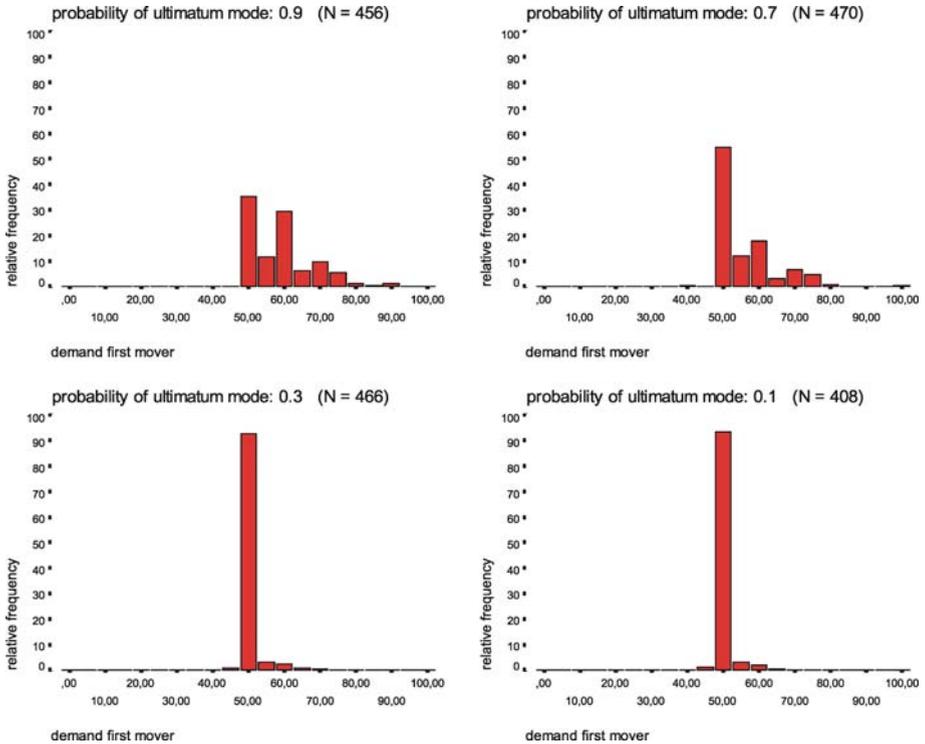


Fig. 1 Frequency distribution of first movers' demands (all rounds)

The histograms in figures 1 and 2 show relative frequencies of demands of first and uninformed second movers for each value of w separately. In both roles, demanding exactly 50 is the mode for each level of w . Though this is generally true for all w 's and both roles, for first movers the pure dominance of this mode for the two higher levels of w is less pronounced than for second movers what also explains the high conflict rates for higher w -values ($w \geq 0.7$). The equal split is also the median demand of uninformed second movers for all w -values and of first movers for all given w -values strictly smaller than 0.9. (For $w = 0.9$ the median is 60.) The distributions of both, first and uninformed second mover demands are more dispersed for higher levels of probability w . But it is not symmetrically dispersed: it rather moves to the right for first movers and to the left for second movers.

Over all, bargaining ended in conflict in 13.6% of all encounters. For the U-mode (14.43%) this number was slightly higher than for the N-mode (12.8%). Since first-mover demands tend to increase more with higher w 's, conflict and rejection rates increase with w . For the U-mode rejection rates increase monotonically with w . Due to different numbers of observations, it is difficult to compare conflict/rejection rates between the U- and the N-mode for each w . Nevertheless it is worth mentioning that in the N-mode the increase in conflict rates is more pronounced than that in rejection rates in the U-mode where subjects know the exact costs of choosing conflict.

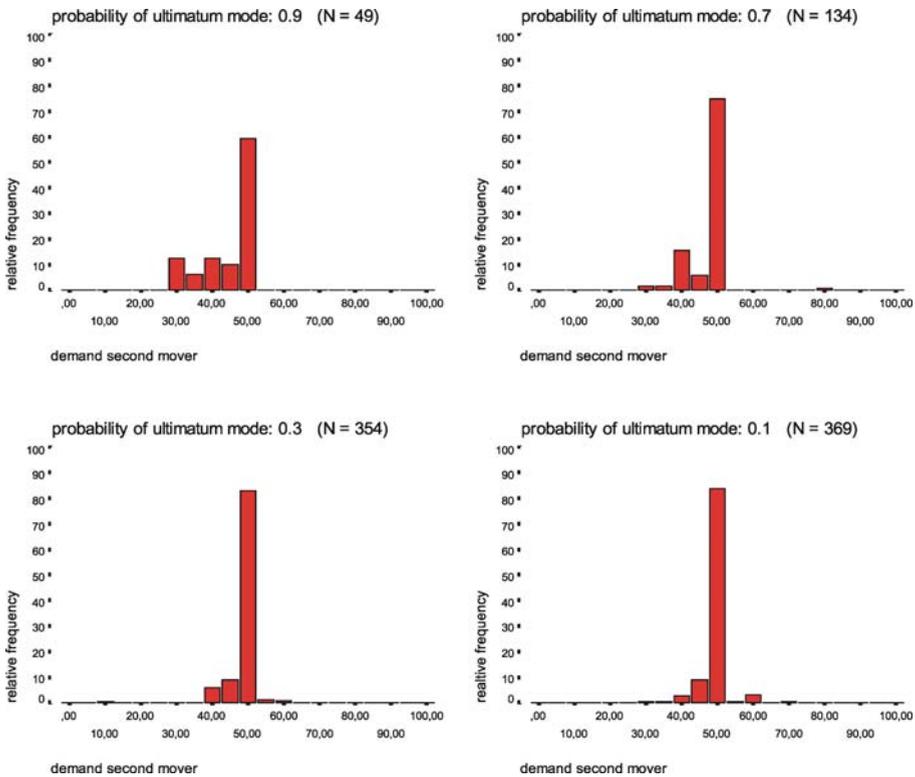


Fig. 2 Frequency distribution of uninformed second movers' demands (all rounds)

4.2. Analysis of first-mover behavior

When first movers state their demands, they only know the probability *w* with which the second mover will be informed about their demand. As the control variable *w* was truly exogenous, random-effects regression models seem to be the most appropriate ones.¹³

In order to assess leaders' behavior, we estimated several versions of the following random-effects model:

$$\begin{aligned}
 x_{it} = & \alpha_0 + \alpha_7 D_7 + \alpha_3 D_3 + \alpha_1 D_1 + \beta_9 (D_9 \times t) + \beta_7 (D_7 \times t) \\
 & + \beta_3 (D_3 \times t) + \beta_1 (D_1 \times t) + \delta_c Conflict_{t-1} + c_i + u_{it},
 \end{aligned}
 \tag{1}$$

where x_{it} is leader *i*'s demand in round *t* and $v_{it} = c_i + u_{it}$ is a composite error term with the usual assumptions made in random effects regression models. D_k is a dummy variable which is equal to 1 if subjects confronted a probability of the U-mode of $w = k/10$ ($k \in \{1, 3, 7\}$) and 0 otherwise. Thus, the behavior under treatment condition $w = 0.9$ serves as the reference

¹³ A Breusch-Pagan LM test was used to test for the necessity of subject effects and a Hausman test was applied to test for differences between the fixed-effects and the random-effects model. In all regressions, a panel regression with subject effects was preferable to a classic regression model and among the panel regression models a random-effects model was preferable to a fixed-effects model.

Table 2 Results of first-mover regressions

	Regression FM1	Regression FM2	Regression FM3	Regression FM4
α_0	58.997*** (0.557)	60.933*** (0.594)	62.486*** (0.683)	62.422*** (0.722)
$\alpha_7 (D_7)$	-3.194*** (0.337)	-3.175*** (0.330)	-3.946*** (0.645)	-3.791*** (0.643)
$\alpha_3 (D_3)$	-8.288*** (0.386)	-8.386*** (0.377)	-10.998*** (0.674)	-10.691*** (0.786)
$\alpha_1 (D_1)$	-8.446*** (0.417)	-8.520*** (0.408)	-11.570*** (0.710)	-11.103*** (0.705)
$\beta_0 (t)$		-0.061*** (0.007)		
$\beta_9 (D_9 \times t)$			-0.110*** (0.013)	-0.104*** (0.013)
$\beta_7 (D_7 \times t)$			-0.085*** (0.013)	-0.083*** (0.013)
$\beta_3 (D_3 \times t)$			-0.025* (0.013)	-0.026** (0.013)
$\beta_1 (D_1 \times t)$			-0.011 (0.014)	-0.016 (0.014)
$\delta_c (Conflict_{t-1})$				-0.066 (0.337)
Adj. pseudo R^2	0.240	0.259	0.263	0.300

Note: Standard errors of estimators in parentheses. * $p < 0.1$, ** $p < .05$, *** $p < .01$.

case. The variable $t = 1, \dots, 60$ indexes the rounds. Finally, $Conflict_{t-1}$ is a one-period lagged dummy variable that is equal to 1 if in the preceding round a demand was rejected in the U-mode or play resulted in conflict in the N-mode, and is equal to 0 otherwise. All first-mover regressions were adjusted for heteroscedasticity.¹⁴ The results of first-mover regressions are presented in Table 2. Regressions FM1–FM4 differ in terms of independent variables that were included.

Considering first the effect of probability w on first mover demands, regressions FM1–FM4 show that without exception the coefficients of the treatment dummies are negative and highly significant (with $0 > \alpha_7 > \alpha_3 > \alpha_1$). Hence demands are significantly smaller than for the reference case of $w = 0.9$. Furthermore, in all four regressions the restriction $\alpha_7 = \alpha_3$ must be rejected whereas the restriction $\alpha_3 = \alpha_1$ can not be rejected.¹⁵ Thus first movers do not react to changes in w if the level of w is relatively small ($w \leq 0.3$). But for high probabilities of the U-mode, statistically significant reactions can be measured which are in line with our qualitative predictions. From Table 1 we already concluded that first movers tended to demand less than what is prescribed by the theory. The results of regressions FM2

¹⁴ A Breusch Pagan test for heteroscedasticity revealed heteroscedasticity between sessions for first-mover data.

¹⁵ If not explicitly mentioned, significance levels in hypothesis tests are set equal to $p \leq 5\%$.

Table 3 Identified individual patterns of first movers' demands

Observed pattern	Number of cases (percentage)		
	All rounds	Rounds 1–20	Rounds 41–60
$x_{0.9}^m = x_{0.7}^m = x_{0.3}^m = x_{0.1}^m$	9 (30.0)	6 (20.0)	13 (43.3)
$x_{0.9}^m > x_{0.7}^m = x_{0.3}^m = x_{0.1}^m$	8 (26.7)	4 (13.3)	4 (13.3)
$x_{0.9}^m = x_{0.7}^m > x_{0.3}^m = x_{0.1}^m$	6 (20.0)	5 (16.7)	5 (16.7)
$x_{0.9}^m > x_{0.7}^m > x_{0.3}^m = x_{0.1}^m$	6 (20.0)	7 (23.3)	6 (20.0)
$x_{0.9}^m > x_{0.7}^m = x_{0.3}^m > x_{0.1}^m$	–	4 (13.3)	–
$x_{0.9}^m > x_{0.7}^m > x_{0.3}^m > x_{0.1}^m$	–	1 (3.3)	–
Other	1 (3.3)	3 (10.0)	2 (6.7)
Total	30 (100)	30 (100)	30 (100)

Note: x_w^m stands for individual median demands at given w -levels.

and FM4 imply that this tendency became stronger as sessions progressed. Regression FM2 shows that there is a negative and significant effect ($\beta_0 = -0.061^{**}$) of time across all treatment conditions. Regression FM 3 and FM4 measure time effects for each of the treatment conditions separately. Since $\beta_9 < \beta_7 < \beta_3 < \beta_1 < 0$, we see that the magnitude of this effect monotonically varies with the probability of the ultimatum mode. Note that one must (separately) reject the restrictions $\beta_9 = \beta_7$ and $\beta_7 = \beta_3$ and that β_1 is insignificant. Finally, since the coefficient δ_c is small in magnitude and statistically insignificant, we conclude that conflict (N-mode) or rejection of an offer (U-mode) in the preceding round has no immediate effect on first mover behavior.¹⁶

We also analyzed individual patterns of first movers' median demands. We identified 6 monotone patterns which are shown in the first column in Table 3. In this Table, x_w^m denotes an individual's median demand for a given value of w . We categorize individual behavior separately for all rounds (column 2), the first 20 rounds (column 3) and the last 20 rounds (columns 4).

Consider, for example, column 2 in Table 3 counting patterns with regard to all rounds. Nine subjects (30%) have median demands that are identical for all w 's. As it turns out, all median demands of these subjects are the equal split. Another 8 subjects (27%) only differentiate between the two high values of w and display the same median demand for all other probabilities. The next 6 subjects (20%) only have two different median demands: one for the two high probabilities and a lower one for the two small probabilities. Another 6 subjects have 3 different median demands. They appear to treat the two low probabilities alike. The one subject appearing in category "other" in Table 3 states median demands that monotonically increase with a decreasing probability w . Note that for no subject median demands decreased with a falling probability w in a strictly monotonic way as predicted by theory. Finally, all subjects have the same median demand for the two low w -values, namely $x_{0.3}^m = x_{0.1}^m = 50$.

When comparing behavior in the first and the last 20 rounds, the clearest effect we observe is a shift towards more w -invariant equal-split demands. Their share increases from 20% to about 43%.

¹⁶ We tested for autocorrelation in residuals in model FM4 but didn't find any evidence for it.

Observation 1. The behavior of first movers can be summarized as follows:

- (i) Average demands (weakly) increase with the probability w of the ultimatum mode. However, demands do not differ significantly for the two small levels of w .
- (ii) In all treatment conditions, the equal-split demand constitutes the mode. Moreover, about 30% of all subjects tend to state w -invariant demands.
- (iii) On average, first movers demand less than suggested by the risk dominant solution in all treatment conditions. This deviation is higher the higher the probability of the ultimatum mode.
- (iv) First mover demands decrease as sessions progressed. This effect is more pronounced the higher the probability of the ultimatum mode. Also, the share of equal-split demands increased over time.

4.3. Analysis of uninformed second-mover behavior

We now turn to the behavior of uninformed second movers, i.e., of second movers facing the Nash demand mode (N). In this case, second movers were asked to state their demand $y(w)$ being only aware of probability $1 - w$ by which first movers expected the N-mode. We estimated random-effects models that were similar to the ones we estimated for first movers (see Eq. (1)), except for the fact that for second movers no adjustment for heteroscedasticity was necessary. The results of second-mover regressions are presented in Table 4. Regressions USM1-USM4 only differ in terms of the independent variables that were included.

Considering first the effect of the probability w on uninformed second movers' behavior, regressions USM1 and USM2 show that without exception the coefficients of the treatment dummies are positive and significant (with $\alpha_7 < \alpha_3 < \alpha_1$). While the hypothesis that $\alpha_7 = \alpha_3$ can be rejected, the hypothesis that $\alpha_3 = \alpha_1$ can not. Hence parallel to our findings about first-mover demands, the two small probabilities of information transmission are treated alike whereas in the range of higher levels of w , participants are more sensitive to w . Regression USM2 shows that there is a small but highly significant time effect across all treatment conditions as $\beta_0 = 0.03$. Regression USM3 (and also USM4) measures time effects for each of the treatment conditions separately. As it turns out, while in some cases there is a significant combined effect of time and treatment condition, in other cases there is none. Finally, we note that the coefficient δ_c is again statistically insignificant (see USM4).¹⁷

Also for uninformed second movers we identified individual patterns of median demands. The results are summarized in Table 5. Due to the all-or-nothing signal technology explored in this experiment, there are some subjects for which we do not have any observation for higher probabilities ($w = 0.7$ or more often $w = 0.9$). In these cases we base our categorization on the available observations. This is indicated in Table 5 by writing $\{y_{0.9}^m$ or $y_{0.7}^m\}$.¹⁸

Demanding 50 reflects in almost all cases individual median behavior at $w = 0.1$ and $w = 0.3$. For example, regarding behavior in all rounds, for 28 out of 30 uninformed second movers the median demand at the two smaller w -values is 50. Also with respect to all rounds, the rate of uninformed second movers who show invariance in their median demands

¹⁷ We tested for autocorrelation in residuals in model USM4 but didn't find any evidence for it.

¹⁸ This means that sometimes the comparison is made only to observations for $w = 0.9$ in case an observation for $w = 0.7$ is missing and vice versa. Furthermore, for some of the subjects we only have data for $w = 0.1$ and $w = 0.3$ or only for $w = 0.1$ and $w = 0.7$ for the first respectively the last 20 rounds (see third to last row in Table 4).

Table 4 Results of uninformed second-mover regressions

	Regression USM1	Regression USM2	Regression USM3	Regression USM4
α_0	44.701*** (0.622)	43.807*** (0.657)	46.7510*** (1.278)	47.265*** (1.306)
$\alpha_7 (D_7)$	3.244*** (0.633)	3.138*** (0.629)	0.938 (1.435)	0.857 (1.430)
$\alpha_3 (D_3)$	4.292*** (0.576)	4.292*** (.571)	0.431 (1.303)	0.233 (1.306)
$\alpha_1 (D_1)$	4.806*** (0.575)	4.781*** (0.570)	2.159* (1.308)	2.208* (1.307)
$\beta_0 (t)$		0.030*** (.007)		
$\beta_9 (D_9 \times t)$			-0.069* (0.038)	-0.069* (0.037)
$\beta_7 (D_7 \times t)$			0.008 (0.019)	0.010 (0.019)
$\beta_3 (D_3 \times t)$			0.060*** (0.012)	0.066*** (0.012)
$\beta_1 (D_1 \times t)$			0.019* (0.011)	0.018 (0.116)
$\delta_c (Conflict_{t-1})$				-0.600 (0.380)
Adj. pseudo R^2	0.065	0.079	0.090	0.110

Note: Standard errors of estimators in parentheses; * $p < 0.1$, *** $p < .01$.

Table 5 Identified individual patterns of uninformed second movers' demands

Observed patterns	Number of cases (percentage)		
	All rounds	Rounds 1 to 20	Rounds 41 to 60
$y_{0.9}^m = y_{0.7}^m = y_{0.3}^m = y_{0.1}^m$	13 (43.3)	4 (13.3)	2 (6.7)
$y_{0.9}^m < y_{0.7}^m = y_{0.3}^m = y_{0.1}^m$	4 (13.3)	-	-
$y_{0.9}^m = y_{0.7}^m < y_{0.3}^m = y_{0.1}^m$	1 (3.3)	-	-
$y_{0.9}^m = y_{0.7}^m = y_{0.3}^m < y_{0.1}^m$	-	1 (3.3)	-
$y_{0.9}^m < y_{0.7}^m < y_{0.3}^m = y_{0.1}^m$	3 (10.0)	1 (3.3)	2 (6.7)
$\{y_{0.9}^m \text{ or } y_{0.7}^m\} = y_{0.3}^m = y_{0.1}^m$	6 (20.0)	7 (23.3)	17 (56.7)
$\{y_{0.9}^m \text{ or } y_{0.7}^m\} < y_{0.3}^m = y_{0.1}^m$	2 (6.7)	3 (10.0)	6 (20.0)
$\{y_{0.9}^m \text{ or } y_{0.7}^m\} = y_{0.3}^m < y_{0.1}^m$	-	3(10.0)	-
$\{y_{0.9}^m \text{ or } y_{0.7}^m\} < y_{0.3}^m < y_{0.1}^m$	-	-	-
$y_{0.3}^m = y_{0.1}^m \text{ or } y_{0.7}^m = y_{0.1}^m$	-	4(13.3)	3 (10.0)
Other	1 (3.3)	7 (23.3)	-
Total	30 (100)	30 (100)	30 (100)

Note: y_w^m stands for individual median demands at given w -levels.

over all four w 's is with 43.3% higher than for first mover data (30%). Including those subjects who only confronted three different w 's the share of w -invariant patterns is even 63.3%(= 43.3% + 20%).

For 7 subjects we could not identify a stable (or monotonically increasing) pattern in median behavior over the first 20 rounds. No such case occurred for data of the last 20 rounds. Neither over all rounds nor over the last 20 rounds, there is a subject with a median demand pattern that increases monotonically with decreasing probability w , as predicted by the risk-dominant solution.

Observation 2. The behavior of uninformed second movers can be summarized as follows:

- (i) Demands of uninformed second movers (weakly) decrease with the probability w of the ultimatum mode. However, demands do hardly react to small levels of w .
- (ii) Demanding 50 is focal and 43% (63%) of all subjects have w -invariant median demands over all w 's (over those values of w they confronted).
- (iii) On average, uninformed second movers demand more than their risk-dominant solution demand in all treatment conditions. This deviation is larger the higher the probability of the ultimatum mode.

4.4. Analysis of informed second-mover behavior

We finally analyze the behavior of informed second movers who were aware of demand x of first movers (U-mode). As can be seen in Table 1, all but one demand was accepted by informed second movers in case of $w = 0.1$ and $w = 0.3$.¹⁹ By and large this is due to the fact that first movers proposed the equal split ($x = 50$) in these cases. Therefore, in all informed second mover regressions in Table 6 acceptance for both $w = 0.1$ and $w = 0.3$ is

Table 6 Results of informed second-mover regressions

	Regression ISM1	Regression ISM2	Regression ISM3
$\alpha_9 (D_9)$	0.687 (0.618)	0.191 (0.678)	0.308 (0.704)
$\alpha_7 (D_7)$	0.728 (0.620)	0.631 (0.683)	0.904 (0.718)
$\gamma (x_{jt})$	0.262*** (0.024)	0.271*** (0.025)	0.282*** (0.027)
$\beta_9 (D_9 \times t)$		0.013* (0.007)	0.010 (0.008)
$\beta_7 (D_7 \times t)$		0.001 (0.010)	0.007 (0.011)
$\delta_c (\text{Conflict}_{t-1})$			0.303 (0.256)
Adj. pseudo R^2	0.693	0.697	0.717

Note: Standard errors of estimators in parentheses. * $p < 0.1$, *** $p < .01$.

¹⁹ The only exception is a rejected demand of $x = 57$ in treatment condition $w = 0.3$.

the reference point. Note that 64% (47%) of first-mover demands exceeded $x = 50$ in case of $w = 0.9$ ($w = 0.7$). Table 6 shows the results of several fixed-effects probit regressions of rejection behavior of informed second movers. The underlying model of regression ISM3 for example is:

$$\text{Prob}[Reject_{it} = 1] = F(\alpha_9 D_9 + \alpha_7 D_7 + \gamma x_{jt} + \beta_9 (D_9 \times t) + \beta_7 (D_7 \times t) + \delta_c \text{Conflict}_{t-1} + \alpha_i)$$

where $Reject_{it}$ equals one if second mover i rejected the offer $y_{it} = 100 - x_{jt}$ of first mover j in period t ; and α_i is the subject specific effect to be estimated. All other variables are defined as in equation (1) and as in all binary probit regressions the underlying distribution is the standard normal.²⁰

All demands by first movers in treatment conditions $w = 0.7$ and $w = 0.9$ were smaller than 100. Thus, subgame perfection predicts that all demands should be accepted suggesting the null hypothesis $\gamma = 0$. If, however, higher demands by first movers are rejected more often, we expect $\gamma > 0$. In all models in Table 6, the coefficient γ is significantly greater than 0, meaning that higher demands are rejected more often. Given the evidence reported in the vast literature on the ultimatum game this result is of course not surprising.

In all three regressions coefficients α_9 and α_7 are insignificant (and insignificantly different from another), i.e. an identical offer is not more acceptable for $w = 0.9$ than for $w = 0.7$ or $0.3/0.1$. Finally, we can not reject that there are no time effects (at $p < 5\%$) and again we do not find a significant influence of variable Conflict_{t-1} .

Observation 3. The behavior of informed second movers can be summarized as follows:

- (i) Rejection behavior is mainly driven by first-mover demands. Higher demands (or smaller offers) by first movers are more likely rejected.
- (ii) There are no significant direct effects of treatment probability w on rejection behavior.
- (iii) Rejection behavior is stable across rounds.

5. Discussion

The results of our experiments show that first movers and uninformed second movers (weakly) react to the control variable w as predicted. For the three higher levels of w we observe statistically significant differences in demands. Furthermore, for the highest level of w given in the experiment ($w = 0.9$) our results resemble stylized facts about behavior in ultimatum games: Informed second movers frequently reject payoff shares of $1/3$ and even above. (The average rejected offer is 33.45). First mover demands are concentrated in the range $1/2 \leq x \leq 3/4$ of relative demands with the equal split $1/2$ being modal. The frequency of equal split demands is higher the lower the value of w . For the probabilities $w = 0.1$ and $w = 0.3$ the median demand of all first movers and of over 90% of second movers is the equal split. Individual demands only become more dispersed for high probabilities w ($w = 0.7$ and $w = 0.9$).

²⁰ All coefficients including the 30 subject effects were estimated by maximizing the unconditional log likelihood using Newton's iteration method.

Although mean demands vary with probability w , median demands of about 30% of all first movers and of more than 40% of uninformed second movers are invariant over all four w 's. Most first and uninformed second movers who do not show w -invariance in their medians, only have two different median demands for all four levels of w , i.e., they demanded the equal split for low probabilities w and adjusted their demand only once. This suggests that for relatively high levels of w , participants treat w -games like an ultimatum game and for relatively low levels of w like a Nash demand game.

Parallel to findings in market experiments with errors in communication (see Güth et al., 2001) we observe that for a high probability w , first movers enjoy a first-mover advantage. However, this first-mover advantage is not as strong as predicted by theory. Nevertheless it is statistically significant although with repeated interaction demands become increasingly invariant with respect to w . A possible reason for this is that from the very beginning of the experiment second movers reject considerable offers in the U-mode and appear to insist on the equal split in the N-mode. Conflict and rejection rates increase with w . Though demands adjust with experience the effect is considerably stronger for first-mover demands.

Altogether, like in other pie-sharing experiments we observe a strong focus on the equal split. The significant effects regarding our control variable w rely on a rather small fraction of subjects reacting to the probability of revelation, but over time they also learned and were taught to play fair.

Appendix A: Proof of Proposition 1

Let us consider two different strict equilibria of the N-truncation: (x, y) and (\tilde{x}, \tilde{y}) with $x, y, \tilde{x}, \tilde{y} > 0, x + y = \tilde{x} + \tilde{y} = p, x \neq \tilde{x}$ and thus $y \neq \tilde{y}$. The 2×2 -bimatrix game

	y	\tilde{y}	
x	x, y	$wx, w(p - x)$	(2)
\tilde{x}	$\tilde{x}, w(p - \tilde{x}) + (1 - w)y$	\tilde{x}, \tilde{y}	

is the minimal formation²¹ spanned by (x, y) and (\tilde{x}, \tilde{y}) for $x > \tilde{x}$. For all $w \in [0, 1]$, strictness of (x, y) and (\tilde{x}, \tilde{y}) implies that they are also strict equilibria of this bimatrix game. Risk dominance for 2×2 -bimatrix games with two strict equilibria is axiomatically characterized by

- invariance with respect to isomorphic transformations,
- best reply invariance,
- monotonicity.

A best reply preserving transformation of bimatrix in (2) is²²:

	y	\tilde{y}
x	$x - \tilde{x}, (1 - w)y$	$0, 0$
\tilde{x}	$0, 0$	$\tilde{x} - wx, (1 - w)(\tilde{y} - y)$

²¹ A formation is a substructure which results from excluding strategies and which is closed with respect to the best reply correspondence in the original game. It is minimal if it contains no proper subformation.

²² Best reply-preserving transformation in detail: $u_x(x, y) - u_x(\tilde{x}, y) = x - \tilde{x}$, respectively $u_x(\tilde{x}, \tilde{y}) - u_x(x, \tilde{y}) = \tilde{x} - wx$ and $u_y(x, y) - u_y(x, \tilde{y}) = y - w(p - x) = y - wy$, respectively $u_y(\tilde{x}, \tilde{y}) - u_y(\tilde{x}, y) = \tilde{y} - w(p - \tilde{x}) - (1 - w)y = (1 - w)(\tilde{y} - y)$.

An isomorphic transformation finally yields²³:

	y	\tilde{y}
x	$\frac{x-\tilde{x}}{\tilde{x}-wx}, 1$	$0, 0$
\tilde{x}	$0, 0$	$1, \frac{\tilde{y}-y}{y}$

Invariance with respect to isomorphisms and best reply–preserving transformations implies that neither (x, y) risk dominates (\tilde{x}, \tilde{y}) nor vice versa whenever

$$\frac{x - \tilde{x}}{\tilde{x} - wx} = \frac{\tilde{y} - y}{y}$$

Thus monotonicity²⁴ implies that (x, y) risk dominates (\tilde{x}, \tilde{y}) for

$$\frac{x - \tilde{x}}{\tilde{x} - wx} > \frac{x - \tilde{x}}{p - x}$$

Rearranging and substituting \tilde{y} by $p - \tilde{x}$ and y by $p - x$ yields

$$\frac{x + \tilde{x} - p}{\tilde{x}} < w \tag{3}$$

Due to the symmetry properties between the formation spanned by (x, y) and (\tilde{x}, \tilde{y}) for $x > \tilde{x}$ and that for $x < \tilde{x}$, one obtains a similar result for the latter case. For $x < \tilde{x}$, (x, y) risk dominates (\tilde{x}, \tilde{y}) if

$$\frac{\tilde{x} + x - p}{x} > w \tag{4}$$

Now for $x = x^*(w)$ where $x^*(w) = p/(2 - w)$, condition (3) is equivalent to $\tilde{x} < x^*(w)$ and condition (4) to $\tilde{x} > x^*(w)$ proving that the strict equilibrium corresponding to $x^*(w)$ risk dominates all other strict equilibria of the N-truncation. Thus $x^*(w)$ is the solution demand for all $w \in [0, 1]$, what proves Proposition 1.

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²³ Isomorphic transformation in detail: $u_x(x, y)/u_x(\tilde{x}, \tilde{y})$ and $u_y(\tilde{x}, \tilde{y})/u_y(x, y)$.

²⁴ If a risk undominated strict equilibrium is “strengthened” by increasing one of its equilibrium payoffs this is then risk dominating.

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