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# Strategies, heuristics, and the relevance of risk-aversion in a dynamic decision problem

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**Abstract**

In this paper a complex decision problem where subjects have to cope with a time horizon of uncertain duration and must update their termination probabilities which depend on stochastic events during "life" is considered. First it is described how economic theory suggests to solve the decision problem. But since real decision makers can hardly be expected to behave according to the theoretical solution in the problem at hand, several heuristics or rules of thumb are described and their theoretical performance investigated. Then observed behavior and the way how people tackled the problem are described. In the second part of the paper I discuss how much of the data can be explained by assuming that experimental subjects are risk-averse. © 2001 Published by Elsevier Science B.V.

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## 1. Introduction

How people solve dynamic decision problems does not seem to be a well-developed area of experimental research, but recently things have begun to change. There are experimental studies like Johnson, Kotlikoff, and Samuelson (1987), Hey and Dardanoni (1988) and more recently Fehr and Zych (1995) and Anderhub, Güth, Müller, and Strobel (2000) that investigate dynamic decision making in saving and consumption contexts in either a deterministic or a stochastic environment. These studies provide interesting insights into human decision making in dynamic situations but their main focus is on whether people behave according to theoretical predictions.

It is only recently that attention has been paid to the question of how people actually solve dynamic decision problems. For example, Carbone and Hey (1998) investigate what people do when faced with a fairly easy dynamic decision problem. The authors find that only a few subjects use Backward Induction (BI) in a consistent and thorough manner, although most people try to use BI. Koehler (1996) gives a report of an experimental investigation of consumption under uncertainty. Subjects are offered a trial facility which enables them to try out different consumption patterns before the actual decision is made. Koehler is able to identify a number of different strategies used by subjects within the trial calculations.

This study tries to contribute to the discussion of intertemporal decision making by further investigating an experiment by Anderhub et al. (2000). While this study explores whether subjects' behavior is at least qualitatively as prescribed by the optimal solution, we here focus on strategies or heuristics that have been used and subjects' attitudes toward risk.

In their experiment Anderhub et al. (2000) basically design a consumption environment with a finite but uncertain time horizon. Moreover, a new feature is added to this more traditional setup by only successively revealing the individual termination probability. This was implemented by using three dice representing different termination probabilities, of which one was excluded after the first choice and another after the second choice. The remaining die then represents the constant termination probability after the third choice. Focussing on subject's qualitative behavior, Anderhub et al. (2000) find that: (i) average observed behavior displays similar effects as the benchmark solution, based on risk neutral utility maximization and (ii) in the complex stochastic environment, on average subjects react in a qualitatively correct way to "good" or "bad" news, i.e. on average subjects make use of particular information concerning their length of "life".

When solving such a dynamic decision problem economic theory expects subjects to rely on BI. The analytical solution of this specific problem, however, is extremely difficult to obtain. In fact one has to rely on numerical techniques in order to derive the optimal solution assuming a risk-neutral decision maker (see Section 3.1). For the optimal solution to be a useful prediction of actual play, subjects should be able to arrive at this solution, though not necessarily by using the same techniques (Friedman, 1953). In the light of the difficulties involved with the derivation of optimal behavior for the problem at hand, it is questionable if the optimal solution is a useful predictor or a "useful approximation" (see Roth, 1996) for the behavior of boundedly rational decision makers.

However, since many economically relevant situations that occur in real life are dynamic in nature, e.g., savings and consumption decisions, it is both interesting and important to find out how human decision makers behave in dynamic situations as complex as the one at hand. The findings of this study provide further evidence (see also e.g., Hey, 1983) that subjects faced with a dynamic decision problem rely on heuristics and apply some forward-looking procedures instead of using BI.

The remainder of the paper is organized as follows: After introducing the experimental design in Section 2, in Section 3 the optimal solution of the decision problem will be presented as well as several simpler strategies and heuristics. In Section 4 observed (average and individual) behavior will be described. Section 5 theoretically investigates the effects of risk aversion for the experimental situation and shows that these results are relevant for explaining observed data. Finally, the findings are summarized and discussed in Section 6.

## 2. The experiment

The experiment consists of 12 rounds. At the beginning of each round a participant is given an amount of 11.92 DM. (There is no further income in a round.) In each round the task is to distribute this amount over an uncertain number of periods. All a participant knows at the beginning of each round is that the round will consist of at least 3 and at most of 6 periods, i.e. the actual number of periods, $T$, in one round of the game is a stochastic variable whose range is the set of numbers $\{3,4,5,6\}$. After the third period of a round an identical and independent random move decides whether or not a subject reaches the next period. Thus after deciding on the consumption level

$x_i$ $(3 \leqslant i \leqslant 5)$ with probability $w \in (0,1)$ the round ends in which case the subject earns the payoff $U = \prod_{k=1}^{i} x_k$ in that round. With probability $1 - w$ the round continues with period $i + 1$. In the case where a subject reaches the sixth period of a round the computer invests any money left over. There are three different termination probabilities, namely $w \in \{1/2, 1/3, 1/6\}$. At the beginning of each round the players do not know which of the three termination probabilities will be applied from the third period on. The information about this is only successively revealed during the first two periods of a round. The players are told that after confirming the choice of $x_1$ one of the three probabilities is randomly excluded and that after confirming the choice of $x_2$ one of the remaining two probabilities is randomly excluded. Thus, a player does not know before the third period which of the three probabilities will be applied from then on. The three different termination probabilities were represented by dice of different colors: $w = 1/2$ was represented by a red die, $w = 1/3$ by a yellow die and $w = 1/6$ by a green die. Players are told that from the third period on the relevant die will be thrown by the computer and they are informed which points the die has to show in order to reach a new period. For instance, if the red die shows the numbers 4, 5 or 6 the participant reaches the next period (see Appendix B for details).

Owing to the three different dice or termination probabilities there are altogether six possible sequences of initial chance moves (first red die excl. then yellow die excl. such that the green die applies, and so on ...). Each participant plays all six sequences in a random order before they are repeated in another random order. [1] The first (second) random order including rounds 1–6 (7–12) will be referred to as the first (second) cycle. The random orders were separately drawn for each subject.

Moreover, before the first decision, participants are asked whether they want to be paid according to the average payoff of all 12 rounds or according to the payoff of one randomly selected round which, of course, is drawn after the experiment. The fully computerized experiment was run in several sessions with a total of 50 participants who were mainly undergraduate or graduate students of business administration or economics. During the experiment subjects could examine both their choices made in a previous round and the choices already made in the actual round. Furthermore, in each period of a round the computer screen showed the provisional payoff as

[1] This procedure was chosen in order to obtain comparable decisions. We did not find evidence that participants noticed this weak regularity.

implied by the choices already made along with the residual fund and the color(s) of the excluded and/or remaining die (dice). The software offered access to a calculator so that participants could easily check the numerical consequences of certain choices. [2] Without imposing time constraints, subjects needed on average 90 minutes to complete the task. Average earnings were DM 27.62.

## 3. How to solve the decision problem?

### 3.1. Economic theory

#### 3.1.1. Backward induction & principle of optimality (BI)

Standard economic theory expects a decision maker to work backward through the decision tree taking into account the principle of optimality. This principle states that the decision at any node in the tree has to be optimal given optimal behavior thereafter. For the problem at hand this implies that a subject first has to determine optimal behavior in the fifth period for every possible history in the game. Then a subject has to determine optimal behavior in the fourth period taking into account the optimal behavior in the fifth period and so on until the first period is reached. Note that when deciding on the optimal behavior in the second (in the first) period the termination probability can assume two (three) values. This together with the special form of the payoff function makes it extremely difficult to solve this game analytically (even by assuming a risk-neutral decision maker). Anderhub et al. (2000) rely on numerical methods to find the solution for a risk-neutral decision maker. The trajectories of the solution are displayed in Fig. 1. (The small numbers on top of the boxes indicate the residual funds available to the decision maker at the beginning of a new period.)

The BI solution will serve as a benchmark for good performance. Throughout the paper this solution will be referred to as the *optimal* solution. Note that the optimal solution reacts sensitively to all chance moves. For example if the order of die exclusion is ¬green, ¬yellow such that the red die (highest termination probability) is the remaining one the termination probability has to be gradually updated upwards. The obvious consequence is that consumption must increase during the first three periods. Similarly, if

[2] We did not record how often and to what purpose the calculator was used.

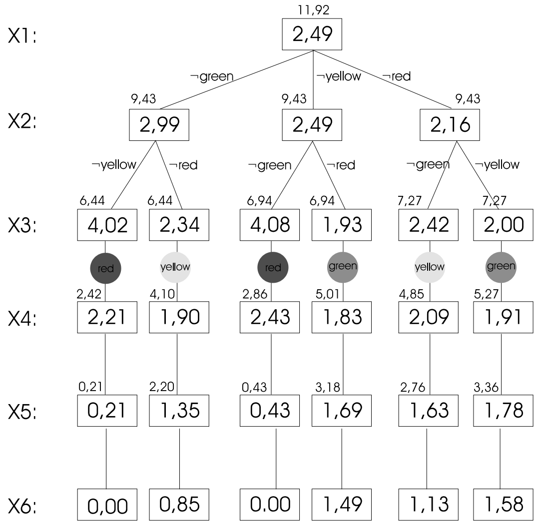


Fig. 1. Optimal consumption behavior.

the order of exclusion is ¬red, ¬yellow such that the green die (lowest termination probability) is the remaining one the termination probability has to be gradually updated downwards with the consequence that consumption must decrease during the first three periods. Furthermore, consumption decreases from the third period on for all six sequences of initial chance moves.

### 3.2. (Simple) strategies and heuristics

In this section several strategies and heuristics will be described that are tailored to the problem at hand.

#### 3.2.1. Expected number of periods (ENP)

The idea of this strategy is to set current consumption equal to current wealth divided by the expected number of remaining periods (see Fig. 2). For example, at the beginning of each round the expected number of periods [3] is 4.46. Thus, this strategy suggests to set $x_1 = 11.92/4.46 \approx 2.67$. Note that this strategy proceeds forward when determining the decision. The most difficult

[3] These numbers appear at the top left-hand side of the boxes in Fig. 2.

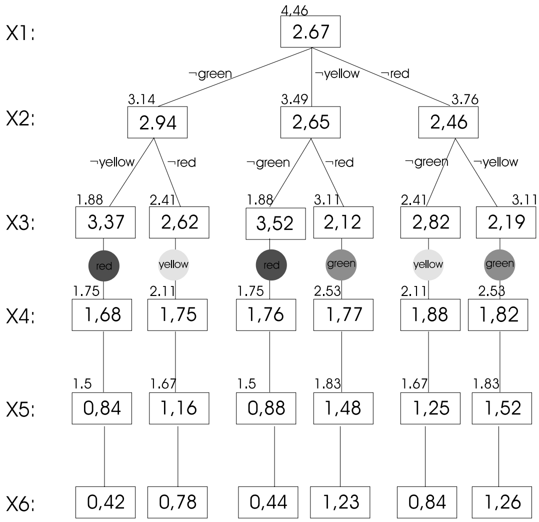


Fig. 2. Expected number of period strategy.

task is of course to compute the expected number of remaining periods. Whereas there is little chance that the BI solution might be computed or heuristically approximated by an experimental subject – at least in the earlier periods – this is more likely for ENP. This is especially true for later periods for which the exact computation of the expected number of remaining periods becomes easier.

### 3.2.2. *Equiprobability (EQP)*

The EQP heuristic that is often mentioned in the literature (see for instance Johnson & Payne, 1985) ignores the fact that different outcomes occur with different probabilities and treats all events as equally likely. Mathematically, the problem can be stated as follows:

$$\frac{1}{4}(x_1x_2x_3 + x_1x_2x_3x_4 + x_1x_2x_3x_4x_5 + x_1x_2x_3x_4x_5x_6) \to \max!$$

$$\text{subject to } x_i \geqslant 0 \ (i = 1, 2, \ldots, 6) \quad \text{and} \quad \sum_{i=1}^{6} x_i \leqslant 11.92.$$

The necessary and sufficient conditions imply

$$x_1^* = x_2^* = x_3^* = 2.50, \quad x_4^* = 2.22, \quad x_5^* = 1.60, \quad x_6^* = 0.60.$$

Table 1
Strategies and heuristics

| Strategy/heuristic | Exp. payoff | S.D. | Eff. | Min. payoff | Max. payoff | $\delta_{opt}$ | $\delta_{obs}$ |
|---|---|---|---|---|---|---|---|
| BI | 35.16 | 18.25 | 1.00 | 0 | 57.78 | 0 | 0.35 |
| ENP | 33.59 | 11.44 | 0.96 | 14.38 | 50.14 | 0.31 | 0.09 |
| EQP | 30.64 | 12.69 | 0.87 | 15.25 | 55.15 | 0.48 | 0.37 |
| APU | 26.29 | 0 | 0.75 | 26.29 | 26.29 | 0.74 | 0.41 |
| 2 | 30.55 | 23.04 | 0.87 | 8.00 | 61.44 | 0.70 | 0.68 |
| MAX | 24.89 | 29.68 | 0.71 | 0 | 78.86 | 0.89 | 0.89 |
| Av. Obs. Beh. | 27.62 | 15.6 | 0.79 | 0 | 78.86 | 0.35 | 0 |

### 3.2.3. *Avoidance of payoff uncertainty (APU)*

Adopting this strategy a player tries to achieve the same payoff for all possible courses of the game. Thus he has to make sure that $x_1x_2x_3 = x_1x_2x_3x_4 = x_1x_2x_3x_4x_5 = x_1x_2x_3x_4x_5x_6$ from which one concludes $x_4^* = x_5^* = x_6^* = 1$ and $x_1 + x_2 + x_3 = 11.92 - 3 = 8.92$ implying $x_1^* = x_2^* = x_3^* = 8.92/3 = 2.97$ $(x_1 = 2.98)$. [4]

In order to relate the various strategies and heuristics described above to the optimal solution, BI, Table 1 shows some of their characteristics such as expected, minimal, and maximal payoff and efficiency which is defined as the expected payoff of a given strategy/heuristic divided by the expected payoff of the optimal solution implying, for example, the value 1 for the latter. Furthermore, the value $\delta_{opt}$ gives the mean of the absolute deviation (cell by cell) of a given strategy/heuristic from the optimal solution. Formally,

$$\delta_{opt} = (1/28)\sum_{k=1}^{28}\left|x_k^{(\cdot)} - x_k^{BI}\right|,$$

where $x_k^{(\cdot)}(x_k^{BI})$ are the consumption levels of a given strategy or heuristic (of the BI solution) for each of the 28 nodes of the decision tree. For example, the ENP strategy differs on average by 0.31 payoff units per cell from the optimal solution. (For the rows "2" and "MAX" and the value $\delta_{obs}$ see the next section.)

[4] Note that in lottery choice experiments, subjects often choose the lottery (if offered) that pays a certain amount for sure (see e.g. Camerer, 1995; Huck & Weizsäcker, 1999).

# 4. Observed behavior

## 4.1. Average observed behavior

Fig. 3 shows the mean, minimum, maximum and the variance of observed decisions at all nodes (the numbers of cases are given above each box). The line above or below each box indicates whether the given mean value lies above or below the corresponding value of the optimal solution.

At the end of the previous section the measure $\delta_{opt}$ was introduced in order to relate the various strategies and heuristics to the optimal solution. The measure $\delta_{obs}$ is similarly constructed but this time with reference to average observed behavior as shown in Fig. 3. Thus, $\delta_{obs}$ measures the mean absolute deviation (per cell) of a particular strategy/heuristic from average observed behavior (see Table 1). The result is striking since according to this measure the strategy ENP fits the data best and is about four times closer to (average) observed behavior than the optimal solution.

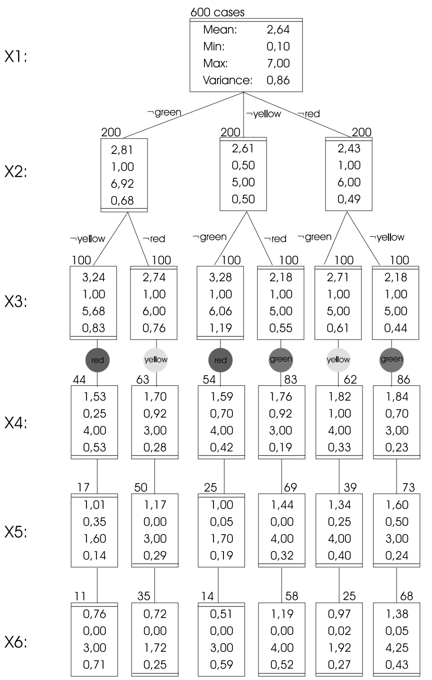


Fig. 3. Average observed behavior.

The fact that the strategy ENP explains average decisions much better than the optimal solution remains true if one uses the dynamically adjusted solutions in order to compute the $\delta_{\mathrm{obs}}$-measure, i.e. observed data were compared with the decisions that would have been the correct ones (according to both strategies) given the – for the most part – wrong decisions in former periods. The values of the measure $\delta_{\mathrm{obs}}$ are now 0.09 and 0.27 payoff units for the dynamically adjusted strategy ENP and the dynamically adjusted optimal solution, respectively. [5] Thus the forward working strategy ENP seems to be a serious alternative to the backward working optimal solution.

### 4.2. Behavioral patterns

Although aggregated behavior is quite impressive in the sense that it reacts to information qualitatively in the same way as the optimal solution does, behavior on the individual level varies considerably. First of all individuals seem to differ in the degree to which they understand the rules and the highly stochastic nature of the decision problem. There are only few participants who act consistently and at least in a qualitatively correct way across rounds and periods. Some participants start out very consistently at the beginning of one round but then their decisions become less thought-out.

In this section observed behavior will be described in more detail. This is done in order to find out whether there is evidence on the individual level for the strategies and heuristics described above and whether there are other rules of thumb used by experimental subjects. Initially, it seems almost impossible to categorize the individual data. But by looking very closely at the individual decisions it is possible to roughly characterize about half of the data.

Owing to the complexity of the problem it is reasonable to concentrate attention on the second cycle of the game. Although one cannot assume that play has converged in the second cycle it is at least more likely that at this stage subjects have understood the rules of the game and that they are familiar using the computer.

The patterns of behavior as described below are not always based on clear definitions. They solely try to reflect certain features of behavior that become apparent when one looks at the data. The hope, however, is to provide some

[5] Note that this time only the decisions up to the fifth round were included in order to compute the measure $\delta_{\mathrm{obs}}$.

Table 2
Example for qualitatively optimal behavior

| Random | $x_1$ | 1st die | $x_2$ | 2nd die | $x_3$ | Rem. die | $S_4$ | $x_4$ | $x_5$ | $x_6$ | Payoff |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 2.93 | Green | 3.30 | Yellow | 4.10 | Red | 1.59 | 1.00 | 0.45 | | 17.84 |
| 8 | 2.93 | Yellow | 2.93 | Green | 4.23 | Red | 1.83 | 1.20 | | | 43.58 |
| 9 | 2.93 | Green | 3.30 | Red | 3.20 | Yellow | 2.49 | | | | 30.94 |
| 11 | 2.93 | Red | 2.80 | Green | 3.30 | Yellow | 2.89 | | | | 27.07 |
| 10 | 2.93 | Yellow | 2.93 | Red | 2.50 | Green | 3.56 | 1.67 | 1.11 | 0.78 | 31.03 |
| 12 | 2.93 | Red | 2.80 | Yellow | 2.50 | Green | 3.69 | 1.60 | 1.20 | 0.89 | 35.00 |

stylized facts about the important question of how subjects try to solve the dynamic decision problem.

#### *4.2.1. Qualitatively optimal behavior*

Subjects in this group (two subjects, i.e., 4% of all participants) display the same qualitative behavior as the optimal solution. They start in each of the six rounds with the same moderate value for $x_1$ and their reactions to all the information are qualitatively correct (and also consistent with respect to $x_2$) i.e., they increase/decrease consumption after "bad news"/"good news". Furthermore, consumption decreases from the third period on. An example is shown in Table 2. [6]

#### *4.2.2. Consistent but not qualitatively optimal behavior*

Subjects in this group (eight subjects, 16%) meet three optimality criteria: (1) they rely on the same $x_1$-choice in all six rounds of the second cycle, (2) they react consistently to the exclusion of the first die, i.e., after the exclusion of a particular die at the first chance move they rely on the same [7] choice for $x_2$ and (3) consumption is monotonically decreasing from the third period on. Because of the first two features it is very likely that subjects in this group had a certain plan when generating their decisions. But despite these consistencies subjects in this group are different from subjects in the above group since their reactions to information are not qualitatively correct. They either choose a rather low/high $x_1$-value such that consumption in the second period

[6] Note that the decisions are not ordered according to the number of rounds but according to the sequences of initial chance moves. Furthermore, $S_4 := 11.92 - \sum_{i=1}^{3} x_i$, i.e. $S_4$ denotes the remaining fund available before deciding about $x_4$.

[7] Actually, these values vary at most by 0.10 units, if at all.

always increases/decreases or their reactions to the exclusion of the second die are qualitatively incorrect.

Some subjects in this group display another interesting tendency: they partially ignore information. One example is shown in Table 3. This subject totally ignores the information concerning the exclusion of the first die and partially ignores the information concerning the exclusion of the second die. (S)He relies on the same choice in the second period and on only two different choices in the third period. It is likely that the representation of the decision tree in this subject's mind is as shown in Fig. 4. This subject's behavior is also interesting in the uncertain periods: Within the first four sequences (s)he saves exactly DM 3 for the uncertain periods and spends exactly DM 1 in each of the uncertain periods with the result that the payoff that was built during the first periods is maintained. This is a variant of the APU strategy. However, the behavior of this subject is different if the green die applies. Now

Table 3
Example for consistent but qualitatively not optimal behavior

| Random | $x_1$ | 1st die | $x_2$ | 2nd die | $x_3$ | Rem. die | $S_4$ | $x_4$ | $x_5$ | $x_6$ | payoff |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 3.92 | Green | 2.00 | Yellow | 3.00 | Red | 3.00 | 1.00 | 1.00 | | 23.52 |
| 10 | 3.92 | Yellow | 2.00 | Green | 3.00 | Red | 3.00 | 1.00 | | | 23.52 |
| 7 | 3.92 | Green | 2.00 | Red | 3.00 | Yellow | 3.00 | | | | 23.52 |
| 12 | 3.92 | Red | 2.00 | Green | 3.00 | Yellow | 3.00 | 1.00 | 1.00 | 1.00 | 23.52 |
| 9 | 3.92 | Yellow | 2.00 | Red | 2.00 | Green | 4.00 | 2.00 | 1.00 | 1.00 | 31.36 |
| 11 | 3.92 | Red | 2.00 | Yellow | 2.00 | Green | 4.00 | 2.00 | | | 31.36 |

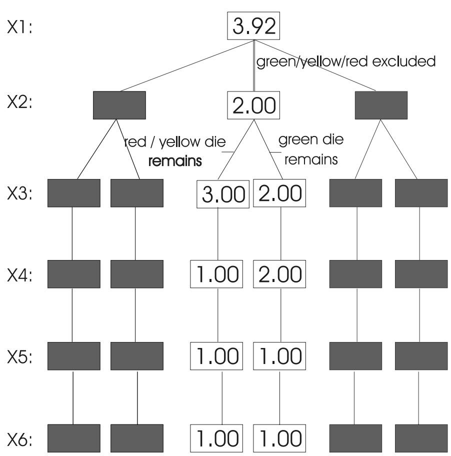


Fig. 4. Representation of the problem by the subject of Table 3.

DM 4 are saved for the uncertain periods in order to spend DM 2 in the fourth period and DM 1 each time the fifth or sixth period is reached. This might be inferred from this subject's decisions.

#### *4.2.3. "Go-for-the-maximum" policy*

The highest possible payoff that can be earned in one round is DM 78.86 by (correctly) guessing that the current round will consist of exactly four periods and allocating the initial amount of DM 11.92 evenly ($11.92/4 = 2.98$) to these periods. There was one subject who was apparently aware of this fact and who tried to reach the maximal payoff at least when the red or yellow die applied. In the course of the 12 rounds this subject earned three times the maximal payoff. [8] This subject deviates from the risky behavior only within the first two rounds where (s)he experiments a bit or when the green die applies. [9] Some characteristics of this strategy are shown in Table 1.

#### *4.2.4. Cautious policy*

The behavior of subjects in this group (two subjects, 4%) is best described as "cautious". These subjects save far more during the certain periods than prescribed by the optimal solution. Subjects in this group also show very irrational behavior. For example one subject chose only DM 0.10 in the first period of round 4. Another subject had two times left DM 3.00 for the sixth period which (s)he reached two times with the red die.

#### *4.2.5. Wait-and-see policy*

The subjects in this group (two subjects, 4%) act cautiously but reasonably during the first two periods. They almost entirely ignore the information concerning the exclusion of the first die and it seems as if they wait until the uncertainty about the termination probability is resolved. Then they strongly react to the information regarding the remaining die via the $x_3$-choice.

#### *4.2.6. "2"-heuristic*

This heuristic simply prescribes $x_1^* = x_2^* = x_3^* = x_4^* = x_5^* = 2.00$ and $x_6^* = 1.92$ disregarding the possible sequences of initial chance moves and

[8] In one of these three instances (4th period of round 7) this subject chose only 2.90 instead of 2.98 implying a payoff of only 76.74.

[9] For those readers interested in such details: this subject decided before the game to be paid according to the result of a randomly chosen round and although this subject earned the highest average payoff (DM 40.88) the actual payment was only DM 2.73.

probabilities. This heuristic is based on the observation that if one experiences six periods in one round it would be optimal (ex post) to choose $11.92/6 \approx 1.99$ in every period (see also Table 1).

This simple strategy is of significance with regard to observed behavior. There is one subject who applied this strategy in all 12 rounds of the game. [10] Furthermore, if one relaxes the above conditions and allows for $1.90 \leqslant x_1, x_2, x_3, x_4, x_5, x_6 \leqslant 2.10$ one observes that this strategy is used in 43 cases by 18 different subjects including the one who used this strategy throughout the experiment. [11] Altogether, this strategy was used in 7.2% of all cases.

#### 4.2.7. *Trial policy*

Subjects in this group (four subjects, 8%) give the impression that they did not fully understand the rules of the game or that it was too much for them to deal with the problem reasonably at least at the beginning of each round. The standard deviations of the decisions in the first two periods confirm what immediately seems to be the case: These subjects' behavior varies much more as compared to all of the other subjects. [12] It seems as if they simply try out certain kinds of behavior and that they are still experimenting in the second cycle.

Other patterns observed include a strong reaction to the first excluded die by choosing above-average $x_2$ values or mixing up the probabilities of the dice. In the latter case subjects save more than twice as much for the uncertain periods in case the red die (highest termination probability) remains than in case the green die (lowest termination probability) remains. Another policy observed is that some subjects allocate their money during the first three periods in such a way that they have a certain (and different) amount left for each of the three different remaining termination probabilities.

The patterns described above suggest that subjects do not work backward through the decision tree in order to solve the problem but rather apply some forward-looking strategies and/or heuristics. Recall that with regard to the $\delta_{\mathrm{obs}}$-measure (see Section 4.1) the strategy ENP fits average observed data about three (four) times better than the optimal BI solution. [13] It is true that

[10] In the first period (s)he actually chose DM 1.99 when called upon to act.

[11] This kind of strategy can be clearly observed in sequences where the green die remained (29 cases). It was used in nine cases when the yellow die remained and in five cases when the red die remained.

[12] The mean of the standard deviations of the decisions in the first two periods for all subjects is 0.34, and excluding that group 0.24, whereas it is 1.49 for the group of these four subjects alone.

[13] Note that in a related study in which the three dice represent individual, but deterministic "life" expectations, Anderhub (1998) also finds evidence for ENP being used.

Table 4
Results of the regression

Estimating equation:
$x_t^{\mathrm{obs}} = \beta_0 + \beta_1 x_t^{\mathrm{ENP}} + \beta_2 x_t^{\mathrm{BI}} + \varepsilon; \quad t = 2, \ldots, 5$

| | $\beta_0$ | $\beta_1$ | $\beta_2$ | adj. $R^2$ |
|---|---|---|---|---|
| $t = 2$: | −0.182 | 0.538** | 0.140 | 0.317 |
| | (0.490) | (0.206) | (0.161) | |
| $t = 3$: | −0.417 | 0.743** | 0.091 | 0.556 |
| | (0.334) | (0.108) | (0.083) | |
| $t = 4$: | 0.874** | 0.657** | −0.234** | 0.639 |
| | (0.175) | (0.076) | (0.078) | |
| $t = 5$: | 0.261 | 0.567* | 0.266 | 0.846 |
| | (0.133) | (0.270) | (0.332) | |

S.D. in parentheses. Subject dummies were used.
* Significant at 5% level.
** Significant at 1% level.

we could not find strong evidence that subjects used ENP properly. There are several possible explanations for this. It might be that subjects were simply not willing to spend the effort necessary to compute the expected number of remaining periods exactly. They may have used rough estimates of these numbers that varied from round to round. However, lacking another serious alternative it is useful to examine how well the two competing strategies – BI and ENP – do against the data. For each subject the predictions provided by the two strategies for periods 2–5 were calculated using the observed decisions of earlier periods i.e., it was calculated what the strategies would have prescribed for a certain decision node given the actual wealth. For periods 2–5 the adjustment equation given in Table 4 was estimated for the pooled sample with all but two subjects. The decisions of the two subjects who adopted the "go-for-the-maximum" and the pure "2" strategy, resp. (as described above) were removed from the data set since these strategies do not react on information about the exclusion of dice. Furthermore, only decisions made in the second cycle, i.e. in rounds 7–12 were included. The independent variable in the adjustment equation given in Table 4, $x_t^{\mathrm{obs}}$, is the observed decision in round $t$ ($t = 2, \ldots, 5$). The two explanatory variables included are the decisions predicted by ENP, $x_t^{\mathrm{ENP}}$, and BI, $x_t^{\mathrm{BI}}$. The coefficients in Table 4 indicate how important the explanatory variables are for each period.

Note first of all that the explanatory variables become more and more relevant the higher the number of periods which is indicated by the adjusted $R^2$ measure. Furthermore, the coefficient $\beta_1$ – measuring the relative importance of the ENP strategy – is always positive and much higher than the coefficient $\beta_2$ suggesting that subjects use at least a variant of the forward-looking strategy ENP rather than the backward working strategy BI. The coefficient $\beta_1$ is especially high (and rather close to 1) in the third period. In later periods this coefficient decreases and in the fifth period the optimal solution becomes more important for explaining observed behavior.

This section's findings are summarized by formulating

**Result 1.** *The observed behavior suggests that most subjects do not work backward through the decision tree in order to solve the decision problem but apply some forward-looking strategies or heuristics. In particular, the regression results demonstrate that subjects' behavior is closer to the one implied by the ENP strategy rather than the BI strategy.*

## 5. Attitudes toward risk

### 5.1. Aggregated behavior

This section will address the question of whether risk-aversion can at least explain the direction of deviations from the optimal solution observed in subjects' average decisions. For example, Fig. 1 shows that when the red die remains the optimal solution prescribes the investment of all the money left in the fifth period when the residual fund falls below a certain level (see below for the derivation of this result). Should one expect a risk-averse subject to choose such an extreme action? After all this choice would result in a zero payoff if the subject experiences the sixth period. Thus one may expect that the optimal solution that allows for risk-aversion would never prescribe the investment of all remaining money until the last period is reached. [14]

In order to see which consequences risk aversion has, the optimal solution was recomputed [15] using the utility function $u(x) = x^{\alpha}$ which exhibits constant relative risk-aversion (or decreasing absolute risk-aversion). This can be

[14] There were 9 out of 600 cases where a subject experienced a period without having money left.

[15] I thank Martin Strobel for adopting the computer program.

seen if we note that $u'(x) = \alpha x^{\alpha-1}$ and $u''(x) = \alpha(\alpha-1)x^{\alpha-2}$ implying $r_R(x,u) := -xu''(x)/u'(x) = 1-\alpha$, i.e. the coefficient of relative risk-aversion at $x$, $r_R(x,u)$, equals the constant $1-\alpha$. Fig. 5 shows the result of the computation for $\alpha = 0.5$.

The line above or below each box in Fig. 5 again indicates whether the corresponding value lies above or below the optimal value for risk-neutral agents shown in Fig. 1. If one compares these deviations from optimality with the deviations of observed behavior from optimality (as presented in Fig. 3) it turns out that there is a complete correspondence; i.e., whenever risk aversion predicts less (more) spending than the risk neutral benchmark at a given decision node, a deviation at the same direction can be observed in the experimental data. As computer simulations have shown, this is true for *all* $\alpha \in [0,1)$. Thus we can state

**Result 2.** *Risk-aversion predicts the direction of deviations of average observed behavior from the optimal path, i.e. from the optimal solution that assumes risk-neutral behavior.*

It is interesting to note that the optimal solution that allows for risk-aversion (using the utility function over money $u(x) = x^{\alpha}$) converges to the

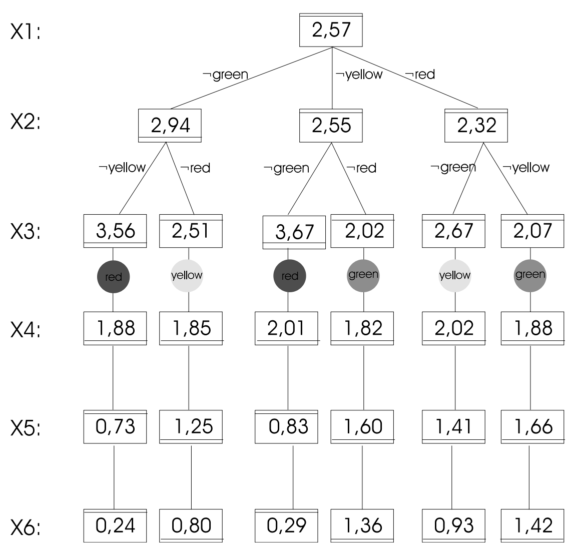


Fig. 5. Optimal solution for $\alpha = 0.5$.

Table 5
Ranked sequences

| Sequence | First chance move | Second chance move | $\overline{\mu_k}$ (obs.) | $\mu_k(\alpha=0)\cdots\mu_k(\alpha=1)$ |
|---|---|---|---|---|
| 1 | ¬green | ¬yellow | 0.73 | $0.75\cdots0.80$ |
| 2 | ¬yellow | ¬green | 0.72 | $0.74\cdots0.76$ |
| 3 | ¬green | ¬red | 0.69 | $0.69\cdots0.66$ |
| 4 | ¬red | ¬green | 0.65 | $0.67\cdots0.59$ |
| 5 | ¬yellow | ¬red | 0.63 | $0.62\cdots0.58$ |
| 6 | ¬red | ¬yellow | 0.60 | $0.61\cdots0.56$ |

ENP solution shown in Fig. 2 as $\alpha$ goes to 0 and it converges to the strategy that was characterized as "go-for-the-maximum"-policy as $\alpha$ goes to $\infty$. [16]

With the help of Result 2 an observation mentioned in Anderhub et al. (2000) can be explained, namely that with regard to the optimal solution there is underconsumption during the first three certain periods in case the red die remains and overconsumption otherwise.

To be more precise, recall that there are six possible sequences of initial chance moves ((¬green, ¬yellow) or (¬green, ¬red) or ...). Now, let $\mu = (x_1 + x_2 + x_3)/11.92$ denote the relative amount consumed during the three certain periods $t = 1, 2, 3$. Using the utility function over money $u(x) = x^{\alpha}$, $\alpha \in [0, 1]$, and recomputing the optimal solution one can construct the functions $\mu_k(\alpha)$ that assign to each $\alpha$ the corresponding values of $\mu_k$. Here $k = 1, \ldots, 6$ indicates the sequence of initial chance moves. Table 5 shows the means $\overline{\mu_k}$ $(k = 1, \ldots, 6)$ of the observed values together with the range of functions $\mu_k(\alpha)$. (Note that the functions $\mu_k(\alpha)$ vary monotonically from $\mu_k(0)$ to $\mu_k(1)$. This is what the dots in Table 5 indicate. The values for $\alpha = 1$ correspond to risk-neutral behavior.)

The two results that immediately follow are

**Result 3.** *The observed means $\overline{\mu_k}$ are ranked in the same order as the values $\mu_k(\alpha)$ (for any fixed value of $\alpha \in [0, 1]$).* [17]

and

[16] Actually, the latter is already the case for $\alpha = 5.86$ without any changes for greater values of $\alpha$.
[17] This result is already mentioned in Anderhub et al. (2000).

**Result 4.** *Risk-aversion predicts underconsumption* (*when compared to the optimal solution, i.e. for* $\alpha = 1$) *during the first three certain periods when the red die remains and otherwise it predicts overconsumption.*

### *5.2. Behavior in the fifth period*

The last subsection focussed on the relevance of risk aversion for explaining deviations of average observed behavior from the optimal solution. In this section attention is paid to individual decisions. We will, however, restrict ourselves to the decision problem in the fifth period. The reasons for this are simple: First, the decision in the fifth period is – from a theoretical point of view – the most simple one to make in the experiment. Thus one might hope that the backward induction solution for the fifth period yields reliable predictions. Second, allowing for risk aversion makes the derivation of analytical results even more complicated. For the most simple case, i.e., the decision in the fifth period, it is, however, possible to derive at least some comparative static results.

Consider an individual having utility function over money $u = u(x)$ with $u'(x) > 0$ for all $x \geqslant 0$. Given $C := x_1 x_2 x_3 x_4 > 0$, termination probability $w \in (0, 1)$ and

$$S_5 = \left(11.92 - \sum_{i=1}^{4} x_i\right) > 0$$

(wealth at the beginning of period 5), the decision problem in the fifth period is to maximize

$$U(x_5) = wu(C \cdot x_5) + (1 - w)u((C \cdot x_5)(S_5 - x_5)).$$

If $x_5^*$ is optimal it must satisfy the necessary condition

$$wu'(C \cdot x_5) + (1 - w)(S_5 - 2x_5)u'((C \cdot x_5)(S_5 - x_5)) = 0 \tag{5.1}$$

or equivalently

$$S_5 - 2x_5 = -\frac{w}{(1 - w)} \frac{u'(C \cdot x_5)}{u'((C \cdot x_5)(S_5 - x_5))}. \tag{5.2}$$

Since, according to our assumptions, $u'(x) > 0$ for all $x \geqslant 0$ it follows from (5.2) that $S_5 - 2x_5^* \leqslant 0$ or $x_5^* \geqslant S_5/2$ must hold, i.e. choices with $x_5^* < S_5/2$ are not compatible with utility maximization of any kind. This observation together with the budget constraint leads to

**Remark 1.** For any kind of utility maximizing behavior in the fifth period it must hold that $S_5/2 \leqslant x_5^*(S_5) \leqslant S_5$.

Note that the condition given in Remark 1 must be satisfied no matter whether (and to which degree) an individual is risk-averse or risk-loving. In our experiment we observe in 24 out of 273 cases that the condition of Remark 1 is violated.

In order to make sharper predictions about the behavior in the fifth period that is compatible with utility maximization one has to further specify the utility function $u = u(x)$. A class of utility functions that is amenable to some comparative static analysis is the class of utility functions that exhibit constant relative risk-aversion given by

$$u(x) = \begin{cases} (1-\alpha)x^{1-\alpha} & \text{for } \alpha \neq 1, \\ \ln x & \text{for } \alpha = 1. \end{cases}$$

According to this utility function an agent is risk-averse, risk-neutral, risk-loving, respectively, for $\alpha > 0$, $\alpha = 0$, $\alpha < 0$, respectively. From now on let us assume $\alpha \geqslant 0$ such that the necessary condition (5.1) that now reads

$$w + (1-w)(S_5 - x_5)^{-\alpha}(S_5 - 2x_5) = 0, \quad \alpha \neq 1, \tag{5.3}$$

is also sufficient.

From (5.3) it follows that for a risk-neutral individual, i.e. for $\alpha = 0$ the optimal policy in the fifth period is given by

$$x_5^*(S_5, w) = \min\left\{\frac{1}{2}\left(S_5 + \frac{w}{(1-w)}\right),\ S_5\right\}. \tag{5.4}$$

For a slight degree of risk-averse or risk-loving behavior, respectively, i.e. for $\alpha$ in a small neighborhood of 0, Eq. (5.3) is not explicitly solvable for $x_5$. [18] One can, however, derive a comparative static result.

**Remark 2.** For the optimal policy in the fifth period it holds that

$$\left.\frac{\partial x_5^*}{\partial \alpha}\right|_{(S_5,w,0)} \begin{matrix} > \\ = \\ < \end{matrix} \begin{cases} 0 & \text{for } \frac{w}{1-w} < S_5 < \frac{2-w}{1-w} \\ 0 & \text{for } S_5 = \frac{2-w}{1-w} \\ 0 & \text{for } S_5 > \frac{2-w}{1-w}. \end{cases}$$

[18] For $\alpha = 1$ the optimal policy is given by $x_5^*(w, S_5) = S_5/(2-w)$.

For a proof see Appendix A. According to Remark 2, a slightly risk-averse (risk-loving) individual who still "lives" in the fifth period will tend to spend more (less) than a risk-neutral individual when $w/(1-w) < S_5 < (2-w)/(1-w)$ and will tend to spend less (more) than a risk-neutral individual when $S_5 > (2-w)/(1-w)$. The intuition behind this result is clearly that saving might also be risky since the money is lost should the round end in the fifth period.

In order to relate the observed decisions made in the fifth period to the theoretical results derived above, note that for $x_5 \neq S_5, S_5/2, S_5 - 1$ it follows from Eq. (5.3) that

$$\alpha = -\frac{\ln\left(w/((S_5 - 2x_5)(-1 + w))\right)}{\ln\left(S_5 - x_5\right)}. \tag{5.5}$$

With the help of Eq. (5.5) one can check which observed $(x_5, S_5)$-combinations can be rationalized by risk-aversion. This is shown in Figs. 6–8 in which we can see the regions that correspond to risk-averse choices (grey shaded areas) together with all observed $(x_5, S_5)$-combinations. [19] (In these figures the two outer dotted lines indicate $x_5 = S_5$ and $x_5 = S_5/2$, the inner dotted line indicates $x_5 = S_5 - 1$ and the bold line indicates the optimal policy of a risk neutral agent as given in Eq. (5.4).)

When determining the number of $(x_5, S_5)$-combinations that are compatible with utility maximization by risk-averse subjects using Eq. (5.5), one has to exclude certain $(x_5, S_5)$-combinations. This is due to singularities in Eq. (5.5) that arise if $x_5 = S_5$, $x_5 = S_5/2$ or $x_5 = S_5 - 1$. Altogether there are 273 cases in which subjects had to decide in the fifth period. But from Remark 1 it follows that values $x_5$ with $x_5 < S_5/2$ cannot be rationalized by any utility function. As mentioned above there are 24 such cases. Moreover, there are 12 cases with $x_5 = S_5$ (including five cases in which it was optimal to set $x_5 = S_5$), 27 cases with $x_5 = S_5/2$, 23 cases with $x_5 + 1 = S_5$ and 15 cases where the two latter conditions apply at the same time, namely, cases with $S_5 = 2$ and $x_5 = 1$ (see Table 6).

From the remaining 172 $(x_5, S_5)$-combinations 89 cases are compatible with utility maximization plus risk-aversion and 83 cases are not. This is also shown in Table 6. Note that in the case of the red die being applied almost all

[19] It can be shown that the results concerning the areas of risk aversion as shown in Figs. 6–8 are also true for the utility function $u(x) = -\exp(-\alpha x)$, $\alpha > 0$, that depicts constant absolute risk-aversion (see Müller, 1999, footnote 18).

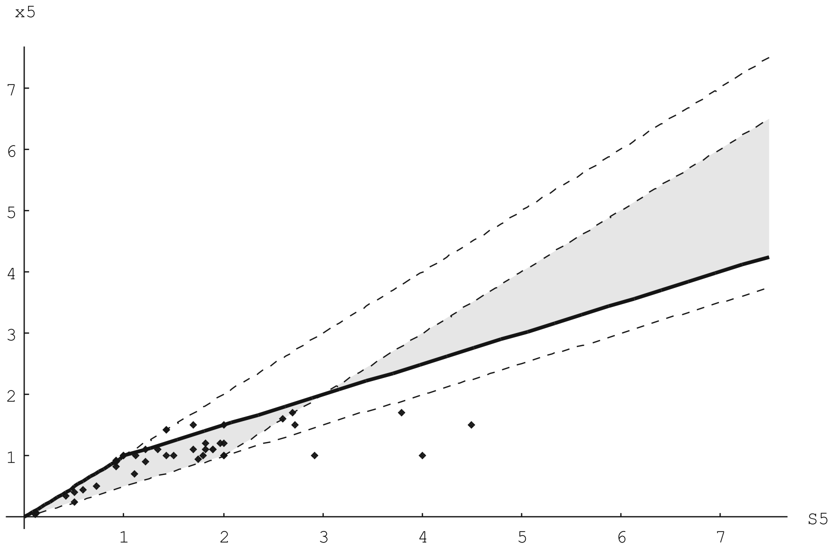


Fig. 6. Observed $(x_5, S_5)$-decisions in case of $w = 1/2$ (red die).

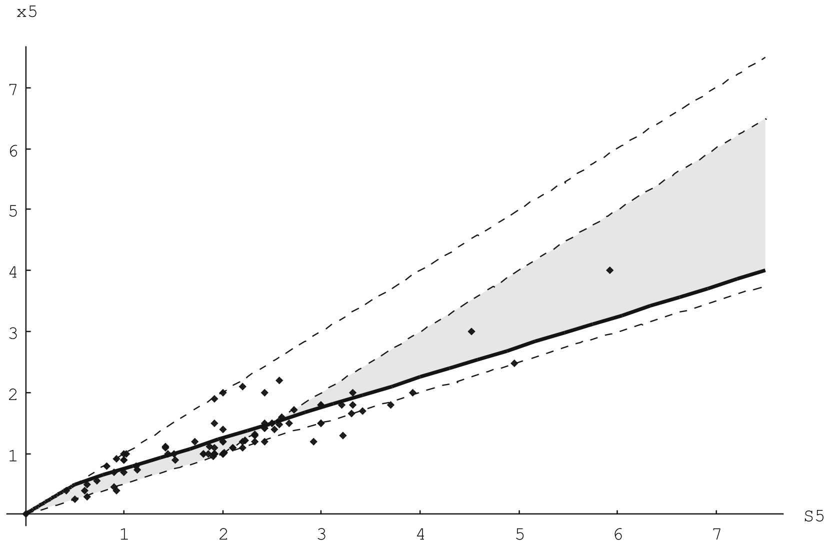


Fig. 7. Observed $(x_5, S_5)$-decisions in case of $w = 1/3$ (yellow die).

cases (from those listed in the last two rows in Table 6) can be rationalized by risk-aversion. This is not true in the case of the green die. In this case only 38% (35 out of 92) of the choices can be rationalized by risk-aversion. This

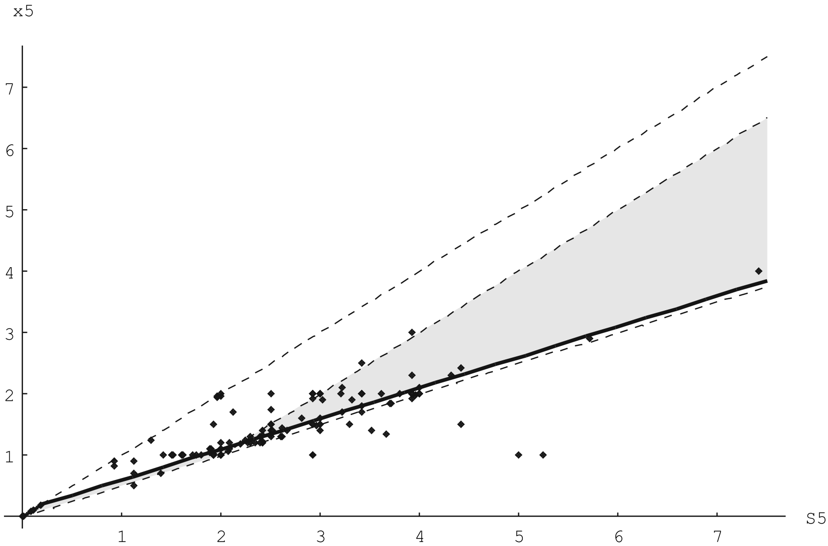


Fig. 8. Observed $(x_5, S_5)$-decisions in case of $w = 1/6$ (green die).

Table 6
Observed decisions in the fifth period

| | Red ($w = 1/2$) | Yellow ($w = 1/3$) | Green ($w = 1/6$) | $\sum$ |
|---|---|---|---|---|
| $\#(x_5 = S_5)$ | 5 | 4 | 3 | 12 |
| $\#(x_5 = S_5/2$ and $x_5 \neq S_5 - 1)$ | 3 | 8 | 16 | 27 |
| $\#(x_5 = S_5 - 1$ and $x_5 \neq S_5/2)$ | 2 | 12 | 9 | 23 |
| $\#(x_5 = S_5/2$ and $x_5 = S_5 - 1 \Rightarrow x_5 = 1,\ S_5 = 2)$ | 2 | 4 | 9 | 15 |
| #(not rationalizable) | 4 | 7 | 13 | 24 |
| #(decisions rationalizable by risk-aversion) | 24 | 30 | 35 | 89 |
| #(other cases) | 2 | 24 | 57 | 83 |

implies that subjects either did not have the assumed preferences or that their attitudes toward risk vary with the termination probability which is puzzling from a normative point of view. This is summarized by

**Result 5.** *If the subjects' preferences can be described by the class of utility functions that exhibit constant relative or constant absolute risk-aversion, then subjects do not act consistently across situations in the sense that they do not show the same attitude toward risk across situations represented by different termination probabilities.*

How can this result be explained? Consider the decision problem to be solved in the fifth period. The choices in periods 1 to 4 may have led to the provisional payoff $C = x_1x_2x_3x_4$. By the choice of $x_5$ a subject creates his/her own lottery in which (s)he is willing to participate. For given $w \in (0, 1)$ and $S_5 > 0$ the choice of $x_5 \leqslant S_5$ leads to the following lottery: with probability $p_1 = w$ the prize is $L_1 := x_1x_2x_3x_4x_5$ and with probability $p_2 = 1 - w$ the prize is $L_2 := x_1x_2x_3x_4x_5(S_5 - x_5)$. For example, as mentioned above, in 38 out of 273 cases (14%) subjects chose $x_5 = S_5 - 1$ and thus made sure they did not lose any of the money accumulated during periods 1–5 ($L_1 = L_2$). Now note that in the case of the red and green die the amount of money left for the fifth period is on average 1.62 and 2.77 DM, respectively (see Fig. 3). Thus in the case of the green dice a subject can choose a lottery such that $L_2 > L_1 > C$, i.e., in this case a subject can successively (strictly) increase the payoff in every period.

This is not possible with the red die since on average $S_5 < 2$ in this case. Here a subject realizes that (s)he can only participate in lotteries with either $L_1 \geqslant C$ and $L_2 < L_1$ or $L_1 < C$ and $L_2 \geqslant L_1$, i.e., it is possible that the payoff shrinks from one period to another. Thus with the green die the experimental situation might usually be perceived as an opportunity to earn successively more money which would make subjects more risk-loving. However, with the red die the possibility of losing some of the accumulated wealth leads subjects to be more risk-averse. Thus a possible explanation for the Result 5 is that the experimental situation in period 5 is perceived differently depending on the termination probability (or depending on the order of die exclusion and the remaining wealth). This triggers different attitudes toward risk not only across but also within subjects.

## 6. Summary and discussion

The main motivation for this study was the wish to uncover how people tackle dynamic decision problems and to understand what role risk aversion plays in these situations. For this purpose an experimental situation was designed comprised of consumption and savings decisions. The combination of an uncertain time horizon with the successive revealing of the termination probability is the specific characteristic of this experimental situation. Anderhub et al. (2000) who also investigate this setup find that on average subjects update their termination probabilities in a qualitatively correct way, i.e. qualitatively in the same way as the optimal solution. Despite these regularities found in average decisions, behavior on the individual level is quite dispersed. However, by closely inspecting individual decisions one is able to make out a number of behavioral patterns that range from consistent and thought-out to very inconsistent and irrational behavior and from very risky to cautious behavior. Other findings concerning behavioral rules found in the data are that some individuals systematically neglect information concerning the exclusion of dice and thereby reduce the complexity of the decision tree. Or that subjects start out investing only a small amount of their wealth at the beginning of a round and reacting strongly to the information regarding the remaining die. Subjects applying the latter behavioral policy seem to wait until the uncertainty about the termination probability has been removed. We find that most subjects perform reasonably well by applying several interacting heuristics. However, these cannot be easily separated. The interacting heuristics and reasoning processes by which most of the decisions are generated can be best approximated by the strategy that sets current consumption equal to current wealth divided by the expected number of periods. This insight is suggested by a distance measure that relates strategies and heuristics to average observed behavior and by results of regressions that are based on individual decisions. We find, furthermore, that risk-aversion completely predicts the direction of deviations of average observed behavior from the optimal path. Moreover, risk-aversion correctly predicts under-consumption (when compared to the solution assuming risk-neutrality) when the actual termination probability is high and overconsumption when the actual termination probability is on a medium or a low level. The inspection of the decision in the fifth period suggests that subjects do not act consistently across situations in the sense that they do not show the same attitude toward risk across situations represented by different termination probabilities.

What general insights do the results of this study provide and what do they mean for economic theories of dynamic decision making? Admittedly, the experimental situation places a subject into a rather artificial environment. Although consumption/savings decisions have to be made repeatedly, the experimental setup is only weakly related to, e.g., the life-time savings problem. [20] But the experiment was not designed to test such real world problems. Rather, it was designed to investigate how subjects cope with the successive revealing of termination probabilities and how they, in general,

[20] Note, for example, that in the experiment the rate of interest equals zero and that there is no uncertainty concerning income.

tackle a problem whose solution requires the application of backward induction. Thus, besides the conclusion that people will react qualitatively correct to changes in their life expectancy, [21] the scope for drawing conclusions from the experimental results to, for example, real world savings problems is of course very limited. However, this study is part of the growing literature (see e.g. the studies cited in the introduction) dedicated to the study of how people actually solve dynamic decision problems. The picture that emerges from these research efforts is that people do not (properly) apply backward induction. [22] Rather, they apply heuristics that proceed forward or simplify the problem by ignoring, for example, information (see Section 4.2) or the fact that they will also subsequently take decisions (see Carbone & Hey, 1998). Of course we are not able yet to develop a behavioral alternative to backward induction. More research detecting empirical regularities is needed before this long-term objective can be tackled. But a step toward this goal, that is worthwhile taking, is to explore the performance and the predictions of alternative strategies and heuristics found so far in economically relevant applications. For example, in the problem at hand, the ENP strategy was seen to be close to the optimal solution in terms of efficiency (see Table 1). Research along this line includes Johnson & Payne (1985) and Pemberton (1993). The former study examines effort and accuracy of several strategies in a production system framework using simulations. And the latter study deals with the stochastic life cycle problem and studies the analytical consequences of the simplifying assumption "that consumers allocate resources between the present and 'the future' but – in contrast to the orthodox model – do not attempt to formulate a detailed *plan* for 'the future' " (p. 19). I think we need more research like this.

## Acknowledgements

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[21] For instance, after consulting with a doctor.

[22] Note, however, that one could read the results presented in the second part of this study as if average observed behavior stems from a population of risk averse decision makers who do apply backward induction. This, however, appears to be driven by the fact that the optimal solution converges to the forward-looking ENP solution as the level of risk aversion increases.

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## Appendix A. Proof of Remark 2

Write Eq. (5.3) as

$$F(S_5, w, \alpha, x_5) = 0, \tag{6.1}$$

where

$$F(S_5, w, \alpha, x_5) = w + (1 - w)(S_5 - x_5)^{-\alpha}(S_5 - 2x_5), \quad \alpha \neq 1,$$

and assume that $x_5 < S_5$ and $S_5 > w/(1 - w)$. Note that the partial derivatives of the function $F = F(S_5, w, \alpha, x_5)$ are continuous with respect to all variables. Consider now the case where $\alpha = 0$. We then know that Eq. (6.1) is satisfied by points of the form $(S_5, w, 0, x_5^*(S_5, w, 0))$. Furthermore, the partial derivative of the function $F$ with respect to $x_5$ evaluated at this point equals $2(w - 1) < 0$. Thus, according to the implicit-function theorem it is justified to write $x_5 = x_5(S_5, w, \alpha)$ emphasizing that in a neighborhood of the point $(S_5, w, 0)$ the optimal policy $x_5^*$ is an implicit function of the variables $S_5, w$ and $\alpha$. Moreover, one has

$$\begin{aligned} \left.\frac{\partial x_5^*}{\partial \alpha}\right|_{(S_5,w,0)} &= -\left.\frac{\frac{\partial F(\alpha,w,S_5,x_5)}{\partial \alpha}}{\frac{\partial F(\alpha,w,S_5,x_5)}{\partial x_5}}\right|_{(S_5,w,0,x_5^*(S_5,w,0))} \\ &= -\frac{1}{2(-1+w)}\left(\ln\frac{1}{2}\,\frac{-S_5 + wS_5 + w}{-1+w}\right)w. \end{aligned}$$

Inspection of the last expression yields the conclusions stated in Remark 2.

## Appendix B. Translated instructions

Your task in every round is to distribute an amount of money as good as possible to several periods. The better you do this, the higher is your payoff. Altogether you play 12 rounds. In the beginning of the experiment you can choose, whether we should draw lots to select one round for which you are paid. Otherwise you will receive the mean of your payoffs of all rounds. In any case you get your payoff in cash after evaluation of the data.

The general task of one round is to distribute a certain amount of money to several periods. Your payoff of one round is calculated by the product of the amounts allocated to the single periods. The difficulty is that there is no certainty about the number of periods you have to distribute your money. The game can last for three, four, five, or six periods. Every round will last at least for three periods. Whether you reach the fourth, fifth or sixth period, will be determined by throwing a die. There are altogether three different dice with the colors red, yellow and green. The following table shows, in which cases you reach a next period.

| Color of die | No further period if die shows | New period if die shows |
|---|---|---|
| Red | 1, 2, 3 | 4, 5, 6 |
| Yellow | 1, 2 | 3, 4, 5, 6 |
| Green | 1 | 2, 3, 4, 5, 6 |

The number of periods of one round cannot be higher than six. In the beginning of a round you do not know which die is used for you. You get this information after you have made some decisions. The general course of the game is as follows:

*1st period.* You will get a total amount of money $S$, which you can spend in the coming periods. Altogether you can only spend this total amount. You can choose an amount $x_1$, which you want to spend in the first period. Think very carefully, how much you want to spend and how much you want to save for the following periods. After your decision one of the three dice is excluded. Now you know that only the two other dices are candidates for the chance move if you reach the fourth, fifth and sixth period.

*2nd period.* You are choosing an amount $x_2$, which you want to spend in the second period. You cannot spend more than you have left from the total amount after the first period. After your decision another die is excluded. Now you know, which die remains to be thrown for the fourth, fifth and sixth period.

*3rd period.* You are choosing an amount $x_3$, which you want to spend in the third period. After this decision the computer will throw the remaining dice in order to decide whether you reach the fourth period. If you do not reach the fourth period, the round ends here. The amount which is not spent until now is lost.

*4th period.* If you have reached the fourth period, you choose an amount $x_4$. For reaching the fifth period, the die will be thrown again.

*5th period*. If you have reached the fifth period, you choose an amount $x_5$. For reaching the sixth period, the die will be thrown again.

*6th period*. If you have reached the sixth period, you do not have to make a decision, because all remaining money is spent automatically.

Your payoff is calculated by the product of all amounts you spent in the periods you reached. For instance if you experienced exactly four periods, your payoff is determined by $G = x_1 \times x_2 \times x_3 \times x_4$. When you have reached for instance all six periods, your payoff is determined by $G = x_1 \times x_2 \times x_3 \times x_4 \times x_5 \times x_6$, where $x_6$ is the amount you have left after the fifth period. Please think about the following: if you spend in one period an amount of 0, your payoff will be also 0, because one of the factors is 0. This can happen, for instance, if you spend all money in the fourth period and reach the fifth period. Then you have to spend 0 in the fifth and perhaps also in the sixth period and therefore you get the payoff 0. You have to weigh up between the risk of spending all your money early or making your money useless if the game ends.

## References


Anderhub, V. (1998). *Savings decisions when uncertainty is reduced – an experimental study*. Discussion Paper 73, SFB 373, Humboldt University Berlin.

Anderhub, V., Güth, W., Müller, W., & Strobel, M. (2000). An experimental analysis of intertemporal allocation behavior. *Experimental Economics*, *3* (2), 137–152.

Camerer, C. (1995). Individual decision making. In J. H. Kagel, & A. E. Roth (Eds.), *Handbook of experimental economics*. Princeton, NJ: Princeton University Press.

Carbone, E. & Hey, J. D. (1998). *A test of the principle of optimality*. Working paper 9/99, University of York.

Fehr, E., & Zych, P. (1995). Die Macht der Versuchung: Irrationaler Überkonsum in einem Suchtexperiment. *Zeitschrift für Wirtschafts- und Sozialwissenschaften*, *114* (Heft 4), 569–604.

Friedman, M. (1953). *Essays in positive economics*. Chicago: University Press.

Hey, J. D. (1983). Whither uncertainty? *Economic Journal*, *93* (conference supplement), 129–138.

Hey, J. D., & Dardanoni, V. (1988). Optimal consumption under uncertainty: An experimental investigation. *Economic Journal*, *98*, 105–116.

Huck, S., & Weizsäcker, G. (1999). Risk, complexity, and deviations from expected-value maximization: results of a lottery choice experiment. *Journal of Economic Psychology*, *20*, 699–715.

Johnson, E. J., & Payne, J. W. (1985). Effort and accuracy in choice. *Management Science*, *31* (4), 395–414.

Johnson, S., Kotlikoff, L., & Samuelson, W. (1987). *Can people compute? An experimental test of the life cycle consumption model*. Working Paper, Harvard University, Cambridge, MA.

Koehler, J. (1996). *Making saving easy: An experimental analysis of savings decisions*. Preliminary version, Department of Economics and Related Studies, University of York.

Müller, W. (1999). *Strategies, heuristics and the relevance of risk aversion in a dynamic decision problem*. Discussion Paper 61, SFB 373, Humboldt-University, Berlin.

Pemberton, J. (1993). Attainable non-optimality or unattainable optimality: A new approach to stochastic life cycle problems. *Economic Journal*, *103*, 1–20.

Roth, A. E. (1996). Comments on Tversky's rational theory and constructive choice. In K. Arrow, E. Colombatto, M. Perlman, & C. Schmidt (Eds.), *The rational foundations of economic behavior* (pp. 198–202). New York: Macmillan.