Conjectural Variations and Evolutionary Stability: A Rationale for Consistency

by

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Adopting an evolutionary approach, we explain the conjectural variations firms may hold in duopoly. Given conjectures, firms play the market game rationally. Success in the market game determines fitness in the evolutionary game. Based on linear heterogeneous Cournot and Bertrand competition models, we show that the unique conjectures that are evolutionarily stable are consistent in that they anticipate the rival's behavior correctly. (JEL: D 43)

1 Introduction

The predictions of oligopoly theory depend crucially on behavioral assumptions on how a firm conjectures other firms will react to its own actions. Cournot made the assumption that firms maximize their profits taking as given the quantity of the rival firms, that is, rivals do not react at all to changes of a firm's own action. Later contributions by Bowley [1924], Stackelberg [1934], Hicks [1935], and Leontieff [1936] varied this assumption and proposed alternative solutions, initiating the conjectural-variations literature.

Interest in conjectural variations grew with the analysis of the consistency criteria. In addition to the individual rationality assumption underlying the notion of Nash equilibrium, consistency requires that conjectures about rivals' behavior be correct. In Bresnahan [1981], the consistency of conjectures occurs whenever the slopes of firms' reaction functions are (locally) equal to the conjectural variations.[1] Applying this definition, Bresnahan [1981] shows that a unique solution exists for duopoly with linear–quadratic costs.

The literature following Bresnahan [1981] pointed out two fundamental problems with the conjectural-variations approach and the consistency criteria in particular: "The heart of the problem is the notion of a conjectural variation. This notion is

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[1] This definition can actually be traced back to Leontieff [1936]. See also Martin [2002].

ad hoc inasmuch as none of the models using a conjectural variation explains how it is formed or whence it came" (DAUGHETY [1985, p. 369]). The second problem is closely related to the first. Conjectures have been found very difficult to rationalize (MAKOWSKI [1987]). Theorists may find consistent conjectures appealing because of the parallel to rational-expectations theory. However, attempts to derive conjectures merely from rationality assumptions have not been successful. Conjectures are essentially "a-rational" (MAKOWSKI [1987]).

Recently, some authors addressed these problems by proposing explicitly dynamic models, usually repeated Cournot settings (DOCKNER [1992], SABOURIAN [1992], CABRAL [1995]). These authors examine conditions under which the outcome of the repeated game equals the outcome of the static conjectural-variations model. For example, CABRAL [1995] proposes an infinitely repeated game with minimax punishments. He shows that, for each discount factor and for any linear oligopoly structure, there is a conjectural variation such that a firm's output in the optimal equilibrium is equal to the quantity of the conjectural-variations solution. In this way, the conjectural-variations models are justified as a "short cut" (SABOURIAN [1992, p. 236]), mimicking the outcome of more complex dynamic games. However, note that only the conjectural-variations outcome is justified; nothing is said about the origin and nature of the conjectures themselves.

In this paper, we propose an evolutionary approach to explain conjectures. We do not impose any rationality or consistency criterion on the conjectures firms may hold. However, given the conjectures, firms play the market game rationally. The link between market performance and conjectures is that profits in the duopoly game determine the success in an evolutionary game. So, what our model does is to impose evolutionary selection of conjectures and rational choice of actions in the basic market game. As a result, we show that the conjectures surviving the evolutionary process are the consistent conjectures proposed by BRESNAHAN [1981]. That is, we do not only justify the market outcome implied by consistent conjectures; we also justify the conjectures themselves.

The evolutionary process we apply has been applied successfully to explain various economic phenomena. The concept was proposed by GÜTH AND YAARI [1992], who called it the "indirect evolutionary approach." As in our paper, the idea is that subjects act rationally in their market transactions, but factors influencing the market game, such as preferences or beliefs, are formed in an evolutionary process. This approach has been used to explain, *e.g.*, monopolistic competition (GÜTH AND HUCK [1997]), altruism (BESTER AND GÜTH [1998]), and behavior in the ultimatum game (HUCK AND OECHSSLER [1999]). KÖNIGSTEIN AND MÜLLER [2000] propose a formal framework for the indirect evolutionary approach. GEHRIG, GÜTH, AND LEVINSKI [2004] analyze the evolution of beliefs about demand expectations.

Independently of this research, DIXON AND SOMMA [2003] have obtained similar results in linear homogeneous-goods Cournot markets. There are several differences

between their study and the current study. First, we analyze both quantity and price[2] competition. Second, while they consider a homogeneous-good market, we consider a heterogeneous-good market, which includes a market for perfect substitutes as a special case. Third, whereas DIXON AND SOMMA [2003] find that the consistent conjecture is not evolutionarily stable in the case of constant marginal costs, we show in this case that the consistent conjectures are evolutionarily stable – as long as goods are not perfect substitutes. Fourth, while they adopt a dynamic approach in their main analysis, we use the straightforward static concept of an evolutionarily stable strategy (ESS).

We proceed as follows. In section 2, we first define the market and then derive the consistent-conjectures equilibrium. In the second part of the section, we determine the evolutionarily stable conjectures. In section 3 we discuss our findings.

2 Assumptions

We consider two firms $i = 1, 2$ in a heterogeneous-goods market for both price and quantity competition. In the Cournot setup, the strategy sets are $S_i = \{q_i \mid q_i \geq 0\}$, $i = 1, 2$, and the inverse demand functions are given by

$$(1) \qquad p_i(q_i, q_j) = a - q_i - \theta q_j, \qquad i, j = 1, 2, \quad i \neq j,$$

with $0 \leq \theta \leq 1$. To analyze price competition, we need to invert the inverse demand functions

$$(2) \qquad q_i(p_i, p_j) = \frac{a(1 - \theta) - p_i + \theta p_j}{1 - \theta^2}, \qquad i, j = 1, 2, \quad i \neq j,$$

and impose $\theta < 1$ strictly. The cost functions are

$$(3) \qquad C(q_i) = c \, (q_i)^2 / 2, \qquad i = 1, 2,$$

with $c \geq 0$. The case of constant marginal cost is obtained by setting $c = 0$. Firm i's profit is given by

$$(4) \qquad \pi_i(q_i, q_j) = \left(a - q_i - \theta q_j\right) q_i - \frac{c}{2} \, (q_i)^2$$

in the Cournot case, and

$$(5) \qquad \pi_i(p_i, p_j) = p_i \left(\frac{a(1 - \theta) - p_i + \theta p_j}{1 - \theta^2} \right) - \frac{c}{2} \left(\frac{a(1 - \theta) - p_i + \theta p_j}{1 - \theta^2} \right)^2$$

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[2] Somewhat surprisingly, with the exception of PFAFFERMAYR [1999], conjectural variations have not been analyzed for price competition. PFAFFERMAYR [1999] shows that the static conjectural-variations model may represent the joint-profit-maximizing collusive Nash equilibrium of a price-setting supergame with differentiated products. He also derives conditions under which the price-setting and the quantity-setting conjectural-variations model yield the same outcome. (On this topic, see also KAMIEN AND SCHWARTZ [1983].)

in the Bertrand case. Our assumptions on demand and cost in the Cournot case are the same as Assumptions 1 and 2 in BRESNAHAN [1981], except that we assume that firms are symmetric.[3]

3 Quantity Competition

3.1 Consistent-Conjectures Equilibrium

We start by reiterating BRESNAHAN's [1981] definition of a consistent-conjectures equilibrium (CCE) with Cournot competition. Let $\rho_i = \rho_i(q_j)$, $i \neq j$, denote firm i's reaction function. From our assumptions, we know that a unique and linear CCE exists (BRESNAHAN [1981, Theorem 1]). We therefore restrict the attention to linear conjectures such that $r_i \in [-1, 1]$, $i = 1, 2$, denotes firm i's conjectures about firm j's reaction to q_i. The restriction $r_i \in [-1, 1]$ is imposed to guarantee equilibrium quantities to be nonnegative. As is well known, the outcomes in a symmetric Cournot duopoly range from perfect competition to joint monopoly when the conjectural variation increases from -1 to 1.

DEFINITION 1 *A consistent-conjectures equilibrium is a pair of quantities,* (q_1^*, q_2^*), *and a pair of conjectures,* (r_1^*, r_2^*), *such that*

$$(6) \qquad q_1^* = \rho_1(q_2^*), \qquad q_2^* = \rho_2(q_1^*),$$

and

$$(7) \qquad r_1^* = \frac{\partial \rho_2(q_1)}{\partial q_1}, \qquad r_2^* = \frac{\partial \rho_1(q_2)}{\partial q_2}.$$

That is, firms' quantities have to be a Nash equilibrium (conditions (6)), and a firm's conjecture about the other firm's behavior has to be equal to the slope of the other firm's reaction function (conditions (7)).

We now compute a closed-form solution of the CCE for the market defined above. From the first-order conditions of profit maximization

$$(8) \qquad \frac{\partial \pi_i(q_i, q_j)}{\partial q_i} = a - \theta q_j - q_i(2 + \theta r_i + c) = 0$$

we derive firm i's reaction function

$$(9) \qquad \rho_i(q_j) = \frac{a - \theta q_j}{2 + \theta r_i + c}.$$

The slope of firm i's reaction function is

$$(10) \qquad \frac{\partial \rho_i(q_j)}{\partial q_j} = -\frac{\theta}{2 + \theta r_i + c}.$$

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[3] Bresnahan only analyzes quantity-setting firms. With asymmetric demand and cost functions, the evolutionary analysis below is extremely messy and cumbersome. BRESNAHAN [1981] shows that the model may also allow for fixed costs, which, from his Assumption 3, should not be too large.

Thus, the consistent conjectures are the solution of the following system of two simultaneous equations:

$$(11) \qquad r_i = -\frac{\theta}{2 + \theta r_j + c}, \qquad i, j = 1, 2, \quad i \neq j,$$

whose two candidate solutions are given by

$$(12) \qquad r := r_i = r_j = \frac{-2 - c \pm A}{2\theta},$$

where

$$(13) \qquad A := \sqrt{(2 + c)^2 - 4\theta^2} > 0.$$

The equilibrium quantities q_i^* are the solution of the system of two simultaneous equations (8). This solution is

$$(14) \qquad q_i^* = \frac{a}{c + \theta r + 2 + \theta}.$$

Note that, using the fact that $r_1 = r_2 = r$, the second-order condition for profit maximization is given by

$$(15) \qquad \frac{\partial^2 \pi_i(q_i, q_j)}{\partial q_i^2} = -2(1 + \theta r) - c < 0.$$

The equilibrium quantities q_i^*, evaluated at $r = (-2 - c \pm A)/2\theta$, equal

$$(16) \qquad q_i^* = \frac{2a}{c + 2 + 2\theta \pm A},$$

which are both strictly positive. However, the second-order condition (15) reads $-2(1 + \theta r) - c = \pm A(-1)$. That is, the second-order condition is negative for the positive root and positive for the negative root, so the positive root yields the maximum.

To summarize the CCE, the unique conjecture is

$$(17) \qquad r^* = \frac{-2 - c + A}{2\theta}.$$

(Note that $r^* < 0$, as expected from the definition of consistency and the fact that we have strategic substitutes.) The equilibrium outputs are

$$(18) \qquad q_i^* = \frac{2a}{2(1 + \theta) + c + A},$$

and the profits are

$$(19) \qquad \pi^*(r^*, r^*) = \frac{2a^2 A}{(2(1 + \theta) + c + A)^2},$$

with A as in (13).

3.2 *Evolutionarily Stable Conjectures*

In this subsection, instead of imposing a consistency condition as in Definition 1, we will make conjectures subject to evolutionary selection. We will first derive firms' outputs given their conjectures. Since conjectures determine profits, they also determine reproductive success, and we can study the evolutionary selection of conjectures in a second step. The underlying assumption is that if firms differ in evolutionary success, the individual characteristics of more successful firms will spread within the population more quickly than the characteristics of the less successful ones. This leads to a dynamic process that determines the long-run distribution of individual characteristics within an economy.

Consider the two steps more formally. We will refer to firm i's (constant) conjecture r_i as to firm i's type (higher polynomial conjectures are analytically not tractable). Firms' types may be completely arbitrary, and the types are known whenever two firms compete against each other. We will derive firms' behavior given their types. Within strategic games this implies that the chosen strategy profile is a Nash equilibrium, denoted by $(q_i^*(r_i, r_j), q_j^*(r_i, r_j))$. In the second step, the types (conjectures) are the strategies, and the evolutionary success function, *i.e.*, the firm's profits

$$(20) \qquad \pi_i^*(r_1, r_2) \equiv \pi_i\big(q_1^*(r_i, r_j), q_2^*(r_i, r_j)\big)$$

evaluated at equilibrium strategies, are the payoff functions. To find the types that survive in the long run, we apply the static concept of an ESS (MAYNARD SMITH [1982]).

DEFINITION 2 *An equilibrium with evolutionarily stable conjectures is a pair of quantities* (q_1^*, q_2^*) *and conjecture* r^*, *such that*

$$(21) \qquad q_1^* = \rho_1\big(q_2^*\big), \qquad q_2^* = \rho_2\big(q_1^*\big), \qquad and$$

$$(22) \qquad \pi_i^*(r^*, r^*) \geq \pi_i^*(r, r^*) \quad for\ all\ r \qquad and$$

$$(23) \qquad \pi_i^*(r^*, r) > \pi_i^*(r, r) \quad for\ all\ r \neq r^* \quad with\ \pi_i^*(r^*, r^*) = \pi_i^*(r, r^*).$$

That is, an equilibrium with evolutionarily stable conjectures requires a Nash equilibrium in outputs given the types (21), and an evolutionarily stable preference type r^* that is a best reply against itself (22) and no r-mutant invading a society of r^*-players may be more successful than r^* (23).

We now solve for an equilibrium of this kind. First, assume that $c = 0$ and $\theta = 1$ do not both hold. The system of first-order conditions (8) can be solved for equilibrium strategies

$$(24) \qquad q_i^*(r_i, r_j) = \frac{a\left(2 + \theta\left(r_j - 1\right) + c\right)}{4\left(1 + c\right) + \theta\left(2 + c\right)\left(r_i + r_j\right) + \theta^2\left(r_i r_j - 1\right) + c^2}.$$

Substituting $q_i^*(r_i, r_j)$ and $q_j^*(r_i, r_j)$ in $\pi_i(\cdot)$ yields the evolutionary success $\pi_i^*(r_i, r_j)$ of type r_i, given that the opponent exhibits type r_j:[4]

$$(25) \qquad \pi_i^*(r_i, r_j) = \pi_i\big(q_i^*(r_i, r_j), q_j^*(r_i, r_j)\big)$$

$$(26) \qquad\qquad\qquad = \frac{\left(q_i^*\right)^2}{2}(c + 2(1 + \theta r_i)).$$

Note that the evolutionary success functions are symmetric (in the sense of $\pi_1^*(r_1, r_2) = \pi_2^*(r_2, r_1)$) and that the function $\pi_i^*(r_i, r_j)$ determines evolutionary success for all combinations of types. Therefore, we can simplify the notation and refer to $\pi^*(r, l)$ as type r's evolutionary success when paired with type l.

In order to satisfy the stability requirement (22), we have to find an r^* which is a best reply against itself. Candidates can be found by taking the first-order condition

$$(27) \qquad\qquad\qquad \frac{\partial}{\partial r}\pi^*(r, l) = 0.$$

This first-order condition can be solved for $r = -\theta/(2 + \theta l + c)$. Setting $r = l = r^*$ and solving the resulting quadratic equation with respect to r^* results in two candidates for an ESS:

$$(28) \qquad\qquad\qquad r^* = (-2 - c \pm A)/2\theta,$$

where A is defined as in (13). We already know that the negative root violates the second-order condition for profit maximization with respect to output. Therefore, only the candidate $r^* = (-2 - c + A)/2\theta$ remains.

To prove that r^* is the unique best preference parameter against itself, consider

$$(29) \quad \pi^*(r^*, r^*) - \pi^*(r, r^*)$$
$$= \frac{4a^2\theta^2 f(r)}{\left(A^2 - 2\theta^2 + \theta r(2 + c) + (2 + \theta r + c)A\right)^2 (2(1 + \theta) + c + A)^2},$$

where $f(r) = a_2 r^2 + a_1 r + a_0$ with

$$(30) \qquad \begin{aligned} a_2 &= 8(1 - \theta^2) + c\left(4(3 - \theta^2) + c(c + 6)\right) + A\left(c^2 + 4c + 4 - 2\theta^2\right), \\ a_1 &= -8\theta^3 + 8\theta + 8c\theta + 2\theta c^2 + A(4\theta + 2\theta c), \\ a_0 &= 2\theta^2 A. \end{aligned}$$

The sign of (29) is determined by the sign of the function $f(r) = a_2 r^2 + a_1 r + a_0$. Note that $a_2 > 0$ for given c and θ. Thus, $f(r)$ is a U-shaped parabola for every given set of c and θ. Solving $\partial f(r)/\partial r = 0$ for r shows that the minimum of the function $f(r)$ occurs at $r = -a_1/2a_2$. Now, note that $f(-a_1/2a_2) = 0$ and that $-a_1/2a_2 = r^*$. That is, the function f and thus the expression $\pi^*(r^*, r^*) - \pi^*(r, r^*)$ in (29) are 0 if

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[4] Note that the game with types $(\widehat{r}_i, \widehat{r}_j)$ does not have an equilibrium if $4(1 + c) + \theta(2 + c)(\widehat{r}_i + \widehat{r}_j) + \theta^2(\widehat{r}_i \widehat{r}_j - 1) + c^2 = 0$. For such $(\widehat{r}_i, \widehat{r}_j)$ we proceed as in POSSA-JENNIKOV [2000] by extending the fitness function by continuity in the first argument in the sense that $\pi_i^*(\widehat{r}_i, \widehat{r}_j) = \lim_{r_i \to \widehat{r}_i} \lim_{r_j \to \widehat{r}_j} \pi_i^*(r_i, r_j)$. This limit does always exist on the extended real line $\mathbb{R} \cup \{\pm\infty\}$, and as a result the function $\pi_i^*(r_i, r_j)$ is differentiable with respect to the first argument at $r_j = r_i$.

and only if $r = r^*$, and otherwise they are positive. This implies that r^* is the unique evolutionarily stable type (conjecture).

Finally, consider the case $c = 0$ and $\theta = 1$, *i.e.*, the case of zero (constant marginal) costs and perfect substitutes. In this case, we have $\pi^*(r^*, r^*) = \pi^*(r, r^*)$ for all $r \in [-1, 1]$ and

$$(31) \qquad \pi^*(r^*, r) - \pi^*(r, r) = \frac{-a^2(r + 1)}{\theta(r + 3)^2} \leq 0$$

for all $r \in [-1, 1]$ such that the condition (23) is violated. Thus, if $c = 0$ and $\theta = 1$ the consistent conjecture is not evolutionarily stable. We have proven

PROPOSITION 1 *The unique evolutionarily stable conjecture of the quantity game is equal to the consistent conjecture and is given by*

$$r^* = \frac{-2 - c + A}{2\theta},$$

unless $c = 0$ and $\theta = 1$, in which case the consistent conjecture is not evolutionarily stable.

Since the evolutionarily stable conjecture is equal to the consistent conjecture, the outputs and profits are also as in (18) and (19) above.

4 Price Competition

4.1 Consistent-Conjectures Equilibrium

As above, we compute a closed-form solution of the consistent-conjectures equilibrium for the price-setting market. Maximizing (5), we take the first-order conditions

$$\frac{\partial \pi_i(p_i, p_j)}{\partial p_i} = \left(\frac{a(1 - \theta) - p_i + \theta p_j}{1 - \theta^2} \right) + p_i \left(\frac{\theta r_i - 1}{1 - \theta^2} \right)$$
$$- c \left(\frac{a(1 - \theta) - p_i + \theta p_j}{1 - \theta^2} \right) \left(\frac{\theta r_i - 1}{1 - \theta^2} \right)$$
$$= 0$$

and derive firm i's reaction function

$$(32) \qquad \rho_i(p_j) = \frac{\left(a(1 - \theta) + \theta p_j \right) \left(1 - \theta^2 + c(1 - \theta r_i) \right)}{2(1 - \theta^2) - \theta r_i(1 - \theta^2) + c(1 - \theta r_i)}.$$

The slope of firm i's reaction function is

$$(33) \qquad \frac{\theta(1 - \theta^2 + c(1 - \theta r_i))}{2(1 - \theta^2) - \theta r_i(1 - \theta^2) + c(1 - \theta r_i)}.$$

We obtain the consistent conjectures by using $r_i = r_j$ and solving

$$(34) \qquad r = \frac{\theta(1 - \theta^2 + c(1 - \theta r))}{2(1 - \theta^2) - \theta r(1 - \theta^2) + c(1 - \theta r)}.$$

There are two solutions. Inspection of the second-order conditions shows that the following consistent conjecture yields the maximum:

$$(35) \qquad r^* = \frac{2(1 - \theta^2) + c(1 + \theta^2) + A(\theta^2 - 1)}{2\theta(1 - \theta^2 + c)},$$

where A is defined as above in the Cournot case. (Note $r^* > 0$.)

The prices implied by these conjectures are

$$(36) \qquad p_i^* = \frac{a(2(1 - \theta^2) + c(c + 2 + A))}{(2 + c + 2\theta)(1 + c - \theta) + (c + 1 + \theta)A},$$

and the equilibrium profits are

$$(37) \qquad \pi_i^* = \frac{a^2(c + A)(4(1 - \theta^2) + c(4 + A + c))}{2((2 + c + 2\theta)(1 + c - \theta) + (c + 1 + \theta)A)^2}.$$

4.2 Evolutionarily Stable Conjectures

We proceed as above with Cournot competition. To compute the equilibrium prices, we have to solve the system of first-order conditions (32). The solution of this system of equations yields $p_1^*(r_1, r_2)$ and $p_2^*(r_1, r_2)$, that is, the optimal prices given the conjectures of the firms. These equilibrium prices are quite complex, so we refrain from writing them down. The profits implied by $p_1^*(r_1, r_2)$ and $p_2^*(r_1, r_2)$ are

$$\pi(p_1^*(r_1, r_2), p_2^*(r_1, r_2))$$
$$= p_1\left(\frac{a - p_1^* - \theta(a - p_2^*)}{1 - \theta^2}\right) - \frac{c}{2}\left(\frac{a - p_1 - \theta(a - p_2)}{1 - \theta^2}\right)^2$$
$$= \frac{a^2(1 - \theta r_1)(c + 2 - 2\theta^2 - \theta r_1 c)(c - c\theta r_2 + 2 - \theta - \theta r_2 - \theta^2 + \theta^2 r_2)^2}{2N^2},$$

where

$$(38) \qquad N = 4 + 4c + c^2 - \theta[(2 + 3c + c^2)(r_1 + r_2)]$$
$$- \theta^2[5 - 2c + r_1 r_2(1 + 2c + c^2)]$$
$$+ \theta^3[(2 + c)(r_1 + r_2)] + \theta^4[1 - r_1 r_2].$$

The first-order condition

$$- \theta^2 a^2(1 - \theta^2)$$
$$\times (2r_1 - \theta r_1 r_2 - \theta + c r_1 - c\theta r_1 r_2 - c\theta - 2\theta^2 r_1 + \theta^3 r_1 r_2 + c\theta^2 r_2 + \theta^3)$$
$$\times (c - c\theta r_2 + 2 - \theta - \theta r_2 - \theta^2 + \theta^2 r_2)^2 = 0$$

can be solved for r_1 as a function of r_2. Setting $r^* = r_1 = r_2$ and solving the resulting quadratic equation for r^* results in two candidates for an ESS. We already know the relevant root, so we obtain

$$(39) \qquad r^* = \frac{2(1 - \theta^2) + c(1 + \theta^2) + A(\theta^2 - 1)}{2\theta(1 - \theta^2 + c)},$$

where A is defined as above.

Note that the only solution of the equation $\partial \pi^* (r, r^*) / \partial r = 0$ is $r = r^*$. Furthermore it holds that $\pi^*(r^*, r^*) > \pi^* (\pm 1, r^*)$. Hence r^* is the unique best reply against itself, which implies that the stability requirement (23) is also fulfilled.

PROPOSITION 2 *The unique evolutionarily stable conjecture of the price game is equal to the consistent conjecture and is given by*

$$(40) \qquad r^* = \frac{2(1 - \theta^2) + c(1 + \theta^2) + A \left(\theta^2 - 1 \right)}{2\theta \left(1 - \theta^2 + c \right)}.$$

In contrast to the quantity-setting case, the limit case of constant marginal cost ($c = 0$) and homogeneous goods is not a problem here, and the explicit solution can be obtained.

5 *Discussion*

In this paper we propose an evolutionary process to select among conjectural variations in Cournot and Bertrand markets. We first determine the unique equilibrium in quantities and prices for all possible combinations of linear conjectures. For the evolutionary game with conjectures as mutants and reproductive success (a firm's profit) as the payoff function, we study conjectures that are evolutionarily stable. It turns out that the equilibrium with evolutionarily stable conjectures is the same as BRESNAHAN's [1981] CCE. In this way, we justify both the outcome implied by consistent conjectures and the conjectures themselves.

Evolution favors firm types with better relative performance. In our Cournot model, a negative conjecture serves as a commitment device in the sense that it yields a profit improvement over a type with a larger conjecture. Therefore, evolution selects generally negative conjectures. In the Bertrand model, the same is true for positive conjectures. However, the result that the evolutionarily stable conjectures coincide with the consistent conjectures is surprising, as there is no obvious analogy between the two concepts.

Recently, POSSAJENNIKOV [2004] has generalized the results of DIXON AND SOMMA [2003] and of this paper to abstract two-player games. It turns out that the equilibrium with evolutionarily stable conjectures is equal to the CCE for a large class of two-player games, provided some regularity conditions are met.[5] POSSAJENNIKOV [2004] also suggests an intuition behind this result: whenever the conjecture is consistent, a player correctly anticipates the reaction of the other player and therefore maximizes the correct profit function.

Our result may be positively interpreted in that it provides support for consistent conjectures. The other side of the medal, the negative interpretation of our result,

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[5] GEHRIG, GÜTH, AND LEVINSKI [2004] obtain a different result. They analyze the evolution of beliefs about demand expectations. They find that the evolution of beliefs about demand does not converge to rational expectations for markets with finitely many competitors.

is that no other conjecture can be justified by arguments based on evolutionary selection. Many empirical researchers use the notion of conjectural variation as a useful shortcut to capture the degree of "competitiveness" that is not reflected in the number of firms, the extent of product differentiation, cost asymmetries, *etc.* The conjecture is supposed to capture something that can be thought of as conduct in the industry but that is hard to model explicitly (see, *e.g.*, KIM AND VALE [2001]). Our result indicates that conjectural variations cannot be used to reflect any degree of competitiveness, as only one specific conjecture is evolutionarily stable. This indicates that more research on the theoretical foundations of conjectural variations is needed.

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