Merger and Collusion in Contests

by

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Competition in some product markets takes the form of a contest. If some firms cooperate in such markets, they must decide how to allocate effort on each of their products and whether to reduce the number of their products in the competition. We show how this decision depends on the convexity properties of the contest success function, and we characterize conditions under which cooperation is profitable. (JEL: D 44, L 11, L 13)

1 Introduction

Competition in product markets is sometimes well described by a contest, particularly if competition via prices is not feasible. In such markets sellers may contest with each other and spend resources in order to attract customers to buy from them and not from another seller. The type of effort can differ from one market to another. It may take the form of visits, gifts, persuasive talking, or invitations to conferences in fancy holiday resorts. The last, for example, is popular in the market for prescription drugs in countries with health care systems. As prescription drugs are covered by health insurance, regardless of whether consumers or physicians make the consumption choice, price competition is more or less ruled out. Pharmaceutical companies' marketing efforts for over-the-counter drugs or prescription drugs are estimated to be in the range between 20 and 40 percent of sales revenues (see, e.g., BREYER AND ZWEIFEL [1999, p. 366], SCHERER [2000, p. 1303], and BERNDT et al. [1995]). In other markets sales effort consists of various types of advertising as in the markets for cigarettes or beverages, or, as in the retail insurance business, of visiting and persuasive talking to customers. Again, this becomes particularly pronounced if price competition is not feasible, which used to be the case in many European insurance markets prior to deregulation on the EU level in 1992. Prior to deregulation, the regulators protected insurance companies from "ruinous competition" by regulating insurance premiums and by restrictions on agents' sales

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commissions and on marketing expenses (see, e.g., REES AND KESSNER [1999]). Other important contest examples are firms competing for a monopoly as in R&D contests (see, e.g., BAGWELL AND STAIGER [1997]), contests for quasimonopoly due to network externalities (BESEN AND FARRELL [1994]), or firms seeking special political favors in rent-seeking contests, and the results in this paper could apply qualitatively to these contests as well.

SCHMALENSEE [1976] observed and characterized promotional competition in markets with few sellers and differentiated products: "[P]rice competition is relatively rare in such markets. Prices generally change infrequently, and sellers compete, if at all, mainly through product variation and promotional expenditures. It is thus of some interest to attempt to model rigorously markets in which the only competition is of this sort" (p. 493). With promotional competition, firms spend effort to attain some payoff or "prize": for instance, a large share in a market in which price exceeds marginal cost. Firms win a customer with some probability (or a share in the total market on the aggregate level) as a function of the various efforts of all competing firms. These contests are all-pay auctions. Efforts are made (and sunk) before the customer makes its decision.

In this paper we consider cooperation among a subgroup of m firms in a market with n ($> m$) firms that is characterized by this type of competition and address two questions. First, we ask what are the factors determining whether the group of cooperating firms will reduce their number of products. For instance, firms often have established brands for close substitutes, and have to decide whether to keep all brands after a merger or to abandon some of them. If they keep all brands, we shall call this *collusion*. If they reduce the number of brands, we shall call this a *merger*. Note that these notions do not refer to the institutional form of cooperation, but simply to whether the cooperating firms decide to reduce the number of their brands. Cigarette markets are an example for what we call "collusion" here: the big firms have multiple brands and, when advertising one of their brands, take into account that they partially cannibalize on their own other brands (NGUYEN [1987]). The U.S. soft-drink industry, in contrast, is an example in which firms seem to concentrate on single brand names. We ask how the type of cooperation is determined by specific characteristics of the contest.

Second, we ask whether cooperation in contests is profitable. The question of profitability of merger or collusion of a subgroup of firms in an industry has received considerable attention for Cournot or Bertrand competition in the absence of sales effort (see, e.g., SALANT, SWITZER, AND REYNOLDS [1983], DENECKERE AND DAVIDSON [1985], GAUDET AND SALANT [1991], and FARRELL AND SHAPIRO [1990] for analyses). These analyses showed that cooperation can harm cooperators and benefit their competitors.[1]

[1] The incentives for divisionalization, which is in some sense the inverse of a merger, have been analysed in BAYE, CROCKER, AND JU [1996] for the case of Cournot competition. A complementary analysis of divisionalization for contests is in HUCK, KONRAD, AND MÜLLER [2001]. Divisionalization generates related strategic

BARROS AND SØRGARD [2000] also consider promotional competition, allowing for some form of collusive price-setting behavior. They consider only merger and study the relationship between advertising and price collusion. Their results are sensitive to the particular contest success function they use for determining market shares. On a more general level, our results relate to the discussion of cooperative rent-seeking. DIJKSTRA [1999] considers several structures of cooperation in contests, allowing for matching grants, delegation, and choices of different roles for different members of a cooperating group of rent-seekers. In our paper the group of contestants collapses into one single decision-maker that maximizes the group's total payoff, which rules out more sophisticated contractual arrangements such as matching grants or strategic delegation.

We proceed as follows. In Section 2 we describe our basic model of promotional competition: sales contests. In Section 3 we consider the determinants for whether firms merge or collude. In Section 4 we consider profitability of merger and collusion, and Section 5 concludes.

2 *Contests*

Consider a market with n identical firms. Each firm offers one product (or brand). Suppose that these firms make efforts in a contest for some prize of size B. A few examples for this type of competition have been discussed in the introduction. Each firm i chooses contest effort $x_i \in [0, \infty)$. These efforts are irreversibly spent by contestants before they know who wins the contest. Contest efforts determine firms' probabilities q_i of winning the prize, according to a contest success function

$$q_i(x_1, ..., x_n) = \frac{(x_i)^a}{\sum_{j=1}^{n}(x_j)^a}. \tag{1}$$

In the context of promotional competition, this parametric form (1) has been used to determine firms' market shares as a function of advertising by SCHMALENSEE [1992, note on p. 131]. This contest success function has been suggested by TULLOCK [1980] in a more general context and is a special case of more general contest success functions, but has gained support from an axiomatization in SKAPERDAS [1996]. Microeconomic underpinnings for the specific form of (1) are provided by MORTENSEN [1982] and, in the context of R&D contests, by FULLERTON AND MCAFEE [1999].

The coefficient a in (1) is called *discriminatory power*. It is a measure of how much the contest outcome can be influenced by contest effort, and how much is left to chance. For instance, if $a \to 0$, each contestant ends up with the same q_i, irrespective of contest efforts. If, instead, $a \to \infty$, (1) approaches a contest success function in which the contestant who makes the highest effort wins the prize. We limit the discriminatory power to $a \in [0, n/(n-1))$ in order to have well-behaved

issues, but the divisionalization analysis does not tell whether firms prefer to merge or to collude, or how profitable these choices are.

optimization problems with equilibria in pure strategies and first-order conditions characterizing these equilibria.[2] (We discuss the case $a \to \infty$ briefly in footnote 3.)

Firms are risk-neutral. Their (expected) payoffs are

$$\pi_i = q_i B - x_i. \tag{2}$$

Firm i wins B with probability q_i and spends contest effort equal to x_i. The first-order condition for firms maximizing their payoffs and symmetry can be used to calculate the contest equilibrium efforts

$$x^*(n) = \frac{aB(n-1)}{n^2}. \tag{3}$$

The equilibrium share is $1/n$ for each contestant, yielding the equilibrium payoffs

$$\pi^*(n) = \frac{B}{n} - \frac{aB(n-1)}{n^2}. \tag{4}$$

Firms contest for contracts with individual customers, whose decisions can be seen as a random function of sales effort. With many identical customers, however, q_i can also be interpreted as firm i's market share, and we will make use of this interpretation in what follows.

3 *Cooperation of a Subgroup of Firms*

Consider a contest of n firms, each firm promoting one product (or brand) in a sales contest. Suppose m firms merge or collude. Let N be the set of all firms, and M be the set of firms that cooperate in one of these ways. Denote by $U = N \backslash M$ the set of firms that do not participate in the cooperation. We consider the following contest game. Each noncooperating firm chooses effort x_k in order to maximize its payoff, and the set of cooperating firms chooses a vector $(x_1, ..., x_m)$ of sales efforts in the m products in order to maximize their joint profits. The total profit of the cooperating firms, π_M, is given by

$$\pi_M = \frac{\sum_{j\in M}(x_j)^a}{\sum_{j\in M}(x_j)^a + \sum_{k\in U}(x_k)^a} B - \sum_{j\in M} x_j, \tag{5}$$

while the profit π_u for each noncooperating firm $u \in U$ is

$$\pi_u = \frac{(x_u)^a}{\sum_{j\in M}(x_j)^a + \sum_{k\in U}(x_k)^a} B - x_u. \tag{6}$$

For the equilibrium we obtain

Proposition 1: The cooperating firms allocate the sum of their efforts equally among all products $i \in M$ if $a < 1$, and concentrate all effort on one product if $a > 1$. If $a = 1$, the allocation of efforts between different products $i \in M$ is indeterminate.

[2] For the equilibrium (in mixed strategies) for the case of $\infty > a > n/(n-1)$ see BAYE, KOVENOCK, AND DE VRIES [1994]. For $a \to \infty$ see BAYE, KOVENOCK, AND DE VRIES [1996].

Proof: Suppose the M-group anticipates the vector of given equilibrium effort choices $(x_{u_1}, ..., x_{u_{n-m}})$ by noncooperating firms. Whatever this vector is, by (5), if $a = 1$, then π_M solely depends on the sum of efforts the cooperating firms exert, i.e., on $\sum_{i \in M} x_i$. Accordingly, it does not matter how they allocate their efforts. If $a > 1$, the cooperating firms maximize the probability of winning by making use of the increasing returns to scale, i.e., by concentrating all efforts on one product. At the same time the cooperating firms' total costs only depend on the sum of efforts. Hence, π_M is maximized if indeed all effort is concentrated on one product. Finally, if $a < 1$ (i.e., with decreasing returns to scale), it is straightforward to see that the total profit of cooperating firms π_M is maximized if the total group effort is spread evenly among all product lines. *Q.E.D.*

Note that the result in Proposition 1 generalizes to the broader class of contest success functions with

$$q_i = \frac{f(x_i)}{\sum_{j=1}^{n} f(x_j)},$$

provided that the equilibrium is in pure strategies and characterized by the first-order conditions: Firms merge if f is convex, and firms collude if f is concave.

An important assumption underlying Proposition 1 is simultaneity: neither the cooperating firms' choice of total equilibrium effort nor the allocation of this amount between different products becomes known to the noncooperating firms before they choose their own efforts. Cooperating firms may sometimes choose to close down a number of products and keep only $h \leq m$ products when they decide to cooperate, and this may be observed by the noncooperating firms before all firms enter the actual contest game of choosing efforts. Proposition 1 states that in this case cooperating firms would choose to spread effort equally among the remaining h product lines if $a < 1$, and to concentrate all effort on one product if $a > 1$. Intuitively, if $a < 1$, there is an advantage in having a large number of products, because the total impact of a given budget $x_M = \sum_{j \in M} x_j$ is larger for a larger number h of products. However, the equilibrium reaction of the noncooperative firms must also be taken into consideration. If the noncooperating firms spend more effort in the equilibrium if h is large, the cooperating firms' optimal choice of h becomes ambiguous. On the other hand, if $a > 1$, the choice of h becomes irrelevant. In that case all firms anticipate that the cooperating firms will concentrate all effort on one product. Hence, the choice of h does not matter, as any choice $h \geq 1$ yields the same payoffs.

4 *Profitability*

Consider now whether cooperation of a subgroup of firms is profitable for this group. From Proposition 1 we know that cooperation essentially leads to a situation in which the set of noncooperating firms contest with one single firm with one product if $a > 1$. If $a < 1$, Proposition 1 tells us that the noncooperating firms contest with

one firm that has m products and spends the same effort on each product. Hence, we can consider profitability of cooperation for the two cases separately.

4.1 High Discriminatory Power ($a > 1$)

Suppose $m < n$ firms cooperate in a contest with $a > 1$. By Proposition 1 they spend effort on only one of their products. Without cooperation the set M of firms received a payoff equal to $m\pi^*(n)$. With cooperation their payoff equals

$$\pi^*(n-m+1) = \frac{B}{n-m+1} - \frac{aB(n-m)}{(n-m+1)^2}.$$

Now let $g(n, m, a)$ be the function that measures the gain (or loss) of m firms that merge in an industry composed of n firms, i.e., $g(n, m, a)$ is given by

$$\begin{aligned} g(n,m,a) &= \pi^*(n-m+1) - m\pi^*(n) \\ &= \frac{B}{n-m+1} - \frac{aB(n-m)}{(n-m+1)^2} - m\left(\frac{B}{n} - \frac{aB(n-1)}{n^2}\right), \end{aligned}$$

and has the following properties:

(i) For all $n \geq 2$ it holds that $g(n, 1, a) = 0$. (If one firm is joined by no other in a merger, the profit does not change.)

(ii) For all $n \geq 2$ and for all $a > 0$ it holds that $g(n, n, a) = (B/n) \cdot a(n-1) > 0$. (Merger to monopoly is always profitable.)

(iii) For all $n \geq 2$ it holds that

$$\left.\frac{\partial g(n,m,a)}{\partial m}\right|_{m=1} = \frac{B}{n^3}(2a + n - 2an + an^2 - n^2) \lesseqgtr 0$$

if and only if

$$a \lesseqgtr \frac{n(n-1)}{(n-1)^2+1} \qquad \left(< \frac{n}{n-1} \text{ for } n \geq 2\right).$$

(iv) For all $n \geq 4$ and for all $a \in \left[0, \frac{n}{n-1}\right)$ it holds that

$$\frac{\partial^2 g(n,m,a)}{\partial m^2} = 2B\frac{n-m+1-a(n-m-2)}{(n-m+1)^4} > 0,$$

i.e., $g(n, m, a)$ is strictly convex (and also continuous) with regard to m.

With the help of properties (i)–(iv) we can prove the following

Proposition 2: Let $n/(n-1) > a \geq 1$.

(A) If there are three firms, then merger of two firms is profitable.

(B) For any number n of firms, there is a critical discriminatory power $a_0(n)$ such that merger of $m \leq n-1$ is never profitable for any contest with $a \leq a_0(n)$.

(C) Let

$$a \in \left(1, \frac{n(n-1)}{(n-1)^2+1}\right) \quad \text{and} \quad n \geq 4.$$

Then the following two statements hold true: If merger by a specified number of firms is not profitable for the merging firms, merger by a smaller number of firms is also not profitable. If merger by a specified number of firms is profitable for them, merger by a larger number of firms is also profitable.

(D) If

$$a \in \left[\frac{n(n-1)}{(n-1)^2+1}, \frac{n}{n-1}\right),$$

then for any number $n \geq 4$ of firms, merger of any number $m = 2, 3, ..., n$ of firms is profitable.

Proof: For part (A) note that $g(3, 2, a) = (B/36) \cdot (7a - 6)$. For part (B) note that

$$\lim_{a \to 0} g(n, m, a) = -\frac{(n-m)(m-1)}{n(n-m+1)} B < 0.$$

The proof of part (C) follows the lines of proof of result D in SALANT, SWITZER, AND REYNOLDS [1983]: properties (i) and (iii) imply that $g(n, m, a)$ becomes negative for small $m > 1$ if $a < n(n-1)/[(n-1)^2 + 1]$. Note that

$$\frac{n(n-1)}{(n-1)^2+1} = \frac{(n-1)^2}{(n-1)^2+1} \frac{n}{n-1}$$

with the first term on the right-hand side being smaller than 1. According to property (iv), $g(n, m, a)$ is continuous and strictly convex in m. Thus, because of property (ii), there is a unique $y^* < n$ such that $g(n, y^*, a) = 0$, and the result follows. Finally, for the proof of (D), it is straightforward to see that, in this case, properties (i), (ii), (iii), and (iv) imply that $g(n, m, a) > 0$ for all $m = 2, 3, ..., n$. *Q.E.D.*

Intuitively, cooperation that makes firms in M concentrate their effort on one of their products has two effects. First, it increases the total profit of the industry, because the total contest effort is reduced with a reduction in the number of contestants. Second, the share of industry profit that goes to the cooperating group of firms is reduced. Proposition 2 shows that the profitability of cooperation depends on the discriminatory power of the contest and on whether the firms that take part in the merger constitute a large share in the total number of firms. If the discriminatory power is not too large, cooperation of many firms can be profitable whereas cooperation of few firms is not. However, if the discriminatory power is sufficiently high, merger – of any number of firms – is always profitable.[3]

[3] We restricted attention to $a < n/(n-1)$ in order to concentrate on pure-strategy equilibria. However, for $a \to \infty$, and with $m < n$, the contest is a symmetric fully discriminatory all-pay auction. It is known (see, e.g., HILLMAN AND RILEY [1989] and BAYE, KOVENOCK, AND DE VRIES [1996]) that all firms' payoffs are zero in the (mixed-strategy) equilibrium for this type of contest, whether firms cooperate or not.

4.2 Low Discriminatory Power ($a < 1$)

Consider next the case in which cooperation does not reduce the number of products. The colluding firms take into account that an increase in contest effort on, say, the product of firm $i \in M$ reduces the market shares of all other firms' products, including the shares of the firms in M. This latter effect will be internalized, leading to a less aggressive effort choice of colluding firms. This, in turn, changes the contest behavior of all other firms. Using Proposition 1 for $a < 1$, we obtain (reduced) payoff functions for the group M of colluding firms as

$$\pi_M = \frac{m(x_\mu)^a}{m(x_\mu)^a + \sum_{k\in U}(x_k)^a} B - mx_\mu \tag{7}$$

and for each noncooperating firm $u \in U$ as

$$\pi_u = \frac{(x_u)^a}{m(x_\mu)^a + \sum_{j\in U}(x_j)^a} B - x_u. \tag{8}$$

Maximization of (7) yields a first-order condition for the choice of x_μ, which, after using the symmetry of the efforts x_u of noncooperating firms, becomes

$$a(x_\mu)^{a-1}(n-m)(x_u)^a B = [m(x_\mu)^a + (n-m)(x_u)^a]^2, \tag{9}$$

and maximization of (8) with respect to x_u for $u \in U$ yields a first-order condition for the choice of noncooperating firms, which, after using symmetry, becomes

$$a(x_u)^{a-1}[(n-m-1)(x_u)^a + m(x_\mu)^a]B = [(n-m)(x_u)^a + m(x_\mu)^a]^2. \tag{10}$$

This system of two equations determines x_μ and x_u, but is not analytically solvable, except for some special cases. This makes it impossible to compare the equilibrium profits $m\pi^*(n)$ of the M-group in the fully noncooperative equilibrium with the equilibrium profits with collusion. However, we can solve three partial problems. First, we find

Proposition 3: At effort values of the fully noncooperative equilibrium, noncooperating firms react to a marginal joint reduction in effort among colluding firms by an increase in their contest effort.

Proof: See Appendix A.1.

If the firms in the colluding group M uniformly choose an effort level that is slightly lower than the effort level $x^*(n)$ in the fully noncooperative equilibrium, the firms outside this group anticipate this, and they choose higher efforts. As this holds for any size of the group M, Proposition 3 states that the efforts of the firms in M and the efforts of the firms that do not cooperate are strategic substitutes locally at the fully noncooperative equilibrium. This result contributes to the discussion on whether advertising redistributes market shares or increases the total market. The empirical study by ROBERTS AND SAMUELSON [1988], for instance, finds "negative conjectural variations": A firm i expects that other firms will reduce their advertising if i increases its advertising effort on some of its brands. This negative slope of reaction functions is considered as counterintuitive if advertising is an activity that

reallocates market shares in a market of given size. The negative slope is, however, in line with advertising being a voluntary contribution to a collective good that increases the size of the whole market. Proposition 3 shows that the empirical finding by ROBERTS AND SAMUELSON [1988] is also compatible with advertising as an activity that reallocates shares in a market of given size: reaction functions in contests can have negative slope in some range of the strategy space, and the slope is negative at the noncooperative equilibrium.[4]

Proposition 4: A marginal joint reduction (increase) in effort among colluding firms that is observed by noncooperating firms before they choose their effort increases their profit if the discriminatory power of the contest is smaller (bigger) than $(n(m-1))/(m(n-2))$.

Proof: See Appendix A.2.

Proposition 4 says that, if the colluding firms can choose effort as a Stackelberg leader, they can always do better than in the fully noncooperative Nash equilibrium. To do this they choose effort that is smaller (larger) than the Nash-equilibrium effort if a is sufficiently small (large). Intuitively, the direct effect of cost savings from reduced effort within the colluding group outweighs the direct effect of reduced market share and the indirect effect of noncooperating firms' changes in effort if the discriminatory power of the contest is sufficiently small.

Let us return to simultaneous effort choices and consider the comparison of profits in the fully noncooperative equilibrium and in the equilibrium with m colluding firms. As pointed out above, for the general case with a, n, and m arbitrary, the problem of comparing these payoffs is not tractable, because it is not possible to calculate closed-form solutions for the efforts in the equilibrium with collusion from (9) and (10). However, closed-form solutions for efforts can be obtained for the case $m = n-1$. Note that this also includes the interesting case with $n = 3$ and $m = 2$. From (9) and (10) we obtain $x_\mu = a(n-1)^a B/[(n-1)^a + (n-1)]^2$ and $x_u = (n-1)x_\mu$. Inserting in (7) and comparing this profit with $(n-1)\pi^*(n)$ yields

$$\pi_M - (n-1)\pi^* = \frac{(n-1)B}{(n-1)+(n-1)^a} - \frac{a(n-1)^a(n-1)B}{[(n-1)^a+(n-1)]^2} - \frac{(n-1)B}{n} + \frac{aB(n-1)^2}{n^2}. \tag{11}$$

This expression is positive for all $a \in (0, 1]$ (as can be seen numerically from Figure 1, which depicts the profit gain from collusion for $B = 1$), and we obtain from (11) $\lim_{n\to\infty}[\pi_M - (n-1)\pi^*] = aB$.

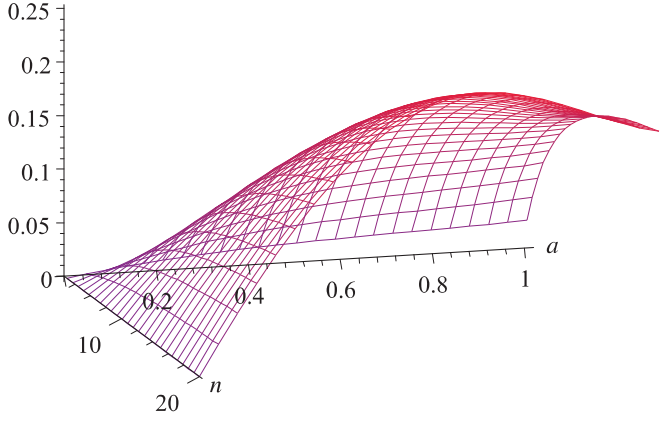
We summarize this result as

Proposition 5: Collusion of $n-1$ firms is always profitable for $a \in (0, 1]$.

[4] Given that strategic complementarity or substitutability of effort choices is not a global property in contests, it is not surprising that the empirical results on strategic substitutability by ROBERTS AND SAMUELSON [1988] are controversial (see, e.g., SELDON, BANERJEE, AND BOYD [1993]).

Figure 1

Profit Gain of $m = n - 1$ Colluding Firms



5 *Discussion and Conclusions*

If we compare cooperation of a subgroup of firms in markets with promotional contests with cooperation in Bertrand or Cournot markets, we first observe that the cooperating group's choice of their number of products becomes important. Firms may or may not want to keep the number of brands they had prior to cooperation. We found that cooperating firms may reduce the number of products on which they spend sales effort. Furthermore, we found that the crucial determinants for this decision are the convexity properties of the contest success function. With high discriminatory power (increasing returns to scale), firms will concentrate their effort on one product (or brand); with low discriminatory power (decreasing returns), they will keep the whole range of products (or brands) and will spread out their efforts equally.

The results on profitability of cooperation with or without a reduction of products are less straightforward than in Bertrand or Cournot competition. As is known from DENECKERE AND DAVIDSON [1985], strategic complementarity as in the Bertrand competition case is sufficient for profitability. In contests, strategic complementarity or substitutability of contest efforts of different contestants is not a global property and changes across the strategy space. This fact makes it impossible to rely on the straightforward reasoning used, for instance, in the Bertrand competition case. Nevertheless, we found that cooperation can be profitable in contests. Generally, cooperation tends to be profitable if the number of cooperating firms is sufficiently large or if the total number of firms is sufficiently small. Also, cooperation tends to be profitable if the discriminatory power in the contest is high.

It would be nice to be able to draw some welfare conclusions on merger and collusion in contests. In the context considered here, cooperation that reduces the number of products (merger) reduces total contest effort. However, whether a reduction in total contest effort reduces or increases welfare depends on the nature of the effort. For instance, if this effort is sales effort, the welfare effect of the reduction depends on how the effort affects consumers. Consumers may appreciate effort for its intrinsic value or for its information value. Also, effort may change customers' rents from consuming the product. Finally, effort can be pure waste or can have characteristics of a transfer. These ambiguities make a welfare analysis difficult.

Appendix

A.1 Proof of Proposition 3

Consider the effect of a symmetric marginal reduction in effort choices by the contestants in M on their equilibrium profits. The first-order condition (10) determines how contestants in U will react to an anticipated reduction in x_μ. Define this function as

$$\text{(A1)}\quad x_u = \xi(x_\mu) \equiv \arg\max_{x_k \geq 0} \left\{ q_k B - x_k \mid x_i = x_\mu \ \forall i \in M \text{ and } x_j = x_u \ \forall j \in U \backslash \{k\} \right\}.$$

It is clear that such x_u exists by standard fixed-point arguments. ξ is implicitly determined by (10). We call ξ the symmetric reaction function of the noncooperating firms for effort choices of the cooperating firms. At the fully noncooperative equilibrium $x^*(n)$, the slope of the function ξ is obtained by total differentiation of (10) and equals

$$\text{(A2)}\qquad \left.\frac{d\xi(x)}{dx}\right|_{x=x^*} = -\frac{am(n-2)}{(n-am)(n-1)+am}.$$

The slope of the reaction function ξ at the fully noncooperative Nash equilibrium as in (A2) is strictly negative for all $n \geq 3$ and $m \leq n-1$. To see this, note that $a \leq n/(n-1)$. This confirms Proposition 3. *Q.E.D.*

A.2 Proof of Proposition 4

As $\partial \pi_k / \partial x_k = 0$ and $\partial \pi_i / \partial x_k = -(n/(n-1))$ for $i \neq k$ at the fully noncooperative Nash equilibrium with efforts as in (3), the profit increase of each firm in the merging group M from a joint reduction in their contest effort x starting in (x^*, x^*) equals

$$\text{(A3)}\qquad -\left.\frac{d\pi_i}{dx}\right|_{x=x^*} = \frac{1}{n-1}\left((m-1) + (n-m)\left.\frac{d\xi(x)}{dx}\right|_{x=x^*}\right).$$

This condition resembles condition (5) in GAUDET AND SALANT [1991], who consider Cournot competition. Inserting (A2) yields

$$-\left.\frac{d\pi_i}{dx}\right|_{x=x^*} > 0 \quad \text{if and only if} \quad a < \frac{n(m-1)}{m(n-2)},$$

which confirms Proposition 4. *Q.E.D.*

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