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The quality of the signal matters — a note on imperfect observability and the timing of moves

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Abstract

In a recent study Huck and Müller [Games Econ. Behav. 31 (2), 2000, 174–190] report that — in contrast to Bagwell's [Games Econ. Behav. 8 (1995) 271–280] prediction — first movers in a simple experimental market do not lose their commitment power in the presence of noise. The present note shows that it is the quality of the signal and not the knowledge about the physical timing of moves that is responsible for these experimental results. Additionally, the findings reported here provide further evidence that the positional order protocol cannot induce non-equilibrium play.

JEL classification: C72; C92

Keywords: Commitment; First-mover advantage; Imperfect observability; Experimental economics

1. Introduction

Bagwell (1995) shows that any noise associated with the observation of the first mover's choice eliminates the first-mover advantage. For that purpose he studies the following two-stage two-player game. First, player 1 chooses an action a. Upon observing a noisy signal (action a perturbed by noise) about the leader's choice, player 2, the follower, chooses an action b. Bagwell's stunning result is that the set of pure-strategy Nash equilibrium outcomes of the so-called noisy-leader game coincides exactly with the set of pure-strategy Nash equilibrium outcomes of the underlying simultaneous-move game.[1] Thus, if the

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[1] This holds whenever the noise has full support and the second mover's best-response correspondence is single-valued.

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observability of the first mover's choice is even slightly in doubt, the strategic benefit of commitment is lost.

In a recent study, Huck and Müller (2000) (henceforth HM) experimentally assess the behavioral relevance of this claim. More precisely, they implemented several versions of a simple two-person, sequential-move game similar to an example given by Bagwell. These versions varied in the quality of the signal informing the second mover. In treatments with noise-levels up to 10%, they observe play settling down close to the Stackelberg outcome favoring the first mover (contrary to Bagwell's prediction). In the treatment with 20% noise, play coincides with the Stackelberg, respectively, Cournot outcome roughly half of the time. The explanation for the behavior in the noisy-leader games is simple: no matter what the level of noise is, second movers tend to identify the signal with the action taken by first movers and play a best response against the assumed action. Whereas this is learned and exploited by first movers in the games with lower levels of noise, this is not the case in the game with high noise. Thus, for experienced players, no support for Bagwell's claim is found. However, there is some support for the so-called noisy Stackelberg equilibrium; an equilibrium in mixed strategies that converges to the Stackelberg outcome as the noise goes to zero. This equilibrium in mixed strategies (one of two such equilibria) has been emphasized by van Damme and Hurkens (1997).

The motivation of the present note is twofold: first, the boundaries of the results reported in HM are explored. If the noise level is strictly between 25 and 75%, the equilibria in mixed strategies no longer exist so that the Cournot equilibrium in pure strategies is the unique game theoretic prediction. What happens in that case? Will second movers still adapt to the signal? Will very high levels of noise finally cause experimental first movers to loose their commitment power, as theory predicts?

Secondly, the extent to which the findings of HM can be explained by the physical timing of decisions is explored. Second movers in HM knew — in addition to the imperfect signal — that the first mover has already taken his action. The mere knowledge about the order of moves has been shown in a number of studies (Cooper et al., 1993; Camerer et al., 1996; Rapoport, 1997; Güth et al., 1998) to affect behavior (in games in which first movers' actions are not observable) in such a way that first movers were favored. So, is it the case that followers in the experiments in HM simply grant leaders the first-mover advantage they would undoubtedly have if their moves were perfectly observable? Is it true that the (quality of the) signal does not matter that much and that all that matters to the results is the knowledge about the physical timing of moves?

Therefore, two further versions of the game mentioned above are implemented: in one version the level of noise is so high that there is no equilibrium in mixed strategies and in another version second movers do not receive a signal at all; they simply know that the first mover has already taken his action. [2] The latter treatment is a further investigation of the positional order protocol that was employed in the studies by Cooper et al., 1993; Camerer et al., 1996; Rapoport, 1997 and Güth et al., 1998. If subjects continue to play near the Stackelberg equilibrium in both cases, this would question the support for the noisy Stackelberg equilibrium found in HM.

[2] Note that this treatment corresponds with a situation in which the noise level equals 50%.

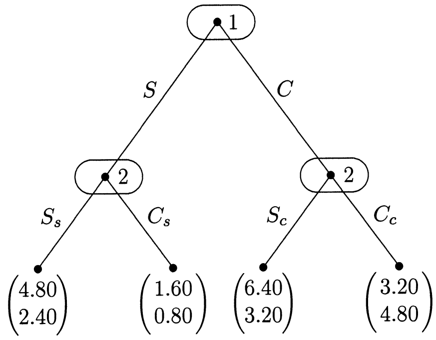

Fig. 1. The game in case of a perfect signal.

Section 2 presents an analysis of the implemented game, introduces the experimental design, and states the hypotheses. In Section 3 the experimental methods and procedures are described. The results of the experiments are presented in Section 4. Finally, the findings will be discussed in Section 5.

2. Analysis, experimental design, and predictions

A two-player game which is similar to the example provided by Bagwell (p. 272) was studied. The first mover (or Stackelberg leader) can choose between S and C. Afterwards the second mover (or follower) receives a signal about the leader's decision. The signal is either s or c. After each signal the follower has two choices called S_s and C_s in case the signal was s and S_c and C_c in case the signal was c. Fig. 1 shows the extensive form game for the case of a perfect signal.

Let $\varepsilon = \text{prob}(c|S) = \text{prob}(s|C)$ be the probability of receiving the wrong signal. If $\varepsilon = 0$ the strategy vector $(S, (S_s, C_c))$ is the unique subgame-perfect equilibrium. As Bagwell has shown, as soon as $\varepsilon > 0$ (and $\varepsilon < 1$), i.e. as soon as there is even the slightest amount of noise, the unique equilibrium in pure strategies is the Cournot equilibrium $(C, (C_s, C_c))$. Here the leader chooses C and — expecting this — the follower ignores his signal and always chooses C. For $0 < \varepsilon < 1/4$, the pure Cournot equilibrium is accompanied by two equilibria in mixed strategies:

$$\text{prob}(S) = 1 - \varepsilon, \quad \text{prob}(S_s) = 1 \quad \text{and} \quad \text{prob}(S_c) = \frac{1 - 4\varepsilon}{2 - 4\varepsilon} \tag{1}$$

and

$$\text{prob}(S) = \varepsilon, \quad \text{prob}(S_s) = \frac{1}{2 - 4\varepsilon} \quad \text{and} \quad \text{prob}(S_c) = 0. \tag{2}$$

Note that the mixed strategy equilibrium (1) converges to the Stackelberg outcome and the mixed strategy equilibrium (2) converges to the Cournot outcome as the noise, ε, goes to zero. If, however, $\varepsilon \in (0.25, 0.75)$ the Cournot equilibrium in pure strategies is the unique game theoretic solution of the game. This can be seen most easily by inspecting the strategic form of the above game.

Two versions of this game were implemented. In treatment NOISE, the parameter $\varepsilon = 0.4$, i.e. the probability of receiving the wrong signal is equal to 40%.[3] In treatment POP (POSITIONAL ORDER PROTOCOL), after the first mover has made his choice, the second mover receives no signal at all. From a game theoretic perspective, the mere knowledge of physical timing is irrelevant. Thus, the game played in treatment POP is equivalent to the game in which both players decide simultaneously. The unique Nash equilibrium in that case is both players choosing action C.

Note that in treatment POP even the concept of 'virtual observability' predicts that both players will choose C. The latter concept has been named by Camerer et al. (1996, p. 5) and was stated in the following way: "Fix a game of imperfect information in which players do not observe earlier moves. Erase the information sets and compute the subgame-perfect equilibria. Then restore the information sets and check if the subgame-perfect equilibrium the first mover prefers is a Nash equilibrium in the restored game. If so, play that equilibrium. If not, ignore timing and play a Nash equilibrium." As already noted by Camerer et al. 'virtual observability' does not select the Stackelberg equilibrium in the game played in treatment POP. This is true, since the (unique) subgame-perfect Stackelberg equilibrium $(S, (S_s, C_c))$ that arises when actions are perfectly observable fails to be a Nash equilibrium in the restored game, i.e. if players decide simultaneously. Thus, the positional order protocol cannot induce non-equilibrium play, a claim supported by the findings of Güth et al. (1998).

In both treatments the game is played for 10 successive rounds with full anonymity between subjects employing a random matching procedure ensuring that nobody would meet the same opponent twice.

Summarizing, the theoretical predictions lead us to expect

2.1. Prediction A

In both treatments, most outcomes will coincide with the Cournot outcome.

If — in contrast to the theoretical predictions — the timing affects the outcomes in both games in such a way that the first mover is favored, one should expect

2.2. Prediction B

A considerable fraction of the outcomes will coincide with the Stackelberg outcome.

3. Method and procedure

The experiments reported in this study were conducted at Humboldt University in November 1998 where 46 subjects participated in the two sessions. Out of them, 24 subjects were allocated to treatment NOISE and another 22 subjects to treatment POP. The participants were undergraduate students of economics or business administration.

[3] In the four treatments reported in HM ε was, respectively, set equal to 0, 0.01, 0.1 and 0.2.

The experiments were run with pen and paper. Subjects were seated in large lecture rooms with enough space between them to rule out communication. The randomly assigned roles were fixed throughout the experiment. The instructions [4] informed participants that there would be 10 rounds of the experiment, with individual feedback between the rounds, and that the matching would be random, but that nobody would meet the same opponent twice. Sessions lasted for about 1 h. Subjects' average total earnings were DM 43.17. [5]

The frame of both treatments was identical and as neutral as possible. The game was illustrated by a graph. Players were labeled A (first mover) and B (second mover), and choices were simply labeled (left) and (right) for the first mover, and L and R for the second mover. In treatment NOISE, first movers received a small white sticker and an envelope. They had to write down their decision on the sticker. Then they stuck it inside the envelope and wrote their codenumber on the envelope. After that subjects carried out the chance move by drawing numbered chips out of an urn containing 100 chips. Depending on the chosen chip, the experimenter wrote the signals on the envelopes, which were then sealed and collected. [6] The sealed envelopes were then handed out to the followers, who had to write code numbers and decisions on them. When all follower subjects had made their decisions, they were allowed to open the envelopes to learn about the actual decisions of their partners. After that, the envelopes were passed back to the leaders in order to inform them about the reaction of the followers. This completed a round.

Treatment POP was conducted similarly with the exception that there was no chance move and that second movers did not get any information about the first mover's choice.

4. Results

Table 1 summarizes the results of the two treatments. For each round, the table shows the total absolute frequencies of first and second movers' decisions at their respective information sets. The two bottom lines show aggregate choices across rounds.

Consider treatment NOISE: in the first round, two thirds of the first movers commit themselves to the Cournot action C while one third choose action S. Only 3 of the 12 second movers do not decide according to the Nash equilibrium prediction in round one by choosing S_s after receiving signal s. In the course of the session, first movers continue to favor action C over action S. In the tenth round, all first movers have learned to choose action C. With regard to second movers most subjects who receive signal c react by choosing action C_c, whereas subjects who receive signal s split roughly half and half between S_s and C_s (see also the bottom line of Table 1). However, in the tenth round, only three second movers violate the unique Nash equilibrium prediction by choosing action S_s after receiving signal s. Thus, subjects in this treatment appear to adjust to the Cournot equilibrium. [7]

[4] The instructions are available from the author upon request.

[5] Note that subjects in treatment NOISE received an additional flat payment of DM 10 to compensate them for the longer time they had to spend.

[6] The second mover got the wrong signal if the chip showed the numbers from 61 to 100 otherwise she got the right signal.

[7] Looking at individual decisions in treatment NOISE we observe that one half (three quarters) of the first movers chose the equilibrium action C in 8(7) or more rounds. However, given last round behavior, individual convergence to equilibrium predictions with regard to second movers seems to require more than 10 rounds.

Table 1
Summary of experimental results: treatment NOISE (left) and treatment POP (right)

Round	NOISE						Wrong signals	POP	
	S_s	S	C_s	S_c	C	C_c		S	C
1st	3	4	3	–	8	6	$2S \to c$	1	10
							$4C \to s$	4	7
2nd	3	6	3	3	6	3	$3S \to c$	2	9
							$3C \to s$	2	9
3rd	4	3	2	1	9	5	$3C \to s$	1	10
								4	7
4th	1	2	2	1	10	8	$2S \to c$	–	11
							$3C \to s$	4	7
5th	1	3	3	4	9	4	$1S \to c$	2	9
							$2C \to s$	2	9
6th	4	3	3	–	9	5	$1S \to c$	2	9
							$5C \to s$	–	11
7th	2	4	1	2	8	7	$3S \to c$	2	9
							$2C \to s$	–	11
8th	4	1	4	1	11	3	$7C \to s$	2	9
								–	11
9th	4	2	1	2	10	5	$3C \to s$	–	11
								–	11
10th	3	–	4	–	12	5	$7C \to s$	–	11
								1	10
Aggregate choices	29	28	26	14	92	51		12	98
								17	93

Next consider treatment POP: with regard to first movers, behavior is rather stable over the rounds. Over all rounds, at most 2 of 11 first movers choose action S. The S-choices in rounds five to eight do not stem from the same subjects.[8] In the last two rounds, all first movers choose action C. Regarding the behavior of second movers, during the first five rounds, there are few subjects choosing S rather than C. However, from round 6 on — with only one exception — all second movers choose action C. It is interesting to note that first movers seem to know that "anteriority" is not advantageous in that game since they do not even seriously attempt to select the Stackelberg outcome (see Table 1).

Summarizing these results one has the foloowing observation.

Observation 1: in both treatments subjects learn to play the unique Nash equilibrium; play clearly converges to the Cournot outcome.

Note that in treatment POP we also observe convergence on the individual level (see Table 1 and footnote 8). In all, the experimental results strongly support Prediction A and falsify Prediction B.

[8] There are 5 out of 11 first movers in treatment POP who choose C in all 10 rounds. Three (respectively two) first movers choose C in nine (respectively eight) rounds. One subject apparently employed a random strategy choosing five times S and five times C.

5. Discussion

Huck and Müller (2000) found that — in contrast to Bagwell's prediction — first movers (in a simple experimental market) do not lose their commitment power in the presence of noise. Instead, they found support for the so-called noisy Stackelberg equilibrium, which favors the first mover, as argued by van Damme and Hurkens. The motivation of the present study is to assess this finding and to explore whether the results can be partially explained by the fact that second movers — although not knowing exactly what happened — do know that first movers have already decided. This question arises because in a number of other studies, the physical timing of decisions serves as a selection device favoring the player who moves first. However, the results of this study point to the following facts. First, the quality of the signal and not (the knowledge of) the timing of moves matters: if the quality of the signal is very low, so that the noisy Stackelberg equilibrium no longer exists, play clearly converges to the Cournot outcome.[9] Second, the results of the previous study cannot be explained by a timing effect favoring the first mover: if second movers do not receive any information about the first movers' action, play clearly converges to the Cournot outcome. Taken together, these results suggest that the evidence for the noisy Stackelberg equilibrium found in Huck and Müller (2000) should be taken seriously. Furthermore, the results of treatment POP confirm the conjecture of Camerer et al. that 'virtual observability' does not select the Stackelberg outcome in the game at hand. On a more general level, this result provides further evidence that the positional order protocol cannot induce non-equilibrium play (Güth et al., 1998).

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[9] As a referee suggested (and I agree), it would be interesting to know more about the domain of games where this insight is relevant. However, since in other games one would naturally be interested in also considering small levels of noise, exploring this issue requires a fully fledged study of its own.

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