Hybridization strategies for routing problems with pairwise synchronization constraints

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This research is motivated by a problem situation commonly encountered in the area of field staff routing and scheduling. It concerns the issue of pairwise route synchronization in both space and time. Many companies plan individual daily tours for each of their field employees since most tasks require only a single staff member. However, both in the service technician field and in the area of mobile care, for some of the tasks to be completed, two staff members are necessary [1]. Pairwise time and space route synchronization introduces timely interrelationships between all those routes serving a client requiring a synchronized visit.

We consider two routing problems with pairwise synchronization constraints. The first problem is a generic problem based on the well-known vehicle routing problem with time windows (VRPTW) [3]. The second problem is motivated by a real-world problem encountered by an Austrian infrastructure and maintenance service provider: the service technician routing and scheduling problem (STRSP) [2], which is closely related to the site-dependent VRP. In the following, we denote the first problem as VRPTW with pairwise synchronization (VRPTWPS) and the second problem as STRSP with pairwise synchronization (STRSPPS).

We propose a hybrid method in order to deal with pairwise route synchronization constraints. The metaheuristic component consists of an adaptive large neighborhood search (ALNS) algorithm which corresponds to the ALNS of [2]. In order to deal with pairwise synchronization issues, we integrate an exact component into the ALNS. During the execution of the ALNS, time windows of synchronized tasks are narrowed to points in time. The exact synchronization component is used to improve this point in time for each synchronized task individually in every newly generated solution if it constitutes a new best solution. In addition, it is employed with a certain probability (controlled by a frequency parameter $i_{freq}$), regardless of the quality of the newly generated solution.

In the synchronization component, for each synchronized task or customer, we first determine the two routes serving it ($r_1$ and $r_2$). Then, we reset the time window of the task to its original value and we simultaneously optimize its positioning on both routes. We model and solve the underlying optimization problem in three ways. In the MIPit approach, we formulate it in terms of a mixed integer program. In the CGit approach, we use a set-partitioning type formulation that is populated by as many routes as there are feasible insertion positions of the synchronized task in $r_1$ and $r_2$. In the DPit approach, the synchronization problem is solved by means of dynamic programming. We first construct a graph $G_{DP}$ of $|r_1| \times |r_2| + 1$ nodes. Each of the nodes in $G_{DP}$ corresponds to one vertex of $r_1$ and one vertex of $r_2$. If the nodes are given in a grid like layout, horizontal arcs correspond to moving to the successor vertex in $r_2$ while staying at the same vertex in $r_1$; vertical arcs correspond to moving to the successor vertex in $r_1$ while staying at the same vertex in $r_2$. Diagonal arcs correspond to moving to the successor vertex in both routes. Diagonally situated nodes may also be connected via a detour through the synchronized node. Thus, each node in $G_{DP}$ is connected to at most four nodes. However, not all four arcs are necessary at all nodes. We solve the shortest path problem on a reduced graph by means of a label setting algorithm. Extensive tests showed that the last approach outperforms the previous two in terms of run time.

In our tests, we used two types of data sets: the well known VRPTW data set [3] and the one proposed in [2] for the STRSP. In our case, we assume that every tenth customer or task demands a synchronized visit. We test several different settings for $i_{freq}$ and compare it to a fixed point in time setting (i.e. ALNS without the synchronization component), and the optimal solutions (for instances with 25 and 50 customers/tasks).

Our results indicated that the proposed hybrid method works well: compared to the fixed point in time setting, solution quality can be improved by 4.5% on average. Using DPit together with an enhanced...
insertion routine (allowing modifications of the fixed point in time setting) within the ALNS, solution quality can be further improved (by about 1.3%). Finally, the deviation from the optimal solutions, where known, amounts to only 0.3%, on average.

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**References**

