Electricity Swing Options

Behavioral Models and Pricing

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- Supply contract
- Exercise rights and constraints
  - Predefined exercise times $\tau_i, i \in \{1, 2, \ldots n\}$, $T_1 \leq \tau_1 < \tau_2 < \tau_3 < \cdots < \tau_n \leq T_2$
  - Variation of consumed amount of electricity (Up-Swing or Down-Swing)
  - Swing number constraint N
  - Freeze time constraint $\Delta\tau$
  - Local boundaries for the requested volume $d_i - d_0 \in [l_i^1, l_i^2) \cup (l_i^3, l_i^4], (l_i^1 \leq l_i^2 \leq 0 \leq l_i^3 \leq l_i^4)$.
  - Global constraints for the total volume $D := \Sigma_{t=1}^{T} d_i \in [Min, Max]$
  - Penalty function e.g.

$$\varphi(D) = \begin{cases} C_1 & \text{if } D < Min \\ 0 & \text{if } Min \leq D \leq Max \\ \xi_t(D - Max) & \text{if } D > Max \end{cases}$$

- Help functions

$$\chi_j^+ = \begin{cases} 1 & \text{Up-swing at time } \tau_j \\ 0 & \text{else} \end{cases}$$

$$\chi_j^- = \begin{cases} 1 & \text{Down-swing at time } \tau_j \\ 0 & \text{else} \end{cases}$$

$$d_j^+ = \begin{cases} d_j - d_0 & \text{when } \chi_j^+ = 1 \\ 0 & \text{else} \end{cases}$$

$$d_j^- = \begin{cases} d_j - d_0 & \text{when } \chi_j^- = 1 \\ 0 & \text{else} \end{cases}$$

- Contract constraints

$$\chi_j^+, \chi_j^- \in \{0,1\}$$

$$0 \leq \chi_j^+ + \chi_j^- \leq 1 \text{ for all } 1 \leq j \leq n$$

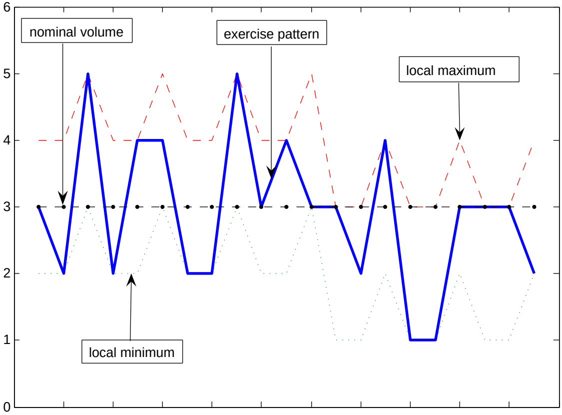
$$(\chi_i^+ + \chi_i^-) + (\chi_j^+ + \chi_j^-) \leq 1 + \frac{\tau_j}{\tau_i + \Delta\tau} \text{ for all } 1 \leq i < j \leq n$$

$$0 \leq \Sigma_{j=1}^n (\chi_j^+ + \chi_j^-) \leq N$$

$$l_j^3 \chi_j^+ \leq d_j^+ \leq l_j^4 \chi_j^+ \text{ for all } 1 \leq j \leq n$$

$$l_j^1 \chi_j^- \leq d_j^- \leq l_j^2 \chi_j^- \text{ for all } 1 \leq j \leq n$$

Figure: Example of feasible exercise path (blue line)



Proposition

The set of feasible exercise paths

$$\mathcal{D} = \mathcal{D}(d_0, n, N, L^{min}, L^{max}, Min, Max, \Delta i)$$

- d_0 *baseline path*
- n *number predefined exercise times*
- N *number of allowed swings*
- L^{min} *and* L^{max} *the sets of local restrictions* l_t^{max} *and* l_t^{min}
- Min *and* Max *for global restrictions*
- Δi *the freeze time as number of whole periods between exercise times*

is under the following assumptions

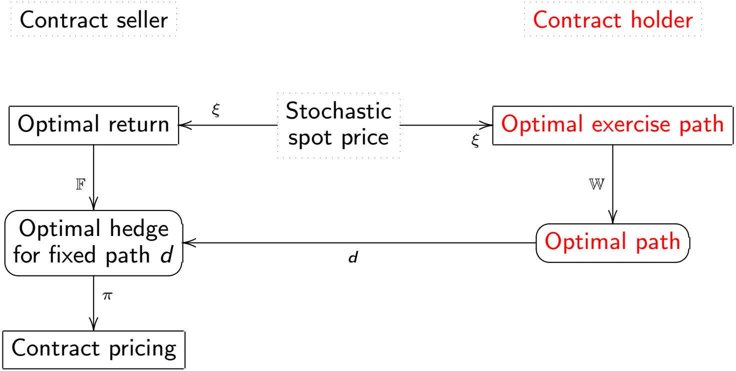
- $N = n$
- $\Delta i_R = 1$

bounded and convex.

- Determining factors
    - spot price
    - behavior of the holder
- Stylized Facts about electricity market
    - electricity is not storable ⇒
        - the spot price is very sensitive to changes in the demand and suply (temperature changes, power plant breakdowns) ⇒ high volatility and spikes
        - electricity market is not a complete market
    - electricity transportation is very expensive ⇒
        - local phenomena
- Possible behavior of the holder
    - own consumption covering
    - profit/speculation

- A swing option is not a typical classical option because there is no unique rational exercise strategy
- The "No-Arbitrage Principle" is a good method for rational pricing of classical options, but it is not fully applicable to swing options because of:
    - electricity is not storable $\Rightarrow$ hedging with the underlying is not possible
    - electricity market is not a complete market
    - there are no rational exercise strategy of the holder
- A "fair" price for the swing contract must be based on a model for the exercise strategy (stochastic game situation)

Valuation flow chart and the relevance of the behavioral model



Contract seller

Contract holder

Optimal return

Stochastic spot price

Optimal exercise path

Optimal hedge for fixed path *d*

Optimal path

Contract pricing

- Finite probability space $(\Omega, \mathcal{P}(\Omega), \mathbb{P})$, $\Omega = \{\omega_1, \dots, \omega_S\}$
- A filtration on this space $\mathbb{F} = (\mathcal{F}_t)_{t\in\{1,\dots,T\}}$, with $\mathcal{F}_1 = \{\emptyset, \Omega\}$ and $\mathcal{F}_T = \mathcal{P}(\Omega)$
- An adapted to $\mathbb{F}$ spot price process $(X_t)_{t\in\{1,\dots,T\}}$
- At each node of the corresponding to $\mathbb{F}$ tree, the holder chooses his exercise volume to maximize his expected profit under the constraints coming from the contract and his past decisions. Further his decision in moment t may be dependent on the information about the spot prices only up to moment $t-1$ which means the exercise process $(D_t)_{t\in\{1,\dots,T\}}$ should be a previsible regarding the filtration $\mathbb{F}$ process. Thus we can represent the optimal exercise process as the solution of the following multistage stochastic program.

$$(D_t^{max}) \in \operatorname{argmax}\{\mathbb{E}_{\mathbb{P}} Z((D_t),(X_t)) : (D_t) \in \mathcal{D}, (D_t) \text{ previsibel regarding } \mathbb{F}\}$$

Here Z is the (random) profit if the exercise process (D_t) is chosen.

$$Z((D_t),(X_t)) := \sum_{i=t}^{T} (X_t - K) D_t$$

where K is the strike price in the contract.

- The stochastic program has the following LP form

$$
\left\| \begin{array}{l}
\text{Max: } \sum_{s \in \mathcal{S}} p_s \sum_{n \in s} (X_n - K) d_n = \sum_{n \in \mathcal{N}} (X_n - K) (\sum_{s \in S(n)} p_s) d_n \\
\text{subject to:} \\
d_n - d_0 \leq l^4_{t(n)} \quad \forall n \in \mathcal{N} \\
d_n - d_0 \geq l^1_{t(n)} \quad \forall n \in \mathcal{N} \\
\sum_{n \in s} d_n \leq Max \quad \forall s \in \mathcal{S} \\
\sum_{n \in s} d_n \geq Min \quad \forall s \in \mathcal{S}
\end{array} \right.
$$

- where
  - $\mathcal{N}$ The set of all nodes
  - $\mathcal{S}$ The set of all scenarios
  - p_s The occurrence probability of scenario s
  - X_n The spot price in node n
  - $t(n)$ The time horizon of node n
  - $S(n)$ The set of all scenarios to which node n belongs
  - $n(s,t)$ The node to scenario s and time horizon t

- The value of the maximum as well the d_n, $n \in \mathcal{N}$ for that this maximum is attained are functions of K which we denote with E and D

$$E(K) := \begin{cases} \mathbb{R} \to \mathbb{R} \\ K \to \max\{\sum_{n \in \mathcal{N}} (S_n - K)(\sum_{s \in S(n)} p_s) d_n : d_n \in \mathcal{D}\} \end{cases}$$

$$D(K) := \begin{cases} \mathbb{R} \to \mathbb{R}^{|\mathcal{N}|} \\ K \to \ \operatorname{argmax} \{\sum_{n \in \mathcal{N}} (S_n - K)(\sum_{s \in S(n)} p_s) d_n : d_n \in \mathcal{D}\} \end{cases}$$

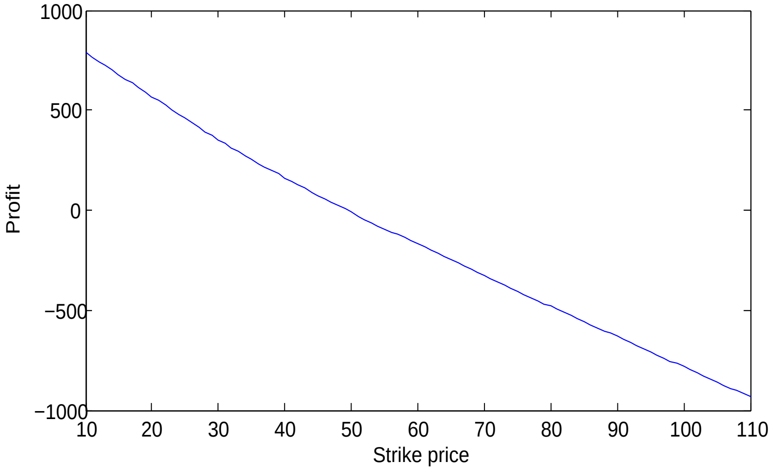
Proposition

The function $D(K)$ is a piecewise constant function.

Proposition

The real function $E(K)$ is a continuous piecewise linear decreasing convex function.

Figure: The dependence of the expected maximal profit from the strike price



- Further analysis of the dependence of the expected maximal achievable profit from the flexibility of the contract we take a special form of our linear program

$$\begin{array}{l} \text{Maximize: } \sum_{s\in\mathcal{S}} p_s \sum_{n\in s}(S_n - K)d_n = \sum_{n\in\mathcal{N}}(S_n - K)(\sum_{s\in S(n)} p_s)d_n \\ \text{subject to:} \\ d_n \le d_0 + l \quad \forall n \in \mathcal{N} \\ d_n \ge d_0 - l \quad \forall n \in \mathcal{N} \\ \sum_{n\in s} d_n \le L + Td_0 \quad \forall s \in \mathcal{S} \\ \sum_{n\in s} d_n \ge L - Td_0 \quad \forall s \in \mathcal{S} \end{array}$$

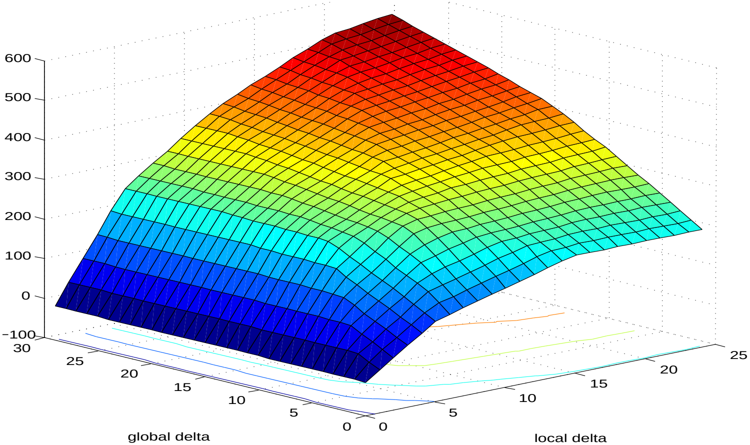
- In an analogous manner as we did this for $E(K)$ we can define the following function:

$$E(l, L) := \left\{ \begin{array}{l} \mathbb{R}^2 \to \mathbb{R} \\ K \to \max\{\sum_{n\in\mathcal{N}}(S_n - K)(\sum_{s\in S(n)} p_s)d_n : d_n \in \mathcal{D}(l, L)\} \end{array} \right.$$

Proposition

The real function $E(l, L)$ is a continuous piecewise linear non-decreasing concave function in both arguments.

Figure: The function $E(l, L)$ for some example data



- The option holder uses the price information from the spot market when taking his exercise decision ⇔ The issuer has to make his offer before the beginning of the contract (information asymmetry)
- The risk management strategy for the issuer is to hedge the contract by other available contracts
- Problem formulation

$s = 1, \ldots, S$	spot-price scenarios
$t = 1, \ldots, T$	the time granularity of the decision period
$m = 1, \ldots M$	the indices of the hedging instruments
$\xi = (\xi_{s,t})$	$\mathbb{R}^{S \times T}$ matrix of the spot prices
$p = (p_s)$	$\mathbb{R}^{S}$ vector of scenario probabilities
$\pi = (\pi_m)$	$\mathbb{R}^{M}$ price vector of the hedging instruments
$\tau(m, t)$	$\mathbb{R}^{M \times T}$ matrix of volumes of hedge contract m
$d = (d_{s,t})$	$\mathbb{R}^{S \times T}$ matrix of scenario dependent demands
$x = (x_m)$	$\mathbb{R}^{M}$ (decision) vector of amount of m-th hedge
K	is the Strike price per unit in the contract

- Expectation-neutrality (avoid unboundedness of the hedging problem)

$$\pi_m = \sum_{t=1}^{T} \tau(m, t) \sum_{s=1}^{S} p_s \xi_{s,t}$$

- Given a hedge x, the *surplus/shortage* in hour t is given by

$$\sum_{m=1}^{M} x_m \tau(m,t) - d_{s,t}.$$

- The surplus/shortage has to be cleared on the spot market. The random monetary value of the surplus/shortage position is the *value process*

$$\Delta_{s,t}(x) = \xi_{s,t}[\sum_{m=1}^{M} x_m \tau(m,t) - d_{s,t}]$$

- The total revenue $Y_s(x,K)$ is a random variable. It takes the value

$$Y_s(x,K) = K\sum_{t=1}^{T} d_{s,t} + \sum_{t=1}^{T} \xi_{s,t}\left[\sum_{m=1}^{M} x_m \tau(m,t) - d_{s,t}\right] - \sum_{m=1}^{M} \pi_m x_m$$

with probability p_s.

- Due to the expectation-neutrality, the expected profit does not depend on the hedging decisions

$$\mathbb{E}[Y(x,K)] = \sum_{s=1}^{S} p_s \sum_{t=1}^{T} d_{s,t}[K - \xi_{s,t}]$$

- The option seller wants to limit the probability of making losses. He offers the contract only if this probability is smaller than α, where α is 5-10%. The random return variable Y is only acceptable to him, if $P\{Y < 0\} \leq \alpha$. Among all acceptable return variables, he chooses the smallest price for the option and solves the so called *quantile restricted option price* problem

$$\min K \quad \text{subject to} \quad P\{Y(x, K) < 0\} \leq \alpha$$

- Replacing the $P\{Y < 0\} \leq \alpha$ by the stronger condition $\mathbb{A}\mathrm{V@R}_\alpha(Y) \geq 0$, where

$$\mathbb{A}\mathrm{V@R}_\alpha[Y] = \frac{1}{\alpha} \int_0^\alpha G_Y^{-1}(u)\, du$$

  with $G_Y(u) = P\{Y \leq u\}$. Notice that $\mathbb{A}\mathrm{V@R}_\alpha(Y) \geq 0$ implies that $G_Y^{-1}(\alpha) \geq 0$, i.e. $P\{Y < 0\} \leq G(0) \leq \alpha$.
- *$\mathbb{A}$V@R restricted option price* problem

$$\min K \quad \text{subject to} \quad \mathbb{A}\mathrm{V@R}_\alpha[Y(x, K)] \geq 0$$

- Structure of the program ((Rockefellar and Uryasev, 2000) and (Pflug, 2000))

$$\left\| \begin{array}{l} \text{Minimize (in } a, x, z) : \;\; K \\ a - \frac{1}{\alpha} \sum_{s=1}^{S} p_s z_s \geq 0 \\ \sum_{t=1}^{T} [\sum_{m=1}^{M} x_m \tau(m, t) - d_{s,t}] \xi_{s,t} \\ + \sum_{t=1}^{T} d_{s,t} K - \sum_{m=1}^{M} x_m \pi_m - a + z_s \geq 0 \\ x, z > 0 \end{array} \right.$$

- The option holder determines his exercise pattern by solving, for a fixed unit price K, the problem

$$(D_t^{max}) \in \mathrm{argmax}\{\mathbb{E}_{\mathbb{P}} Z_K((D_t),(X_t)) : (D_t) \in \mathcal{D}, (D_t) \text{ previsibel regarding } \mathbb{F}\} \tag{1}$$

- The option seller solves for a given scenario-dependent demand d the problem

$$\min K \quad \text{subject to} \quad \mathbb{A}\mathrm{V@R}_\alpha[Y(x,K)] \geq 0 \tag{2}$$

- We call a pair (K^*, d^*) an *equilibrium point* or a *Nash-equilibrium*, if d^* is the solution of (1) for K^* and K^* is the solution of (2) for given d^*.
- We have (1) is a linear, hence concave maximization problem and (2) is a linear, hence convex minimization problem ⇒ the existence of an equilibrium point is guaranteed (Rosen, 1965).

- Stochastic multistage game theoretical behavioral model for the holder
    - Dynamical choice of exercise patterns
    - Having the information about spot price development on a binary scenario tree, we suppose the possibility to change the future part of the exercise path at every moment, to maximize the expected profit
- Pricing model of scenario dependent demands
    - Evaluating the price of a variable demand pattern given a number of hedging instruments
- Combination of the behavioral with the fixed demand pricing model (The contract price influence the Option holder behavior as well the behavior influence the contract price)

- Extension of the behavioral model to holders which act as well as dealer and customer
- Analysis the price as a function of the holder's degree of disposition to speculate
- To solve the behavioral model for other kind of Swing Option restrictions like:
    - $l_i \leq \sum_{i=1}^{N} d_i \leq L_i$

Thank you for your attention!