Swing Options in Electricity Markets:

Behavioural Models and Pricing

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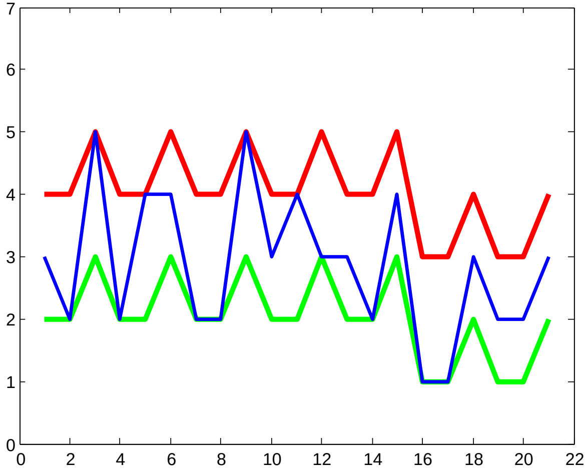
July 10, 2006

- Supply contract
- Exercise rights and Restrictions
  - Predefined exercise times $t = 1, \dots, T$
  - Variation of consumed amount of electricity (Up-Swing or Down-Swing)
  - Restriction of the total number of swings N
  - Freeze time Δt_R
  - Local restrictions of the consumed amount per swing l_t^{max} and l_t^{min}
  - Global restriction of the total volume $D := \Sigma_{t=1}^{T} d_i \in [Min, Max]$ or penalty function e.g.

$$\varphi(D) = \begin{cases} C_1 & \text{if } D < Min \\ 0 & \text{if } Min \leq D \leq Max \\ \xi_t(D - Max) & \text{if } D > Max \end{cases}$$

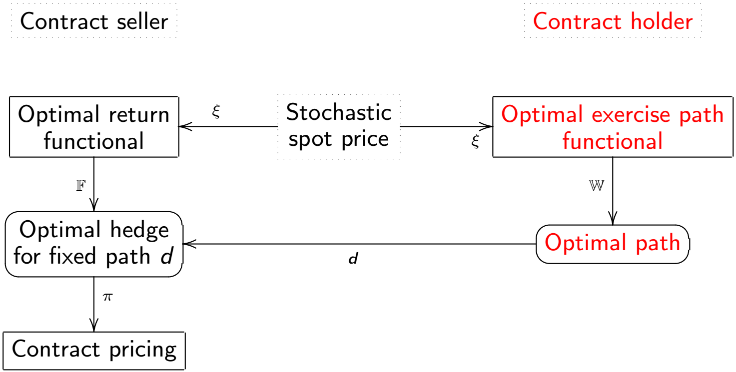
Figure: Example of feasible exercise path (blue line)



- Determining factors
  - spot price
  - behaviour of the buyer
- Stylized Facts about electricity market
  - electricity is not storable ⇒
    - the spot price is very sensitive to changes in the demand and suply (temperature changes, power plant breakdowns) ⇒ high volatility and spikes
    - electricity market is not a complete market
  - electricity transportation is very expensive ⇒
    - local phenomena
- Possible behaviour of the buyer
  - own consumption covering
  - profit/speculation

- A swing option is not a typical classical option because there is no unique rational exercise strategy
- The "No-Arbitrage Principle" is a good method for rational pricing of classical options, but it is not fully applicable to swing options because of:
  - electricity is not storable ⇒ hedging with the underlying is not possible
  - electricity market is not a complete market
  - there are no rational exercise strategy of the buyer
- A "fair" price for the swing contract must be based on a model for the exercise strategy (stochastic game situation)

Valuation flow chart and the relevance of the behavioural model



- The set of feasible exercise paths

$$D = D(d_0, n, T, N, L^{min}, L^{max}, Min, Max, \Delta i_R)$$

  - d_0 baseline path
  - n number predefined exercise times
  - N number of allowed swings
  - L^{min} and L^{max} the sets of local restrictions l_t^{max} and l_t^{min}
  - Min and Max for global restrictions
  - Δi_R the freeze time as number of whole periods between exercise times

  is under the folowing assumptions

  - $N = n$
  - $\Delta i_R = 1$

  bounded and convex.
- Knowing the properties of this set is helpful for
  - Generating random exercise paths in the simulation
  - Solving optimisation problems over this set
  - Existence questions about optimal exercise paths

- Let see the following example of 3 exercise paths and 3 possible hedging strategies with the pertaining costs:

	Hedging 1	Hedging 2	Hedging 3
Path 1	1	7	8
Path 2	2	6	3
Path 3	9	5	4
Max	9	7	8
Average	4	6	5

- Minimax strategy: If the contract seller want to minimize his maximal costs, he has to choose hedging strategy 2
- Min expectation strategy: If we assume the probabilities of $1/3$ for all exercise paths and he wants to minimize his expectation costs he should take strategy 1

- The constant sum matrix game: $C \in \mathbb{R}^{m \times n}$ of the costs (hedging costs + shortage costs - surplus profit)

$$\mathbf{C} := \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \dots & c_{mn} \end{pmatrix}$$

  - c_{ij} and c_{ij} are the costs when the customer takes path i and the seller choose strategy j

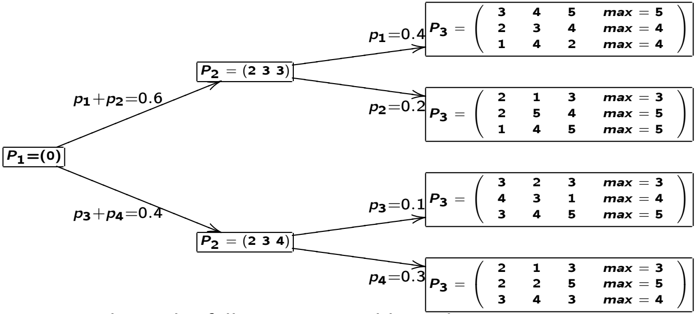
- The obvious relation between the two strategies and their prices

$$\pi_{exp} := \min_j \sum_{k=1}^{m} p_k c_{kj} \leq \min_j \max_i c_{ij} =: \pi_{minimax}$$

shows that the costs without assuming a probabilistic model for the behaviour of the buyer are higher

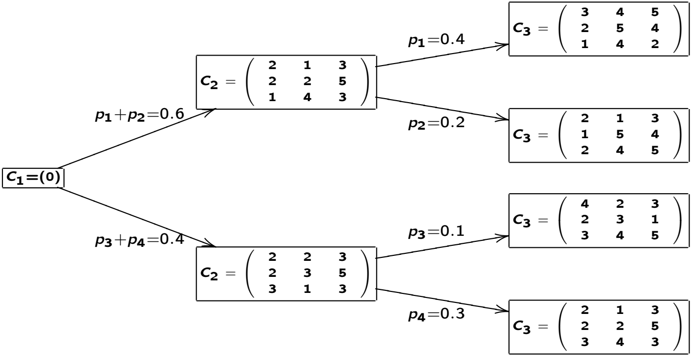
- The simple matrix game model is based on a couple of assumptions
  - The holder of the option takes his decision at the same moment as the seller chooses his hedging strategy(here and now)
  - The holder is acting against us like a player in a constant sum game (maximizing buyers costs) or is acting following some probability distribution
  - The corresponding cost are $\pi_{minimax}$ and π_{exp}
- A more sophisticated model should consider the following points
  - The holder of the option takes his decision much later then the the seller about his hedging strategy
  - In this later moment the holder has more information (wait and see) about the spot price market then the seller at the beginning (asymmetric game in time and information)
  - The holder is not acting directly against the seller but there could be an other relation between his strategy and the hedging strategy of the seller.

- A possible model could be the following
  - Discrete (in time) scenario tree of the spot price scenarios and there probabilities
  - In all decision moments of the holder he takes his decision trying to maximize the sum of the profit from the actual moment plus the expected maximal reachable profit at the later moments
  - A simple example could be the following model with 3 moments and a holder taking decision at moments 2 and 3



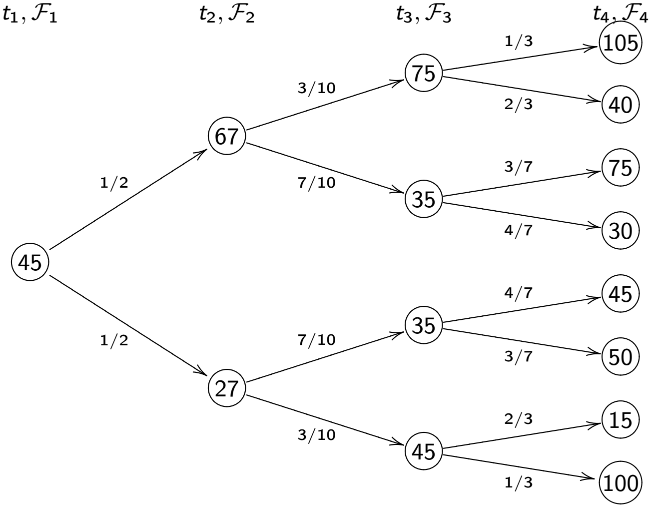
- This means we have the following 4 possible paths
  - 1: (3 2)
  - 2: (3 3)
  - 3: (3 3)
  - 4: (3 2)

- Let see the situation from the seller point of view. We assume he has 3 possible hedging strategies with the respective costs matrices depending on the scenario and holder strategy in all time moments



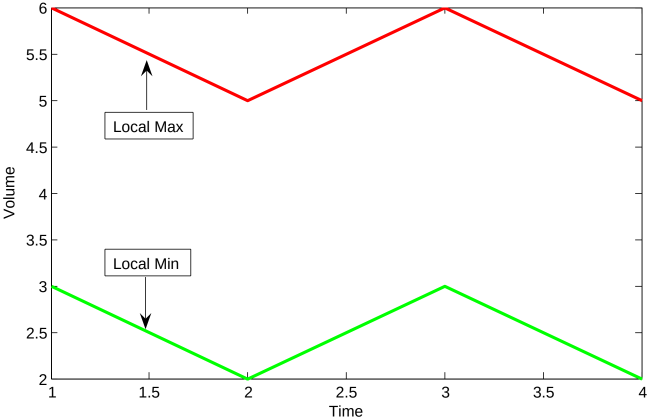
- If we assume the rational strategy of the holder from the above model we can find the optimal hedging strategy for the seller

Figure: Binary Spotprice Scenario Tree



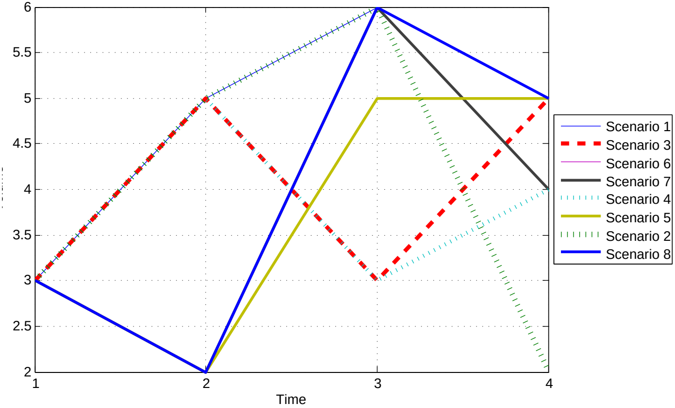
Global Max=25 and global Min=15

Figure: Swing Option Local Restrictions



Price per Unit = 46

Figure: The Solution for all 8 Scenarios



Contract price = 50.50

Thank you for your attention