Functions of Variable Bandwidth - a Time-Frequency Approach

PhD thesis topic proposal of Mag. Roza Aceska

February 27, 2007

member of the IK Modern Mathematical Analysis and Applications - Time-Frequency and Microlocal Analysis

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The well-developed theory of band-limited functions was induced by the real-life fact that frequencies higher then some finite cut-off practically are not relevant (due to the limited sensitivity of the measuring devices, for one instance). From a mathematical point of view, functions of limited bandwidth are extremely smooth; in fact, they are entire functions of time, hence such signals cannot have compact support. Real-world signals are, by necessity, time-limited, so it seems more appropriate to describe them as such; by the uncertainty principle, this implies using functions which are not band-limited. However, functions with unbounded spectrum are not a good fit as explained before - for a signal happening in reality.

The topic of this thesis is motivated by a suggestion by D.Slepian for resolving this seeming paradox. We consider functions of variable bandwidth (some authors label them as functions with time-varying bands, or, only approximately bandlimited). This thesis goes beyond the naive description of these signals as only smooth deformations of band-limited functions: we suggest to pursue functions with essential time-frequency support around a strip, bordered by a moderate function m(t), which mildly varies through time. We are not excluding the possibility that the Short-Time Fourier Transform (STFT) of the considered function has non-zero values outside this strip; but we minimize the importance of such occurrences with a particular, very strong weight. In that way, the spectrum would involve all possible frequencies, but with graded importance. Of course, the 'out of band' values if occurring - would lead to a reasonably large norm, due to the large weight. All this imposes working with co-orbit spaces, modulation spaces in particular, whose theory has been developed within the NuHAG group by H. Feichtinger and K. Gröchenig. Modulation spaces are described in terms of the behavior of the STFT of their elements. Atomic decompositions of Gabor-type for the members of these function spaces have already been derived, so a Gabor analysis approach to these functions of variable bandwidth is only natural. Using a weighted L2 norm on the STFT also implies that we are obtaining a reproducing kernel Hilbert space.

One of the goals of this thesis is to show that (our customized weight) co-orbit spaces are best so far for describing functions of variable bandwidth. Each of these spaces would certainly involve the appropriate bandlimited functions as well, when a suitable weight is applied. Some of the questions considered within this thesis would be: What are the conditions that ensure that a function belongs to one of these spaces? What is the smallest natural space that would contain a particular function? What is the criteria for two of these spaces to be equal? In addition, such co-orbit spaces are translation invariant and have atomic decompositions, which is a useful information. Among the tasks that need to be realized is to study the reconstruction from samples, which might not be a perfect reconstruction as in the bandlimited case. We expect the reconstruction to be possible and almost perfect - up to a fairly small error, if samples occur at least at the local Nyquist rate. The Hilbert space structure of the L2 co-orbit spaces would allow us to look for a minimal norm solution for the corresponding interpolation problem, that will also be considered.