

Test on ‘Econometric forecasting’

December 2007

1. A time series of five observations is given as $(x_1, \dots, x_5) = (1, 2, 3, 2, 1)$.
 - (a) Apply SES (*single exponential smoothing*) with the specification $\hat{x}_1 = x_1$, $\alpha = 0.5$. First determine $\hat{x}_2, \dots, \hat{x}_5$. [Hint: The formula for SES is: $\hat{x}_t = \alpha x_t + (1 - \alpha) \hat{x}_{t-1}$]
 - (b) Determine $\hat{x}_5(1)$ and $\hat{x}_5(2)$, i.e. the predictions for the unknown values x_6, x_7 .

2. You assume that the time series given in # 1 was generated by an ARMA process

$$x_t = 0.5x_{t-1} + \varepsilon_t + 0.5\varepsilon_{t-1},$$

for $t = 2, \dots, 5$, and you also assume $E(\varepsilon_t | \varepsilon_{t-1}, \dots) = 0$. You assume for simplicity $\varepsilon_1 = 0$.

- (a) Evaluate the model forecast $\hat{x}_3(1)$ as the conditional expectation $E(x_4 | x_1, x_2, x_3)$. [Hint: first determine ε_2 , then ε_3 . All numbers are given, the answer is a number, not a formula.]
 - (b) Suppose now you make the simplifying assumption that the data are generated by the AR(1) process $x_t = 0.5x_{t-1} + \varepsilon_t$. Determine $\hat{x}_3(1)$ and $\hat{x}_3(2)$ accordingly. Where do these forecasts converge to? Do you feel this is a realistic prediction, given the pattern in the five given observations?
3. A clever person suggests the following nonlinear time-series model as the data-generation mechanism of the # 1 data:

$$x_t = \begin{cases} x_{t-1} + 1 + \varepsilon_t & \text{if } x_{t-1} > x_{t-2} \\ x_{t-1} - 1 + \varepsilon_t & \text{if } x_{t-1} < x_{t-2} \end{cases}$$

with ε_t defined as ‘white noise’.

- (a) Do you have a creative alternative suggestion? Specify your own prediction (maybe just an intuitive guess) for the unknown x_6 .
 - (b) Assume you really consider the clever person’s model. How would you modify the specification of the error process? What would $\hat{x}_5(1)$ be?