Does the Quantity Theory of Money hold for the Euro Area?

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Contents

1 Introduction

The European central bank as well as the federal reserve do not control the monetary stock explicitly as their policy target. In contrast, proponents of the Quantity Theory of Money state that inflation is a monetary phenomenon and targeting money supply might be useful to achieve stable prices in the long run. Bachmeier, Swanson (2005) provide evidence that the Quantity Theory of Money could indeed help to forecast inflation in the US, so the theory might be still useful in monetary policy analysis. We use a cointegrated VAR to examine if the Quantity Theory of Money does hold in the euro area, i.e. if there is a long run relationship between real output, prices and money.

2 Quantity Theory of Money

The equation of exchange

$$M_t V_t = P_t Q_t \tag{1}$$

relates money supply (M_t) times the velocity (V_t) of money to the quantity of output produced (Q_t) times the price level (P_t). Taking the natural logarithms of equation (1) and rearranging yields:

$$ln(V_t) = ln(P_t) - ln(M_t) + ln(Q_t) \tag{2}$$

The Quantity Theory of Money states that V_t is stable in the long run. This is equivalent to the assumption that V_t is I(0) and implies a stable long-run relationship between M_t, Q_t and P_t. The Quantity Theory of Money can be examined for a specific currency union by testing for cointegration of the true values of M_t, Q_t and P_t. If the natural logarithms of the series M_t, Q_t and P_t are I(0) or I(1) we can use Johansen's procedure in order to test whether there is a cointegrating relationship between $ln(P_t)$, $ln(M_t)$ and $ln(Q_t)$ with a cointegrating vector $(1, -1, 1)$[1]. These considerations constitute the structure for this project. The remainder is organized as follows. The next section describes the data used. Section 4 outlines the used methods as well as the empirical findings while section 5 concludes.

3 Data

We obtained our data for the Euro Area from the International Monetary Fund - International Financial Statistics (IFS) and from the European Central Bank (ECB). In the remainder we use real output Y_t for the quantity of output Q_t. For the real output Y_t we use the gross domestic product volume index (2005=100) in constant prices of 2005 from the IFS database[2]. For the price level P_t we use the gross domestic product price deflator index (2005=100) from the IFS database. For the amount of money M_t we use an index variable (Dez. 2008=100) for the notional amount of M2 from the ECB[3]. All series are seasonally adjusted in the original data source. The notional amount of M2 is reported monthly so we transform it to quarterly data by taking averages. After transformation of M2 all series contain quarterly data ranging from period 1999:1 to period 2011:2. The three variables P_t, Y_t and M_t are plotted against time in Figure 1. For the further analysis we use the logarithm of the series.

[1] cf. also (Bachmeier, Swanson 2005: 573)

[2] Downloaded from Forschungsschwerpunkt Internationale Wirtschaft FIW Database Tool: `http://data.fiw.ac.at/FiwDat/FiwDatServlet`

[3] Downloaded from ECB Statistical Data Warehouse: `http://sdw.ecb.europa.eu/`

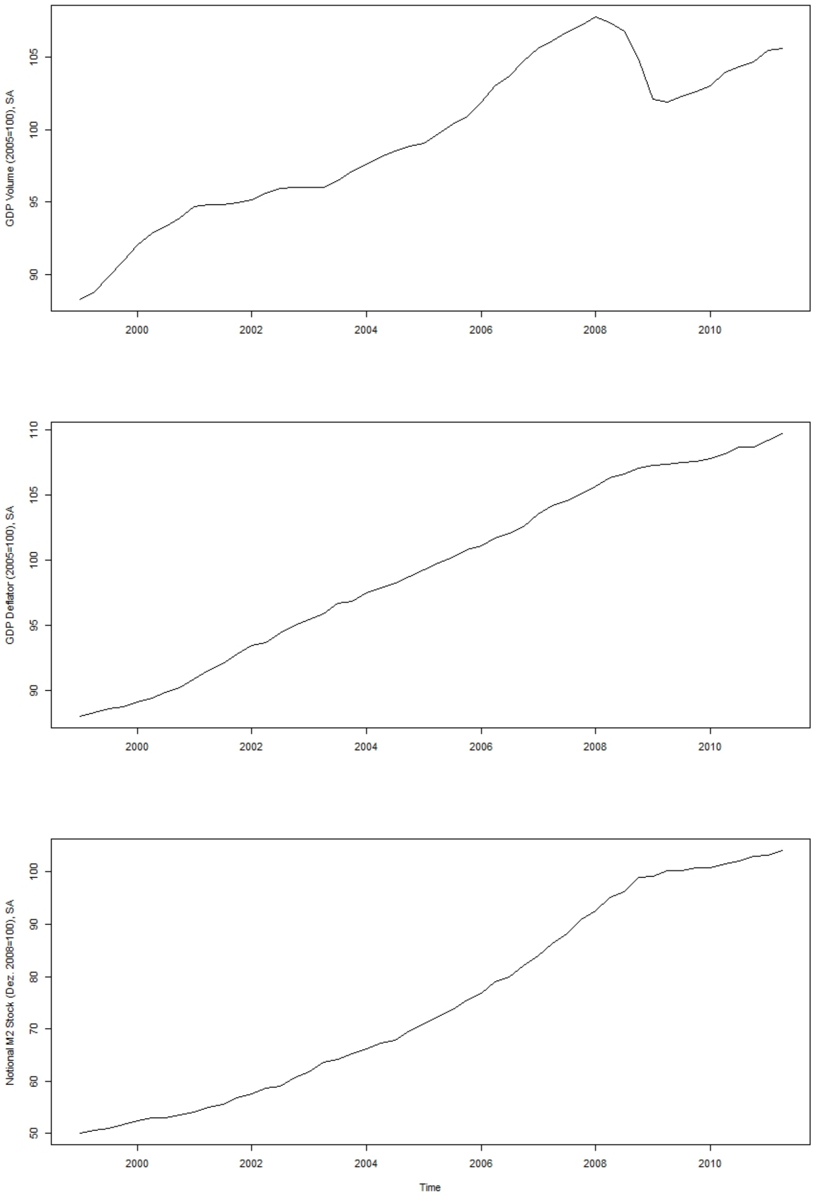

Figure 1: Quarterly level series Y_t, P_t and M_t from 1999:1 to 2011:2

4 Estimation

4.1 Integration order of the individual series

We use an Augmented Dickey Fuller test (ADF) and a Phillips Perron test (PP) to evaluate whether the level data are I(1), i.e. to examine the order of integration. The tests are described in the appendix. The decision upon lag selection order p for the ADF test is based on the Akaike information criteria (AIC) and on the Schwartz information criteria (SIC). We set the Newey-West (Bartlett) weights L for computation of the the PP model to $L =$*integer part of* $[4 * (N/100)^{0.25}] = 3$ where $N = 50$ is the number of observations. This specification for L=3 is in line with the suggestion of Brown, Cronin (2006)[4].
The results for the level and difference variables are summarized in Table 1. We use the ADF with trend and PP with trend in order to evaluate the integration order of the level variables $ln(Y_t)$, $ln(P_t)$ and $ln(M_t)$. The ADF as well as the PP do not reject the null hypothesis of unit root for the level variables $ln(Y_t)$, $ln(P_t)$ and $ln(M_t)$ at a 5 % significance level. In order to evaluate the integration order of the difference variables $\Delta ln(Y_t)$, $\Delta ln(P_t)$ and $\Delta ln(M_t)$ we use the ADF with constant and the PP with constant. The ADF does reject the null for the difference variables $\Delta ln(Y_t)$ and $\Delta ln(P_t)$ but does not reject the null for $\Delta ln(M_t)$ at a 5 % significance level. The PP does reject the null for all three variables.

	Real GDP	**GDP Deflator**	**Nominal M2**
Levels			
$ADF(AIC)$	-2. 65	-1.60	-1.83
	(1 lag)	(3 lags)	(2 lags)
$ADF(BIC)$	-2.65	0.54	-1.83
	(1 lag)	(0 lags)	(2 lags)
PP	-1.83	-0.06	-1.11
critical value: -3.50			
Differences			
$ADF(AIC)$	-2.97	-3.01	-1.50
	(0 lags)	(1 lag)	(4 lags)
$ADF(BIC)$	-2.97	-4.97	-1.60
	(0 lags)	(0 lags)	(1 lag)
PP	-3.04	-5.12	-5.57
critical value: -2.93			

Table 1: Unit Root Test

[4]They use L = smallest integer $\geq N^{0.25}$.

A priori we would expect money to be an I(1) process. The data indicates a change in mean of $\Delta ln(M_t)$ in the aftermath of the financial crisis starting in Q3 2008 (cf. Figure 2 in the appendix). Hence we consider the fact that the ADF would give evidence for an I(2) as an further indication for a structural break. Another problem might be the small sample size. Furthermore the ADF might not be able to distinguish between a unit root and a near unit root process. As the PP supports evidence for integration of order 1 of $ln(M_t)$ we consider all three variables as I(1).

4.2 Cointegration test and vector error correction model

We apply the Johansen procedure (cf. the appendix for details) in order to test if there is a cointegration relationship between the three variables $ln(P_t)$, $ln(M_t)$ and $ln(Y_t)$. The multivariate AIC of the matrix $X_t = (ln(P_t), ln(M_t), ln(Y_t))$ reports an optimal lag number of 3, while the multivariate BIC reports an optimal lag number of 2. We use 2 respectively 1 lag in order to compute the trace statistic with the Johansen procedure and include a constant in the equation. The trace statistic is reported in Table 2. The estimates for the loading coefficient vector α and the cointegrating vector β are reported in table 3.

	r=0	r=1
trace statistic (1 lag)	71.02	18.80
trace statistic (2 lags)	162.32	13.36
critical value (5% level)	35.07	20.16

Table 2: Trace statistics in the Johansen Procedure

	$rank(\Pi)$	$\hat{\alpha}$ $(ln(P_t),ln(M_t),ln(Y_t))$	$\hat{\beta}$ $(ln(P_t),ln(M_t),ln(Y_t))$
1 lag	1	(0.065,0.274,0.031)	(1,-0.526,1.071)
SD		(0.011,0.045,0.038)	(,0.035,0.155)
t-values		(5.731,6.074,0.821)	(,15.059,6.909)
2 lags	1	(0.079,0.107,-0.033)	(1,-0.650,1.593)
SD		(0.012,0.048,0.044)	(,0.056,0.249)
t-coefficient		(6.577,2.215,-0.756)	(,11.631,6.410)

Table 3: Vector error correction model

The Johansen procedure does reject the null for $rank(\Pi) = 0$ but not for $rank(\Pi) = 1$, so we find evidence for one cointegration relationship between prices, money and real output. The cointegration vector $\hat{\beta}$ does support our theoretical assumptions and shows the correct signs. The coefficients of the cointegration vector fit to our theoretical assumption of (1,-1,1) although the coefficient of $ln(M_t)$ is smaller than expected. The loading coefficients are significant for $ln(P_t)$ and $ln(M_t)$ while the loading coefficient for $ln(Y_t)$ is not. This result gives evidence that money and prices do adjust to the steady state while real output does not adjust.

5 Discussion

While the Quantity Theory of Money is widely accepted among neoclassical economists, the theory is by far not beyond criticism in particular from the Keynesian school. From the equation of exchange a simple condition for testing the theory can be derived, which claims a stable long run relationship between notional stock of money (M_t), output (Q_t) and the price level (P_t). Applying different unit root test we find that all series are I(1) and hence can proceed to testing the cointegrating relationship. Using the Johansen procedure a cointegrating vector is found, which indicates that a stable long run relationship indeed existed for the Euro Area between 1999q1 and 2011q2. The estimated values for β – while different from the theoretically expected values (1, -1, 1) – are within a reasonable range of the values derived from theory.
These findings however, are subject to some caveats: the length of the time series considered is rather short and there might be a structural break in the data in the aftermath of the financial crisis 2008, which could affect the unit root tests.

6 References

Bachmeier,L. and N. R. Swanson (2005): Predicting Inflation. Does the Quantity Theory Help?, *Economic Inquiry*, Vol. 43, Issue 3, pp. 570-585, 2005

Brown, F. and D. Cronin, (2007): Commodity Prices, Money and Inflation, *Working Paper series No. 738*, European Central Bank, Frankfurt am Main.

Greene, W.H. (2008): *Econometric Analysis*, $6^{th}edt.$, Pearson International, Upper Saddle River, NJ.

Lütkepohl, H. (2005): *New Introduction to Multiple Time Series Analysis*, Springer, Heidelberg.

7 Appendix

7.1 Unit root test: Augmented Dickey Fuller and Phillips Perron

The Augmented Dickey Fuller tests (ADF) and Phillips Perron test (PP) have unit root of a univariate time series as their null. The **ADF** considers an AR(p) process of the order p

$$X_t = \mu + \beta t + \phi_1 X_{t-1} + ... + \phi_p X_{t-p} + \varepsilon_t \tag{3}$$

Transforming the equation gives the following formulation:

$$\Delta X_t = \mu + \beta t + \phi X_t + \phi_1 \Delta X_{t-1} + ... + \phi_p \Delta X_{t-p} + \varepsilon_t \tag{4}$$

where $(\mu = 0, \beta = 0)$ (no intercept), $(\mu = const., \beta = 0)$ (constant) and $(\mu = const., \beta = const.)$ (trend) are possible parameter constellations for the ADF. The null of the test is unit root which is equivalent to $H_0 : \phi = 0$. We use a standard ADF that carries out a one-sided t-test for all three cases (no intercept, constant, trend) and compare it with the critical values reported by Dickey and Fuller (cf. Greene (2008,p.746)). In the case of a trend a F-Test might be used as well to test the joint hypothesis.

The **PP** uses above formulation of the ADF test with either no intercept or constant or trend. As pointed out in Greene (2008, p. 752) the procedure modifies the DF statistics and corrects for autocorrelated u_t. The critical values of the Dickey Fuller table remain valid. The maximal order for correcting autocorrelation or bandwidth parameter L has to be specified (e.g. as a function of observations).

7.2 Cointegration with Johansen procedure

The Johansen procedure is a multivariate generalization of the the ADF test. Consider the following representation of an vector autoregression:

$$\Delta X_t = \mu + \Pi X_{t-1} + \Psi_1 \Delta X_{t-1} + ... + \Psi_{p-1} \Delta X_{t-p+1} + \varepsilon_t \tag{5}$$

where X_t is a vector of n variables, μ and ε_t (white noise) are n-dimensional vectors and Ψ_{p-1} and Π are n $\times$ n- matrices.

If the coefficient matrix Π has reduce rank $(r < n)$ (in other words: Π is singular) there exists a cointegrating relationship, such that Π can be represented as $\Pi = \alpha\beta'$, with α containing the adjustment parameters (loading matrix) and β the cointegrating matrix. While β describes the equilibrium relationship α contains information on the adjustment behavior of the respective variables in case of deviation from the equilibrium.

The numbers cointegrating vectors which are linearly independent relates to the rank of Π, such that if:

- rank $\Pi = 0$, no cointegration (VAR model in differences)
- rank $\Pi = 1$, there exists one cointegrating vector
- rank $\Pi =$ n, self cointegration, (system is stationary)

In order to determine the rank of Π trace tests – based on the canonical correlations between X_{t-1} and ΔX_t – can be used. Testing upwards, that is: starting the sequence of testing with the null $r = 0$ against the alternative $r > 0$ and moving on to testing the null $r \leq 1$" against alternative $r > 1$ and so on, only if the tests reject (i.e. cointegration is found), is the most common approach in literature.
Similarly to the ADF, the Johansen procedure (which can be considered a generalization of the ADF to the multivariate case) is affected by deterministic terms like trend and intercepts. While theoretically many combinations exist, the relevant specification in the case of variables with trend is the one given in the equation above, which includes a constant term. Given that the rank of Π ($r = rank(\Pi)$) has been determined above equation can be rewritten, such that:

$$\Delta X_t = \mu + \alpha\beta' X_{t-1} + \Psi_1 \Delta X_{t-1} + ... + \Psi_{p-1} \Delta X_{t-p+1} + \varepsilon_t \tag{6}$$

While estimates for β are obtained from the testing procedure, estimates for μ, α and Ψ_1, ... , Ψ_{p-1} can be obtained by replacing β with $\hat{\beta}$.[5]

[5]cf. Lütkepohl (2005:244-258)

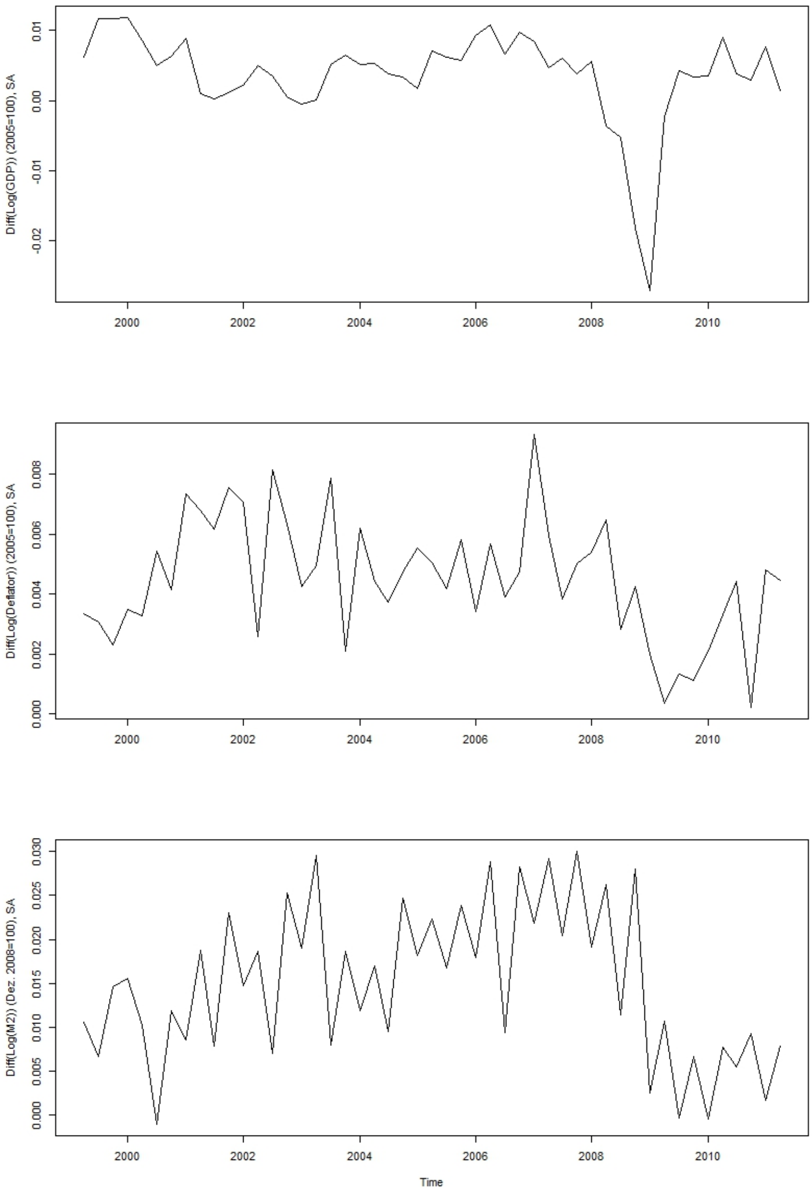

Figure 2: Quarterly logarithmic difference series $\Delta ln(Y_t)$, $\Delta ln(P_t)$ and $\Delta ln(M_t)$ from 1999:1 to 2011:2