Test in Applied Time Series Analysis—Suggested answers to problems

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1. An MA(1) process is given by the following equation:

$$X_t = 5\varepsilon_{t-1} + \varepsilon_t$$

   (a) Find the characteristic polynomial of this MA process and determine its zeros (roots); *Answer:* The characteristic polynomial is $\theta(z) = 1 + 5z$ with the root $\zeta = -0.2$.

   (b) Is the model invertible? If not, determine a process with exactly the same ACF that is invertible; *Answer:* The MA model is not invertible, as the modulus of the root is less than one. An equivalent invertible model is $X_t = 0.2\varepsilon_{t-1} + \varepsilon_t$, with its root $\zeta = -5$.

   (c) Determine the ACF for this process. *Answer:* The ACF for this process is identically 0 for $|k| > 1$, as it is an MA(1) process. $\rho(0) = 1$, trivially. $\rho(1) = \theta_1/(1 + \theta_1^2) = 5/26$ for both equivalent processes.

2. You want to fit an ARMA model to given data.

   (a) Suppose you want to restrict attention to pure models (AR and MA) at first. How would you proceed for identifying lag orders? *Answer:* There are several valid answers. A simple one would be to use the sample ACF for MA models and the sample PACF for AR models. The location of the highest significantly nonzero value could be selected as a tentative lag order. Other valid answers would be to iteratively increase the lag order on the basis of coefficient tests, or to use information criteria.

   (b) If you allow for mixed structures of the type ARMA(p, q), you will probably use information criteria. Suppose the minimum AIC is found for ARMA(1,1) but the AR polynomial has its root less than one. What is your conclusion? *Answer*: If the root is less than one (understood to be as modulus), the identified model is unstable and useless. Reasons may be incorrect specification of deterministic terms, structural breaks etc.

(c) Now suppose the estimated model is perfectly stable. Which additional checks or tests would you apply to the model? *Answer*: There are several valid answers here. You may wish to check the residuals for whiteness, either by visual aids (residual correlogram) or by the portmanteau statistic. Others may prefer testing for the possibility of structural breaks or for normality.

3. You wish to test a given variable for possible unit roots. The data has a clearly recognizable trend. You only have a regression program available, and a table of significance points for the Dickey-Fuller test.

   (a) How do you construct your unit-root test statistic? Also describe how you find the augmentation lag order. *Answer:* Basically, you always run a regression of the type $\Delta X_t = a + bt + cX_{t-1} + \psi_1 \Delta X_{t-1} + \ldots + \psi_k \Delta X_{t-k} + u_t$. The required statistic is the t–statistic on the lagged level coefficient c. With regard to the lag order k, there are several valid answers. For example, you may determine it by minimizing information criteria based on the model $X_t = a + bt + c_1 X_{t-1} + \ldots + c_h X_{t-h} + u_t$ and set k equal to $h-1$.

   (b) Do you use the tables of critical points for the DF-0, DF-μ, or DF-τ test, or would you need different tables? *Answer:* You would simply use the table for the DF-τ test.

   (c) Suppose the test rejects its null. What is your conclusion? *Answer:* It appears that the variable is trend-stationary.