Forecasting Austrian Industrial Production

with Seasonal ARIMA Models in R

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Description of Data 1/4

- We employ historical data for the industrial production in Austria for the last 15 years.
- The raw data is available from the IFS (International Financial Statistics) data base.
- Plotting the series leads to the conclusion, that the data clearly exhibits seasonality.

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Description of Data 2/4

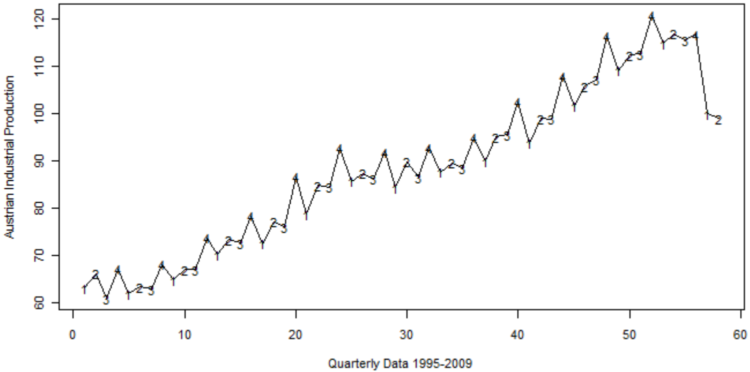

Figure: Austrian industrial production from 1995q1 till 2009q2 with quarterly symbols.

Description of Data 3/4

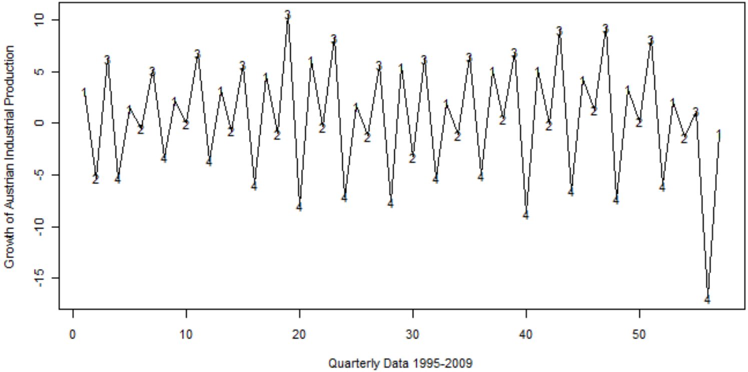

Figure: Growth of austrian industrial production from 1995q1 till 2009q1 with quarterly symbols.

Description of 4/4

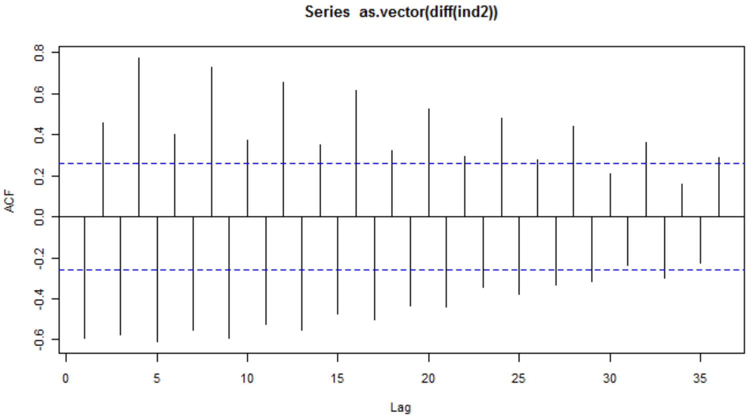

Figure: Sample ACF clearly shows seasonality.

Estimation of Seasonal Model 1/5

- An investigation of the time series plot leads us to the conclusion, that a stationary seasonal ARIMA model can describe the data well.
- The model consists of a seasonal MA(Q) model and a seasonal AR(P) model, with seasonal period s=4, because of quarterly data.
- The MA(Q) model is given by

$$Y_t = e_t - \Theta_1 e_{t-s} - \Theta_2 e_{t-2s} - \ldots - \Theta_Q e_{t-Qs}$$

with seasonal MA characteristic polynomial

$$\Theta(x) = 1 - \Theta_1 x^s - \Theta_2 x^{2s} - \ldots - \Theta_Q x^{Qs}$$

Estimation of Seasonal Model 2/5

- Seasonal MA(Q) models can also be viewed as a special case of nonseasonal MA models of order q=Qs but with all $\theta-$values zero except at the seasonal lags s, 2s, 3s, ... , Qs.
- Such a series is always stationary and the acf will be nonzero only at s, 2s, ... , Qs.
- The seasonal AR(P) model is given by

$$Y_t = \Phi_1 Y_{t-s} + \Phi_2 Y_{t-2s} + \ldots + \Phi_P Y_{t-Ps} + e_t$$

with seasonal characteristic polynomial

$$\Phi(x) = 1 - \Phi_1 x^s - \Phi_2 x^{2s} - \ldots - \Phi_P x^{Ps}$$

Estimation of Seasonal Model 3/5

- This model can also be seen as a special version of the AR(p) model with $p = Ps$ with nonzero $\phi-$coefficients only at the seasonal lags s, 2s, 3s, ... , Ps.
- Again, the ACF of the AR process is nonzero at lags s, 2s...
- A closer investigation of the ACF shows us, that not only the lags are nozero, but also the values close to the lags. To model this, we have to combine the ideas of seasonal and nonseasonal ARMA models. This can be achieved by multiplicative seasonal ARIMA models.

Estimation of Seasonal Model 4/5

- In a multiplicative seasonal ARIMA model the MA characteristic polynomial is given by $(1-\theta x)(1-\Theta x^4)$.
- Multiplying out gives us $1-\theta x-\Theta x^4+\theta\Theta x^5$. Therefore the series is given by

$$Y_t = e_t - \theta e_{t-1} - \Theta e_{t-4} + \theta\Theta e_{t-5}$$

Estimation of Seasonal Model 5/5

- Parameter estimates for the seasonal ARIMA (0,1,1) model are all highly significant

Coefficient	θ	Θ
Estimate	0.2217	-0.6904
Standard error	0.1097	0.1687
$\widehat{\sigma}_e^2 = 5.629$:	log-likel.$= -122.32$	AIC$= 248.64$

Forecasting with Seasonal Model 1/2

- Computing forecasts with seasonal ARIMA models is most easily carried out recursively by employing difference equations for the model.
- The prediction limits for seasonal models are obtained exactly as in the nonseasonal case. We compute the 95% confidence interval for our prediction, which gives us a good feeling for the precision of our prediction.

Forecasting with Seasonal Model 2/2

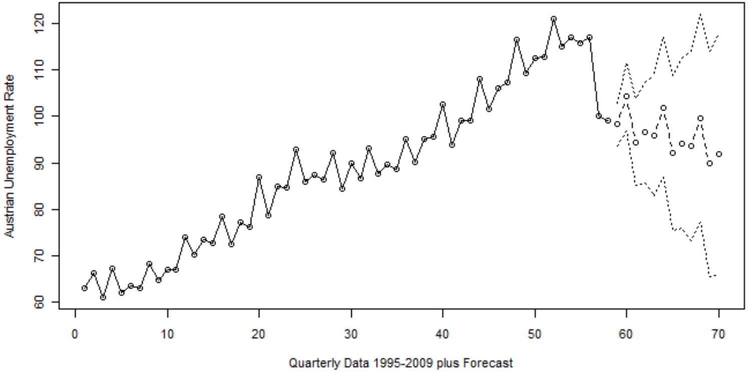

Figure: Forecast of industrial production (4 years).

Thank you for your attention!

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Appendix: R Code

```
> # R Code to accompany 'Forecasting Austrian Industrial Production'
> # Author: Markus Mayer; 05 March 2010
> #
> library(TSA)
> ind <- read.table("d:\\meine dateien\\RFolder\\ind19952009.txt", header=F)
> ind2 <- ind[[3]]
> #
> # Plotting the series
> win.graph(width=7,height=4,pointsize=8)
> ts.plot(ind2,xlab='Quarterly Data 1995-2009', ylab='Austrian Industrial Production')
> Quarters=c('1','2','3','4')
> points(window(ind2),pch=Quarters)
> #
> # Plotting the growth of the series
> ts.plot(diff(ind2),xlab='Quarterly Data 1995-2009', ylab='Growth of Austrian Industrial Production')
> Quarters=c('1','2','3','4')
> points(window(ind2),pch=Quarters)
> #
> # Plotting of sample ACF
> acf(as.vector(diff(ind2)),lag.max=36)
> #
> # Parameter estimates of the model
> m1.ind2=arima(ind2,order=c(0,1,1),seasonal=list(order=c(0,1,1),period=4))
> m1.ind2
> #
> # Forecasting with ARIMA model
> m1.ind2=arima(ind2,order=c(0,1,1),seasonal=list(order=c(0,1,1),period=4))
> win.graph(width=7,height=4,pointsize=8)
> plot(m1.ind2,xlab='Quarterly Data 1995-2009 plus Forecast', ylab='Austrian ...
...Industrial Production')
```

References

- Cowperwait , P. S. P., and A. V., Metcalfe (2009): *Introductory Time Series with R.* Springer, New York.
- Cryer , J. D., and K. S., Chan (2008, 2nd): *Time Series Analysis with Applications in R.* Springer, New York.[1]
- Ghysels , E., and D. R., Osborn (2001): *The Econometric Analysis of Seasonal Time Series.* Cambridge University Press, Cambridge, UK.

[1] Basis for our estimation and forecast.